Physics for the Life Sciences: Fall 2008 Lecture #25

Real fluids:

As we have mentioned several times, real fluids are more complex than the ideal fluids described by the continuity equation and Bernoulli's equation. Fluid dynamics is one of the most complex subjects in physics, and is now addressed using the world's largest supercomputers. John von Neumann, one of the great early physicist/computer scientists, once commented that assuming no friction in fluids is like "working with dry water". Many of the important and interesting phenomena associated with fluids emerge from the "extra" properties of real fluids. As you might expect, these interesting properties of real fluids are regularly used by living things to accomplish their goals, so it is especially important that you should understand some aspects of them.

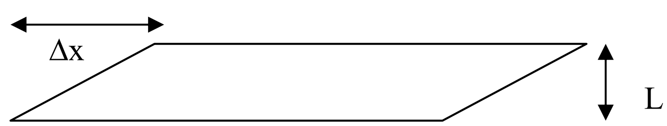
Today we will introduce real fluids, and especially get an idea of how we describe them. Along the way we'll see how even a basic consideration of friction in fluids explains a lot of new phenomena. This is a rich and very current field, important because most of the matter in the universe is in fluid form. Even the Earth itself, the canonical solid object, is mostly a fluid, covered by a thin solid crust. In addition, all life we know moves in fluids, air or water, so the behavior of these real fluids places strong constraints on how living things evolved.

Returning to the definition of fluids and shear stress

Remember how shear stress works in solids:

$$(F/A) = S(\Delta x/L)$$

Where F/A is the shear stress (the force per unit area creating the shear) and $\Delta x/L$ is the shear strain, how much the material distorts under the stress. See the picture below:



This is what happens when you apply a shear stress in a solid. What happens in a fluid? If I apply a shear stress, instead of just distorting and coming to a halt like the solid does, it begins to flow. There is no limit to the shear strain which can be produced in a fluid. *Any* shear stress can produce an *infinite* shear strain.

So, what's the appropriate thing to associate with the varying properties of fluids? If I apply a shear stress to some water, and apply the same stress to tar, they both will flow, and both will flow as far as I like if I wait long enough. It's clear that something is different about these two fluids. How should we quantify this difference?

Viscosity and time rate of change of fluid shear.

Fluids with low internal friction shear rapidly, those with high internal friction shear slowly. This can be expressed in the equation:

$$(F/A) = \eta(v/L)$$

here the F/A is again the shear stress. Now instead of a strain ($\Delta x/L$), we have a time rate of change of a strain (v/L), and instead of the shear modulus S we have the "viscosity" η.

Viscosity is a measure of the internal friction of fluids, how much they resist flow. The units of viscosity are:

$$\eta = N/m^2 / [(m/s)/m] = Nsm^{-2} = (N/m^2)s = \text{Pascal*second}$$

unfortunately this SI unit is not always used, and instead viscosity is usually tabulated in terms of "poise", a unit named for the French scientist Poiseuille, and defined as:

$$1 \text{ poise} = 0.1\ Nsm^{-2}$$

In fact, viscosities are often given in tables in units of "centipoise", or 100ths of a Poise. So you'll have to be careful if you take viscosity values from the literature. Here are some example values for viscosity measured in the SI unit Pascal*second.

Temp °C	Water	Air	Mercury	SAE 10 motor oil	SAE 30 motor oil	Glycerin	Honey
0	1.8×10^{-3}	1.7×10^{-5}	1.7×10^{-3}				
20	1.0×10^{-3}	1.8×10^{-5}	1.6×10^{-3}	7×10^{-2}	0.3	0.5	1.5
40	0.7×10^{-3}	1.9×10^{-5}	1.4×10^{-3}				
60	0.5×10^{-3}	2.0×10^{-5}					
80	0.4×10^{-3}	2.1×10^{-5}					
100	0.3×10^{-3}	2.2×10^{-5}	1.2×10^{-3}				

Note a few facts which are apparent from this table. Liquid viscosities decrease with temperature, while gas viscosities increase. This is because the increasing thermal energy available as temperature rises in liquids makes it easier for atoms to flow past one another. In a gas, like air, the increasing temperature implies more frequent collisions among the atoms, and hence greater viscosity. It's also worth noting that fluid viscosities, even in this short table, can vary a lot. Honey is about 100,000 times as viscous as air.

