Reynolds number.

The purpose of the Reynolds number is to get some sense of the relationship in fluid flow between inertial forces (that is those that keep going by Newton's first law – an object in motion remains in motion) and viscous forces, that is those that cause the fluid to come to a stop because of the viscosity of the fluid. We know from experience that a fast moving stream of water will slow down quickly, but a slow moving one (like the Potomac) keeps right on going. We also know that honey, no matter how slowly flowing will quickly come to a complete stop. How do we convert these notions derived from experience to something that we can calculate and then use a figure of reference.

Well, just in time, we have begun to think about the ideas of kinetic energy and work. The idea that kinetic energy represents the energy content due to inertial motion is not a hard one to grasp. Further, that if something is in motion, it will take work done on it to slow it down, comes directly from experience and is codified the work-energy theorem. How does help us? What if we were to compare the energy content of an element of fluid in a flow with the work required by the viscous force to bring it to a stop? That might be what we are looking for, provided we specified the length over which we are interested. For example, the length of interest for the flow of the Potomac might be 2 meters, if we considered how it is flowing past a canoe, which we are paddling in the river, but not the length of the river from Harper's Ferry to where it empties into the Chesapeake Bay, about 300 miles. Also, since we are looking for a figure of merit that depends upon material quantities, then we will drop constants like ½, and 6π from the answer. These will change the number but not change the comparison say between honey and water.

So lets get started by imagining a cube of liquid flowing by. Its volume is ΔV and it flows with velocity v. Also, it flows past an object (say our boat or through a short tube) of length, L. Or more specificity, let's assume that the object is a sphere so that we can use the Stoke's Drag equation. As we discussed in class, a longer ellipsoid will have a contribution to the drag force due to its length, but be assured that the result derived here for any shape will differ by a constant, geometrical factor from the spherical definition, and since by our original rules, we are ignoring those constant factors, it is ok.

So for our sphere, the kinetic energy of the fluid that moves by (is displaced by) the sphere is given by $K = ½\, mv^2 = ½\, \rho\Delta V v^2. = ½\, \rho 4\pi r^3 v^2/3 \sim \rho 4\pi r^3 v^2$ The viscous force acting against the object is $6\pi\eta r v$, and so the work done moving by the object is $F*d = F*r = \eta r^2 v$, again ignoring the constants by our original rules. If we take the ratio of inertial (kinetic) energy to the work done by the viscous force, we get the Reynolds number:

$$Re = \rho r^3 v^2 / \eta r^2 v = \rho r v / \eta$$

Here, ρ is density of the fluid, η is the fluid's viscosity, and v is the velocity of the object relative to the liquid (also low speed for a stationary object), and r is the length of the object. By choosing a sphere as my object, I have obscured the distinction between the length along vs. against the flow, but this is ok for the purposes of the discussion and in most definitions, r is replaced by L, the object's length along the flow:

$$Re = \rho L v / \eta$$

Compare the Reynolds numbers for a 1-mm diameter steel ball dropped in vegetable shortening (Crisco) at a speed of 1 mm/s to a 3 micron plastic sphere moving in water at 1 micron/s.
For the steel ball:

$$\mathrm{Re} = \frac{800\frac{kg}{m^3}\cdot 10^{-3}m\cdot 10^{-3}\frac{m}{s}}{10^3 Pa\circ s} = 0.8x10^{-6}$$

$$for \quad the \quad plastic \quad sphere:$$

$$\mathrm{Re} = \frac{1000\frac{kg}{m^3}\cdot 1.5x10^{-6}m\cdot 10^{-6}\frac{m}{s}}{0.001 Pa\circ s} = 1.5x10^{-6}$$