Abstract

A classic result in noncommutative ring theory states that a ring $R$ is an $n \times n$ matrix ring if, and only if, $R$ contains $n^2$ matrix units $\{e_{ij}\}_{1 \leq i,j \leq n}$, in which case $R \cong M_n(S)$ where $S$ is a subring of $R$ that can be described completely in terms of the matrix units. A lesser known result states that a ring $R$ is an $(m+n) \times (m+n)$ matrix ring, so $R \cong M_{m+n}(S)$ for some ring $S$, if, and only if, $R$ contains three elements $a, b, f$ satisfying the two relations $af^m + f^n b = 1$ and $f^{m+n} = 0$. In this talk, we investigate algebras over a commutative ring (or field) with elements $c$ and $f$ satisfying the two relations $c^i f^m + f^n c^j = 1$ and $f^{m+n} = 0$. Surprisingly little is known here about the structure of these algebras and about the underlying ring $S$ for most cases of the integers $i, j, m, n$. Questions whether $S$ is non-trivial or not turn out to be surprisingly difficult to answer, let alone describing the structure of these algebras or of $S$ in general.