

## Standard deviations: example

Going back to the game of dice, what is the standard deviation for the average value calculated previously:
There are 6 possible values, and each has equal probability, 1/6. We modify the earlier table:

|  | $S$ (score) | $P_{s}$ |
| :---: | :---: | :---: |
|  | 1 | $1 / 6$ |
| $T_{s} P_{s}$ |  |  |
|  | 2 | $1 / 6$ |
| We | 3 | $1 / 6$ |
| get | 4 | $1 / 6$ |

## Calculating expectation values: $2^{\text {nd }}$ example

Let's give values to certain colors of balls: $B=4, R=2, G=1, V=0$.
What is the probability to get a total score of 4 , if we have 4 boxes of balls?
Think about the probability to get 4, we can only have
4G, 1B3V, 2R2V, or 1R2G1V.
$\mathrm{W}_{4}=\mathrm{W}_{4 \mathrm{G}}+\mathrm{W}_{1 \mathrm{~B}}+\mathrm{W}_{2 \mathrm{R}}+\mathrm{W}_{1 \mathrm{R} 2 \mathrm{G}}$
$=4!/ 4!+4!/ 1!3!+4!/ 2!2!+4!/ 2!=1+4+6+12=23$
$\mathrm{P}_{4}=\mathrm{W}_{4} / 4^{4}=0.090=9 \%$
Now, what are 17 possible scores, $0,1,2,3, \ldots, 14,15,16$
We have to calculate P for each, multiply P by the score and add them all
up. $4 \mathrm{P}_{4}=4(.09)=.36,16 \mathrm{P}_{16}=16\left(1 / 4^{4}\right), 0 \mathrm{P}_{0}=0,1 \mathrm{P}_{1}=1\left(4 / 4^{4}\right)=0.016$
See next slide

Calculating expectation values: example continued
It is easiest to make a table at this point:

| Score $(s)$ | $P_{s}$ | $s P_{s}$ | Score $(s)$ | $P_{s}$ | $s P_{s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $4 / 4^{4}$ | $1 / 64$ | 9 |  |  |
| 2 |  |  | 10 |  |  |
| 3 |  |  | 11 |  |  |
| 4 | $23 / 4^{4}$ | $23 / 64$ | 12 | $10 / 4^{4}$ | $30 / 64$ |
| 5 | $24 / 4^{4}$ | $30 / 64$ | 13 |  |  |
| 6 |  |  | 14 | $4 / 4^{4}$ | $14 / 64$ |
| 7 |  |  | 15 | 0 | 0 |
| 8 |  |  | 16 | $1 / 4^{4}$ | $1 / 16$ |

$\mathrm{W}_{12}=\mathrm{W}_{3 \mathrm{~B} 1 \mathrm{~V}}+\mathrm{W}_{2 \mathrm{~B} 2 \mathrm{G}}=4+6=10$
$W_{5}=W_{1 \mathrm{~B} 1 \mathrm{G} 2 \mathrm{~V}}+\mathrm{W}_{2 \mathrm{R} 1 \mathrm{G} 1 \mathrm{~V}}=12+12=24$
Add up all 16 values of $s P_{s}$ (the $17^{\text {th }}$ value $0 P_{0}=0$ ) to get $<s>$

## Ponderable: Flipping coins III

1. Think about the distribution of coin flips and their probability. For H assign a value 1 , for T , a value of -1 . Now considering what you remember about the properties of the distribution (do not perform any calculations or coin flips), predict your score after:
2. 1 flip
3. 2 flips
4. 4 flips
5. 10 flips
6. 100 flips
7. Now calculate the standard deviation for the above cases. You may have to calculate this for 1,2 , and 4 , but seeing the pattern that is emerging, can you make a reasoned guess for 10 and 100 ?

## Ponderable: Random walks

## Brownian Motion and random walks

This writeup describes the unit on the random walk problem and experiments to measure diffusion and Brownian motion

- Background (from readings on Brownian motion and statistics). What is some history of measurements and theory, what is importance of diffusion as a physical process, some background on the randomwalk model. What is colloidal motion?
Experiment description (materials, protocol)
- Modelling:

1. Run two programs: RandomWalkDataOut -- for multiple objects, diffusion -- and SimpleWalkerDataOut -- for single objects, Brownian motion
2. You can compile (javac) and run (java) these programs using the instructions given in class or on Prof. Simha's website. The computers in the physics help room are setup to run java. You can go there, download the zip file, expand it to the desktop or to the Student folder. Then open the terminal window, change directories to NewRW, the compile and run. Two helpful commands: cd .. changes directory, ls .. gives a list of files.
3. RandomWalkDataOut
4. RandomWalkDataOut is a modification of the program we ran in class that will give you the positions (for less than 30 particles, and the values from the histogram or 20 or more particles.
5. You should run it for larger and larger numbers of particles and time steps
6. Start with $\mathrm{d}=1, \mathrm{t}=1$ (one coin flip) $\mathrm{n}=1,10,30,100$. For $\mathrm{n}=100$, explain the histogram in terms of what you expect from making a single coin flip. What is the size of the step each particle makes?
7. Now try $\mathrm{d}=1, \mathrm{t}=2$ (two coin flips) $\mathrm{n}=30,100,10000$. Discuss. Explain the difference between $n=30$ and $n=100$, and $n=10,000$ ?
8. Now try $\mathrm{d}=1, \mathrm{t}=30$ (two coin flips) $\mathrm{n}=10000$. Discuss. What do you expect the shape to be as t becomes larger?
9. Now try even larger values of $t$ (if you haven't already). Discuss.
10. Class discussion: From the stats point of view, how does the histogram take on the shape of the theoretical probability distibution as the number of particles increases? How does the histogram change as the number of time steps is increased from 1 to 2 to 3 to many Discussed this in terms of the theoretical distribution expected (ie. from one coin flip averaged over N particles, 100 coin flips over N particles)?