Consequences of viscosity

How do fluids with viscosity flow differently from ideal fluids, which would have a viscosity of zero? Here are a few of the key consequences.

1: Loss of pressure in flow: Energy is lost as a fluid flows. This effectively adds another term to Bernoulli's equation:

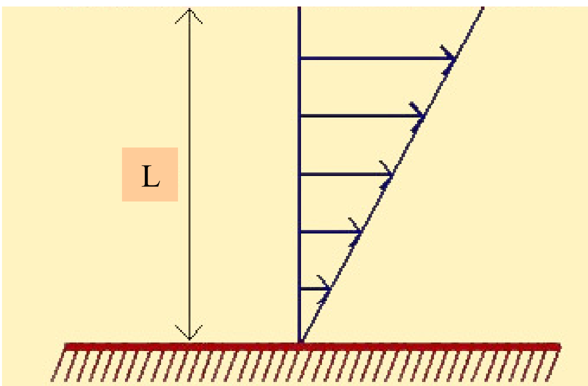
$$P_1 + \rho g h_1 + 1/2\rho v_1^2 = P_2 + \rho g h_2 + 1/2\rho v_2^2 + \text{Losses of energy due to viscosity}$$

So even in constant velocity flow in a horizontal pipe (where h and v remain constant), pressure declines along a flow. This should not be too surprising. In the presence of friction, it costs something to keep a fluid flowing.

2: Flow layering and laminar flow: The velocity of flow at the wall of a pipe is zero. It rises to the center of the flow, and then falls back to zero at the opposite wall. Imagine water flowing down through in a pipe or a river. The shear stress coming from gravity is the same everywhere, so you have:

$$(F/A) = \eta(v/L)$$

where L is the distance from the wall. This means $v = L(F/A\eta)$; the speed of the flow is proportional to the distance from the wall.



You can see this in many flows, such as when a river flows rapidly in the center and slowly at the edges. This layered, “laminar” flow is a widespread phenomenon for smooth flows, and plays an important role in many living systems.

3: Terminal velocity in falling bodies. The rate at which the fluid can move out of the way of the falling body is dependent on the viscosity of the fluid. Under a constant strain (the weight of the falling body) the fluid reaches a constant velocity, the value of which is governed by the viscosity η.

This is actually a bit more complicated. Even motion through a fluid without friction will exhibit terminal velocity. Just accelerating the fluid to get it out of the way requires that a falling object exert a force on the fluid, implying a reciprocal force on the falling object. For ‘large’ objects falling at ‘large’ velocities, like you or I falling through air, this inertial effect is the most important. For small objects falling at low velocities, the frictional effects associated with viscosity are more important.

You can see that in the two fluid friction laws we discussed so much earlier in the class:

$$F_f^{fluid} = 3\pi\eta D v \quad \text{(small/slow)}$$

$$F_f^{fluid} = 1/2 C\rho A v^2 \quad \text{(large/fast)}$$

We will (finally) see how to decide what constitutes ‘large’ objects and velocities below.

Limitations of this picture

This is a kind of first approximation, much better than assuming no friction. But it is limited. The assumption we have made that:

$$(F/A) = \eta(v/L)$$

assumes that η is independent of velocity. Fluids which obey this are called Newtonian fluids (because he first stated this relationship).

There are many non-Newtonian fluids, including most biological fluids, blood for example. As the velocity of blood increases, cells align themselves with the flow, lowering the friction. Such a material is called "shear-thinning", because it becomes less viscous once it starts to flow. This is common in suspensions and otherwise non-uniform liquids.

Many other common examples of non-Newtonian fluids exist, like egg white (albumen), a "visco-elastic" fluid, which is elastic for small shears and can spring back to its original shape, but will flow like any liquid for larger shears. Ketchup is another well known shear-thinning fluid. Once ketchup starts to flow, it does so very freely, blopping out all over your French fries. Then there is the famous corn starch and water concoction you may have played with in elementary school. This is a "shear-thickening" fluid. If you apply a small shear it will flow nicely, but if you apply a large shear it becomes very viscous indeed.

