Multidimensional Poverty Dynamics

Mauricio Apablaza - OPHI, University of Oxford
Multidimensional Poverty Dynamics

The literature on poverty dynamics is vast (poverty transitions, chronic versus transient poverty, expected vulnerability; and, poverty traps). Yet all this literature treats poverty as unidimensional, implicitly or explicitly, and focuses on monetary metrics of wellbeing.

- Decomposition of changes and Transitions (2011). Purpose: To improve the understanding of changes in multidimensional poverty and its components, by linking micro changes (by dimension, subgroup and in terms of probability) with the aggregated results.

- Chronic multidimensional poverty (2012). Purpose: We seek to build a bridge between the Alkire and Foster methodology and the strand of the poverty dynamics literature, specially chronicity (Foster, 2009).
Decomposition of changes and Transitions (2011)
Normalized adjusted headcount ratio

\[ M_0(t - a) \]

\[ \Delta \%_a M_0(t) \]

\[ M_0(t) \]

Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA
Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA

Normalized adjusted headcount ratio

\[ M_0(t-a) \]
\[ \Delta\%_a M_0(t) \]
\[ M_0(t) \]

\[ \Delta\%_a H(t) \]
\[ \Delta\%_a A(t) \]

\[ \Delta\%_a H(t) \times \Delta\%_a A(t) \]
Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA

Normalized adjusted headcount ratio

\[ M_0(t - a) \]
\[ \Delta \% _a M_0(t) \]
\[ \Delta \% _a H(t) \times \Delta \% _a A(t) \]

Decomposition by Subgroup
Decomposition by Dimension
Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA


-12.8%  -8.4%  -5.4%  -5.8%  -9.9%  -7.1%  -12.3%  0.3%

Δ%M

OPHI
Oxford Poverty & Human Development Initiative
Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA

<table>
<thead>
<tr>
<th>Period</th>
<th>Δ%Hx</th>
<th>Δ%A</th>
<th>Δ%H</th>
<th>Δ%M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-1992</td>
<td></td>
<td></td>
<td>-10.5%</td>
<td></td>
</tr>
<tr>
<td>1992-1994</td>
<td>-1.0%</td>
<td>-7.6%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1994-1996</td>
<td>8.4%</td>
<td>-5.6%</td>
<td>-5.4%</td>
<td></td>
</tr>
<tr>
<td>1996-1998</td>
<td>-1.2%</td>
<td>-5.2%</td>
<td>-0.8%</td>
<td></td>
</tr>
<tr>
<td>1998-2000</td>
<td>-1.3%</td>
<td>-8.9%</td>
<td>2.8%</td>
<td></td>
</tr>
<tr>
<td>2000-2003</td>
<td></td>
<td>-6.0%</td>
<td>-2.5%</td>
<td></td>
</tr>
<tr>
<td>2003-2006</td>
<td></td>
<td></td>
<td>-1.3%</td>
<td>-7.1%</td>
</tr>
<tr>
<td>2006-2009</td>
<td></td>
<td></td>
<td></td>
<td>-11.2%</td>
</tr>
</tbody>
</table>
Decomposition and Transitions

REPEATED CROSS SECTIONAL AND PANEL DATA


-10.5% -7.6% -5.6% -5.2% -8.9% -6.0% -11.2% 1.2%

Adjusted Urban Contribution
Adjusted Rural Contribution
Δ%H
Δ%A
Δ%M

Δ%Hx Δ%A

Oxford Poverty & Human Development Initiative
Decomposition and Transitions

PANEL DATA

ALKIRE–FOSTER STATISTICS

\[ \Delta^0 \% M^0 \]
\[ \Delta^0 \% A \]
\[ \Delta^0 \% A_d \]
\[ \Delta^0 \% H \]
\[ \Delta^0 \% C H_d \]

PROBABILITIES OF TRANSITION

\[
\Pr[c^n_t < k \mid c^{t-a}_n \geq k]
\]
\[
\Pr[c^n_t \geq k \mid c^{t-a}_n < k]
\]
\[
\Pr[x^{t}_{nd} \leq z_d \land c^n_t \geq k \mid x^{t-a}_{nd} > z_d \lor c^{t-a}_n < k]
\]
\[
\Pr[x^{t}_{nd} > z_d \lor c^n_t < k \mid x^{t-a}_{nd} \leq z_d \land c^{t-a}_n \geq k]
\]

PROBABILITY OF EXIT FROM MULTIDIMENSIONAL POVERTY

PROBABILITY OF ENTRY INTO MULTIDIMENSIONAL POVERTY

PROBABILITY TO BECOME POOR AND DEPRIVED IN \( d \)

PROBABILITY TO BECOME NON POOR or NON DEPRIVED IN \( d \)
Decomposition and Transitions

PANEL DATA – BASED ON H

Alkire and Foster (2011) also describes $M_0$ as a linear combination of several multidimensional headcounts with different poverty cut-off between $k$ and $D$. 

