Moufang Loops

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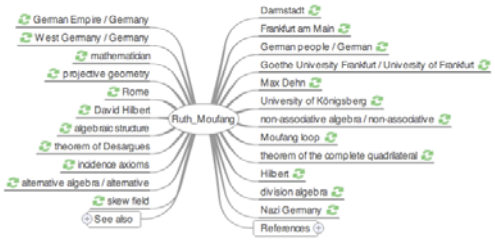
Ruth Moufang

- Born Germany, 1905
- Studied at University of Frankfurt
- Received PhD in 1931
- First woman professor in Germany (1957)

Ruth Moufang's Research

- Used geometry and algebra
- Worked in theoretical physics

Ruth Moufang's First Degrees of Separation



A Quick Review: Groups

Reminder: A **group** is a set defined with a binary operation

1. Associative: for a,b,c in G (ab)c= a(bc)
2. Identity: There is an element "e" such that: ae =a=ea
3. Inverse: for every element a in G, there exists an element a^{-1} in G such that: $aa^{-1} = e = a^{-1}a$

Quasigroups

Definition: A **quasigroup** is set Q that has a binary operation *: Q x Q → Q such that

ax = b

ya = b

Where a,b are both in Q and x,y are **unique** elements of Q.

Note: Quasigroups do not need to be associative

Quasigroups: Examples

Examples:
(Z,-)
(Q\{0},/)

Loops

Definition: A **loop** is a quasigroup with an identity element, e, such that:

$xe = x$
$ex = x$

The identity element is unique and every element of the loop has a left and right inverse.

Loops: Examples

Example:

- Every group is a loop (associative)
- Quaternion group (non-associative)

Moufang Loops!

A **Moufang loop** is a special case of a loop that satisfies the following properties:

1. $z(x(zy)) = ((zx)z)y$
2. $x(z(yz)) = ((xz)y)z$
3. $(zx)(yz) = (z)(xy)z$
4. $(zx)yz = z((xy)z)$

Note: Moufang loops need not be associative

Moufang Loops: Special Properties

- All groups are Moufang loops
- The Moufang loop properties hold in any alternative algebras
  $x(xy) = (xx)y$
  $y(xx) = (yx)x$
- Commutative loops only need the single property:
  $(x^2)yz = xyxz$

Moufang Loops: Two-sided inverse

- Two sided inverse:
  $x^{-1}(xy) = y = (yx)x^{-1}$
- Therefore,
  $(xy)^{-1} = y^{-1}x^{-1}$
  $x(yz) = e$ if and only if $(xy)z = e$

Moufang Loops: Examples

- All Groups
- Quasigroups that satisfies any one of the Moufang properties
- The Octonions (non-zero and under Octonion multiplication)

MOUFANG LOOPS

Thank you Alissa for a great two weeks ☺ !!