The figure shows volume flow rates (in cm³/s) for all but one tube. What is the volume flow rate through the unmarked tube? Is the flow direction in or out?

A. 1 cm³/s, in
B. 1 cm³/s, out
C. 10 cm³/s, in
D. 10 cm³/s, out
E. It depends on the relative size of the tubes.
What goes in must come out, so if 13 cm$^3$/s of fluid are shown going in and cm$^2$/s of fluid coming out. That leaves 1 cm$^3$/s coming out.

**ConcepTest 15.b.1** What goes in must come out

A. 1 cm$^3$/s, in
B. 1 cm$^3$/s, out
C. 10 cm$^3$/s, in
D. 10 cm$^3$/s, out
E. It depends on the relative size of the tubes.

**Fluids in Motion**

**conservation in flow**

**New Topic**
Two main types of fluid flow:

1) streamline (laminar)

2) turbulent

Volume per second here

Why are volumes equal?
- water is incompressible
- no leaks or ruptures

Volume $V = A \times L = A \times vt$

$V / t = A \times v$ is a constant!!

volume flow rate

Continuity Equation

$A_1 v_1 = A_2 v_2$
Water spigot

- Water velocity increases as it drops further from the faucet

⇒ Stream of water must narrow

ConcepTest 15.b.2 Human circulatory system

A single capillary is much smaller than the aorta, but the total cross-sectional area of all the capillaries in the body is several hundred times that of the aorta. Where is the blood flow the slowest?

(1) aorta
(2) capillaries
(3) veins
(4) All are equal
**ConcepTest 15.b.2 Human circulatory system**

A single capillary is much smaller than the aorta, but the total cross-sectional area of all the capillaries in the body is several hundred times that of the aorta. Where is the blood flow the slowest?

What goes in must come out, so the flow gets divided by each branching, which makes it hard to calculate. However, the flow rate depends upon the total area, which means that it is slowest in the capillaries.

---

**Example: Blood flow rate**

The radius of the aorta is 1 cm and that of the capillaries is 7 µm. If the average speed of blood in the aorta is 40 cm/s and in the capillaries is about 0.6 mm/s, what is the number of capillaries in the human body?
Example: Blood flow rate

The radius of the aorta is 1 cm and that of the capillaries is 7 µm. If the average speed of blood in the aorta is 40 cm/s and in the capillaries is about 0.6 mm/s, what is the number of capillaries in the human body?

\[
\text{Blood flow: } \quad \frac{v_A A_a}{v_C A_c} = \frac{v_A A_a}{v_C A_c} = \frac{v_A}{v_C} \frac{A_a}{A_c} = 2.04
\]

\[
\frac{v_A}{v_C} = \frac{A_a}{A_c} = \frac{0.44 \text{ cm/s}}{0.6 \text{ mm/s}} = \frac{4.4 \times 10^{-4} \text{ cm/s}}{0.6 \times 10^{-4} \text{ cm/s}}
\]

\[
n = \frac{v_A A_a}{v_C A_c} = 1.4 \times 10^4 \text{ capillaries}
\]

Ponderable: Fluid flow

Water flows from pipe A, with diameter D, into pipes B and D, each with diameter D/2. From D the water flows into pipes C, E, and F, each of which has a diameter D/6. Rank the speed of the water from least to greatest at points A, B, C, D, E. (Assume there is no viscosity and that all five locations are at the same height.)

For a real fluid, would you expect the flow rate to be the same in B as in D? Why?
Water flows from pipe A, with diameter D, into pipes B and D, each with diameter D/2. From D the water flows into pipes C, E, and F, each of which has a diameter D/6. Rank the speed of the water from least to greatest at points A, B, C, D, E. (Assume all five locations are at the same height.)

To see the largest narrowing effect, should the faucet be running fast or slow?

Calculate the speed at which the water leaves the faucet.

Try this at home.
Fluid speeds up in narrow part of pipe:

\[ V \]

\[ \text{high } P \rightarrow \text{low } P \rightarrow \text{high } P \]

\[ \text{acceleration} \rightarrow \text{deceleration} \]

\[ \text{force} \rightarrow \text{force} \]

**Bernoulli’s Principle**

Apply *energy conservation* to a fluid moving through a pipe:

\[ P \Delta V + \frac{1}{2} \rho \Delta V v^2 + \rho \Delta V g y = \Rightarrow F \Delta x + \frac{1}{2} m v^2 + m g y \]

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = \text{const.} \]

If \( y_2 = y_1 \) we find that:

\[ P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) < P_1 \]
Another way to look at this Summary of Fluid Motion