What we have said about simple works best with uniform, one component fluids. What this means for life is that if it wants to get around the limitations of simple fluids, it can often work up a kind of a cocktail mix of things which will behave the way it needs. A recently published example is the visco-elastic properties of the digestive juices of a carnivorous pitcher plant. This gooeyness greatly increases the chance that a fly landing in the fluid will be unable to escape[8]

Kinematic viscosity

This discussion of fluid flow is still really static; what **stable** state of flow does the fluid reach after we give it time to settle down. The dynamical response of a fluid to a stress depends on both the viscosity and the density of the fluid. This just reflects the difference in inertia between two fluids with the same viscosity. So it is often useful to talk about the kinematic viscosity of a fluid:

$$\text{Kinematic viscosity} = \eta/\rho$$

A whirlpool of air stops spinning much faster than a whirlpool of water, because although the viscosity of air is 50 times less than that of water, the density is 1000 times less. So

[8] Gaume L, Forterre Y 2007 A Viscoelastic Deadly Fluid in Carnivorous Pitcher Plants. PLoS ONE 2(11): e1185 doi:10.1371/journal.pone.0001185

although there is somewhat less viscous force available to stop the air, much less is needed, and it stops much more quickly. The difference in kinematic viscosity of these two expresses this different response to change much better than a simple comparison of their viscosities.

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Real fluid flow and turbulence

We have been discussing fluid flow which is smooth and laminar, neatly layered and without mixing. But that's not what we often see in fluid flow. Instead we see a swirling mixing of fluids, a much more complicated flow in which stability is not evident. What causes this turbulence, and what marks the transition between smooth flow and turbulence?

If the motion of a fluid is dominated by internal friction, by viscous drag, the flow will always be smooth. Any deviations from smoothness will be "damped out" by the friction before they have a chance to become large. This is what you usually see when you pour highly viscous liquids like honey or syrup. Just try mixing a jar of honey with a spoon and you'll see what I mean.

On the other hand, if the friction is small and something disturbs the flow even slightly, the elements of the fluid which are disturbed will be able to travel far from where they would be in smooth flow before the viscous effects stop them. This in turn allows them to affect other parts of the fluid before they are stopped, leading to a kind of cascade of confusion. This is what you usually see in the flow of air, perhaps most clearly when you look at the smoke from a recently extinguished candle. Swirling around, it doesn't take long for the smoke to be well mixed with the air around it.

Reynold's number as the thing which characterizes fluid flow

Somehow the important thing is the balance between the inertia of a bit of the fluid, and the size of the viscous forces acting on it. What follows is a generic discussion of how these two things might vary with the parameters of the material.

First consider the inertia of the object: what size force does it take to stop it?

$$Ma = (\rho L^3)a \approx (\rho L^3)\,(v/\Delta t) \approx (\rho L^3)v/(L/v) = \rho L^2 v^2$$

Where I have assumed $\Delta t = L/v$ is a reasonable approximation for how quickly I might make it stop. That is, I want it to stop before it would travel its own length.

Viscous forces: what size force is available to stop it?

$$F/A = \eta(v/L) \quad \text{implies} \qquad F = \eta(v/L)L^2 = \eta v L$$

The ratio of these two forces is a dimensionless number:

$$R = (\text{inertial properties})/(\text{viscous forces}) = (\rho L^2 v^2)/(\eta v L) = \rho L v/\eta$$

This is the famous Reynolds' number R, names for British fluid mechanic Osborne Reynolds. Note that it is dimensionless, a pure number independent of what units we measure it in:

$(kg/m^3)m(m/s) / (kgm/s^2)sm^{-2}$

What does this dimensionless nature mean? It suggests that R is an important number, because it allows us to compare problems on all kinds of scales, independent of the units we use, so that they can be uniformly understood.

Meaning of Reynolds' number

R is dimensionless, so it only has a value, not units. Remember that it's the ratio of inertial properties to viscous forces. If R is small, then the viscous forces are dominant and the flow is smooth and steady. If R is large, then viscous forces are negligible and the flow rapidly becomes complex and turbulent.

What is "large" and "small"? Can we guess from the way we formulated it?