- **Continuity equation:** $A v = \text{constant}$
  - what goes in must come out – matter conservation
  - wide (BIG area) $\rightarrow$ low speed
  - narrow (small area) $\rightarrow$ high speed

- **Bernoulli’s equation:** $P + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}$
  - $P \Delta V + \frac{1}{2} \rho \Delta V v^2 + \rho \Delta V gy \Rightarrow F \Delta x + \frac{1}{2} mv^2 + mgy$

$W + KE + PE = \text{constant} – \text{energy conservation}$
  - high speed (fast) $\rightarrow$ low pressure
  - low speed (slow) $\rightarrow$ high pressure

**ConceptTest 15.b.3 Rank the gas speeds**

From high to low:
1. a,b,c
2. c,b,a
3. All are equal
4. b,a,c
5. a,c,b
The lower the velocity of fluid, the higher the pressure at a given point in the pipe. The velocities from highest to smallest are b,a,c. This can be seen from Bernoulli's principle. The kinetic energy of a fluid element balances with the work done by the pressure force.

**ConcepTest 15.b.3** Rank the gas speeds

From high to low:
1. a,b,c
2. c,b,a
3. All are equal
4. b,a,c
5. a,c,b

**ConcepTest 15.b.4** Rank the liquid heights

From high to low:
1. d,b,c
2. c,d,b
3. All are equal
4. b,d,c
5. d,c,b
The lower the velocity of fluid, the higher the pressure at a given point in the pipe. The velocity is highest at b, from the continuity equation. Thus the pressure is the lowest there. This combines the idea of continuity with Bernoulli’s principle: Conservation of mass with conservation of energy.

**ConcepTest 15.b.4**  
Rank the liquid heights

From high to low:
1. d,b,c
2. c,d,b
3. All are equal
4. b,d,c
5. d,c,b

Bernoulli’s Principle  
Applications: Airplanes

An airplane wing (airfoil) is shaped so that the air above the wing travels faster than the air below:

\[ P_{\text{top}} < P_{\text{bottom}} \]

So the air over the top of the wing is at a lower pressure. This upward force is called “lift.”
As people get older plaque can build up in the major arteries. In the region of the buildup, how do the blood flow rate and pressure change?

Flow rate, pressure
(1) Increases, increases
(2) increases, decreases
(3) decreases, increases
(4) decreases, decreases

The continuity principle tells us that the velocity must increase, Bernoulli tells us that when the velocity increases, the pressure must decrease, in the region of the constriction.

Oh really?? I thought the pressure went up as people age !!!
Ponderable: Bernoulli

Liquid flows through this pipe and the picture below is an overhead view – all points are at the same height.

1. Rank in order, from largest to smallest, the flow speeds \( v_1 \) to \( v_4 \) at points 1 to 4.
2. Rank in order, from largest to smallest, the pressures \( p_1 \) to \( p_4 \) at points 1 to 4.

\[ A_1 < A_2 = A_3 < A_4 \quad ; \quad V_1 > V_2 = V_3 > V_4 \quad \text{continuity} \]
\[ p_2 > p_1 > p_3 \quad ; \quad v_3 > v_1 > v_2 \quad \text{Bernoulli} \]
\[ A_2 > A_1 > A_3 \quad \text{continuity} \]
Liquid flows through a pipe. You can’t see into the pipe to know how the inner diameter changes. A) Rank in order, from largest to smallest, the fluid speeds \(v_1\) to \(v_3\) at points 1, 2, and 3. B) Rank in order, from largest to smallest, the diameters \(d_1\) to \(d_3\) at points 1, 2, and 3.

---

In an aortic aneurism, a bulge forms where the walls of the aorta are weakened. If blood flowing through the aorta (radius 1.2 cm) enters an aneurism with a radius of 3.1 cm, what is the average increase in the blood pressure inside the aneurism relative to that in the healthy part of the aorta? Assume that the average flow rate through the aorta is 120 cm\(^3\)/s, that the blood is not viscous, and that the patient is lying down (no change in height). The density of blood is 1060 kg/m\(^3\).

The average blood pressure is about 100 mm Hg = 1.32 \(\times\) 10\(^4\) Pa. Why is the pressure change so small?

......Because most of the work done (\(P\Delta V = F\Delta x\)) is used to overcome viscosity, not increase the kinetic energy of the blood.
The diameter of an artery is decreased by 20% due to arteriosclerosis. A) If the same amount of blood flow passes through the artery in any time period as before it was obstructed, will the pressure in the obstructed area increase or decrease? B) By what percentage? C) Will there be a pressure drop across the partially obstructed section? C) Is this realistic?