$R < 1/1000$	Viscous forces completely dominate
$1/10 < R < 10$	Mixed behavior
$1000 < R < 3000$	Drag negligible, but still influences large scale flow patterns
$3000 < R$	Flow is "fully" turbulent

We brushed over a problem though, what is this length scale L we have used in determining R? L is the thing which sets the scale for the problem. Could be the size of an object moving through a fluid, diameter of a pipe through which it flows, etc. It's the scale on which we want to examine the fluid.

We can ask about different length scales within the same problem even. Imagine the flow of water in a pipe of radius r_{pipe}. If we wish to know about the flow in the pipe overall, we might use this length scale. But imagine that we want to know what the flow is like around little bumps on the wall. If these are smaller, with size $r_{bump} << r_{pipe}$, the flow around them may have a different nature. For example, the Reynolds' number for the pipe as a whole might be large (both r_{pipe} and v are large for the whole pipe) while R for the bumps may be small (both r_{bump} and v_{wall} are small). This is very typical of flows; turbulence on large scales accompanied by smooth laminar flow on small scales.

Implications of scale dependence

The scale dependence (the dependence on L) in the Reynold's number shows that what we will see depends on what we look at. Let's consider some large and small things.

Consider a falling dust particle: $\rho=1.3kg/m^3$, $v=0.001m/s$, $L=0.0001m$, $\eta=1.8x10^{-5}\ Nsm^{-2}$
So $R = 1.3(0.001)(0.0001)/1.8x10^{-5} = 0.007$. This means a falling dust grain is in the viscous regime and will always experience the small-slow kind of friction.

Redoing this for a falling raindrop:

$\rho=1.3kg/m^3$, $v=0.1m/s$, $L=0.005m$, $\eta=1.8x10^{-5}Nsm^{-2}$

we get

$R = 50$

Here the motion is dominated by the inertia of the drop, but affected still by the friction which is involved in moving the air out of the way.

Redoing this for a falling person

$\rho=1.3\text{kg/m}^3$, $v=30\text{m/s}$, $L=2\text{m}$, $\eta=1.8\text{x}10^{-5}\text{Nsm}^{-2}$

we get

$R = 4.3\text{x}10^6$

Here the motion is completely dominated by the inertia of the fluid. The fact that there is viscosity, that there is fluid friction, is really not important.

R describes the quality of the flow around an object. So if you want to model a flow, by placing a model in a wind tunnel for example, you have to do it with the same R as you would have in the real system. This means that if you use a 1/10 scale model (L becomes L/10) you have to somehow compensate for this in your test, perhaps by going 10x faster. This presents real problems for the testing of aircraft, as going 10x faster would usually require traveling faster than sound. Other problems occur when you try to blast air over a model at speed greater than the speed of sound.

Life at large and small Reynolds' number

People live in a world where R is large; v is big, η is small, L is large, etc. So we usually are not dominated by viscosity. As a result, we also live in a world where turbulent mixing is easy to achieve. If we want to mix cream into our coffee uniformly we just do it. With no special effort we can make the uniformly smooth liquid we so enjoy.

Bacteria, on the other hand, live in a world totally dominated by viscosity. Their speed v is small, and size L is small, so they're always in the viscous dominated, low Reynolds' number regime. Every time such a creature stops pushing for forward motion, it immediately stops moving. This is a world in which $F \propto v$, rather than $F \propto a$. It's an Aristotelian world in which you have to have a forward force constantly to have motion. As soon as it disappears you stop.

In addition, the fluid in the world of low Reynolds' number does not mix. Since it flows only laminarly, it never gets the kind of turbulent mixing that allows cream to mix into your coffee. This is very important if you're a bacteria hoping that a little food will come your way; basically it won't.

Reynold's number and terminal velocity

Earlier in the term we learned that for a large object moving through a fluid the frictional resistance is given by:

$$\text{Drag} = 1/2C\rho Av^2$$

Where C is a property of the shape of the object, and A is its area. This is approximately true for most large objects, moving in low viscosity air. This is basically saying that the Reynolds' number for the motion must be large.

What happens in the small Reynold's number case? In this case drag is given by Stokes law:

$$\text{Drag} = 3\pi\eta D v$$

This is quite different. It depends on the diameter of the object D, not on its area A, and on v, not on v^2. This is because this drag is dominated by fluid friction, rather than by fluid inertia. Note too that viscosity shows up here, because it matters.