$\Delta% M^0$

$\Delta% H(t,k) ,..., \Delta% H(t,j)$

$P(c_i^t < k | c_i^{t-a} \geq k)$  $P(c_i^t \geq k | c_i^{t-a} < k)$
Decomposition and Transitions

PANEL DATA – BASED ON H
Decomposition and Transitions

Changes in $M^0$ can also be described as a combination of changes in each censored headcount. This changes could be liked to the combine probabilities of become (non) poor and (or non) deprived.

\[
P\left(x_{id}^t \leq z_d \land c_i^t \geq k \mid x_{id}^{t-a} > z_d \lor c_i^{t-a} < k\right)
\]

\[
P\left(x_{id}^t > z_d \lor c_i^t < k \mid x_{id}^{t-a} \leq z_d \land c_i^{t-a} \geq k\right)
\]
Chronic Multidimensional Poverty (2012)
Multidimensional Poverty

**Deprivation Matrix**

<table>
<thead>
<tr>
<th>Edu.</th>
<th>Inc.</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Counting Vector**

<table>
<thead>
<tr>
<th>Inc.</th>
<th>Identification 1</th>
<th>Identification 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>( k_d = 3 )</td>
</tr>
</tbody>
</table>

**Poverty identification**

<table>
<thead>
<tr>
<th>Inc.</th>
<th>Identification 1</th>
<th>Identification 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( c_i \geq k_d )</td>
<td>( k_d = 3 )</td>
</tr>
</tbody>
</table>

**Individual Average Poverty Deprivation** (only among the poor)

\[ H(k_d) = \frac{1}{3} \]

\[ M_0(k_d) = \frac{1}{3} \times \frac{3}{3} = \frac{1}{3} \]

\[
H(k_d, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ \sum_{d=1}^{D} w_d \cdot I(x_{itd} < z_d) \right] \geq k_d
\]

**Multidimensional Poverty**

\[ OPHI \]
Multidimensional Poverty

But, with several periods, there are two alternative strategies of identification.

\[
H(k_d, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ I \left( \sum_{d=1}^{D} w_d \cdot I(x_{itd} < z_d) \geq k_d \right) \right]
\]

Oxford Poverty & Human Development Initiative
Identify by multidimensional poverty in every period… then by chronicity

<table>
<thead>
<tr>
<th>Present (t=1)</th>
<th>Past (t=2)</th>
<th>Past (t=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Educ.</strong></td>
<td><strong>Inc.</strong></td>
<td><strong>Health</strong></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Poverty Condition (t=1)**

**Poverty Condition (t=2)**

**Poverty Condition (t=3)**

**Chronic Multidimensional Poverty Condition**
Identify by multidimensional poverty in every period… then by chronicity

<table>
<thead>
<tr>
<th>Present (t=1)</th>
<th>Past (t=2)</th>
<th>Past (t=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Educ.</strong></td>
<td><strong>Inc.</strong></td>
<td><strong>Health</strong></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Chronic Multidimensional Poverty Condition**

\[
H_{cmp}(k_d, k_t, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ I \left( \sum_{t=1}^{T} w_t * I \left( \sum_{d=1}^{D} w_d * I(x_{itd} < z_d) \right) \geq k_d \right) \geq k_t \right]
\]
Identify by chronicity (deprivation) … then by Multidimensional poverty

<table>
<thead>
<tr>
<th>Aggregated</th>
<th>Chronic Deprivation Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Educ.</td>
<td>Inc.</td>
</tr>
<tr>
<td>$c_{1E}$</td>
<td>$c_{1I}$</td>
</tr>
<tr>
<td>$c_{2E}$</td>
<td>$c_{2I}$</td>
</tr>
<tr>
<td>$c_{3E}$</td>
<td>$c_{3I}$</td>
</tr>
</tbody>
</table>

**Multidimensional Chronic Deprivation Condition**
Identify by chronicity (deprivation) … then by Multidimensional poverty

\[
c_{itd} = \sum_{t=1}^{T} w_t \cdot I(x_{itd} < z_d)
\]

Multidimensional Chronic Deprivation Condition
Identify by chronicity (deprivation) ... then by Multidimensional poverty

\[ c_{itd} = \sum_{t=1}^{T} w_t \cdot I(x_{itd} < z_d) \]

Chronic Deprivation Condition

Multidimensional Chronic Deprivation Condition

\[ H^{mcp}(k_d, k_t, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ I\left( \sum_{d=1}^{D} w_d \cdot I(c_{itd} \geq k_t) \geq k_d \right) \right] \]
A general form of the Adjusted Headcount Ratio

\[
M_{cmp}^{mc}(k_d, k_t, \alpha, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ I \left( \sum_{t=1}^{T} w_t * I \left( \left( \sum_{d=1}^{D} w_d * I(x_{itd} < z_d) \right) \geq k_d \right) \geq k_t \right) \right] * AD_i(\alpha)
\]

\[
M_{mcp}^{mc}(k_d, k_t, \alpha, z_d) = \frac{1}{N} \sum_{i=1}^{N} \left[ I \left( \left( \sum_{d=1}^{D} w_d * I \left( \left( \sum_{t=1}^{T} w_t * I(x_{itd} < z_d) \right) \geq k_t \right) \right) \geq k_d \right) \right] * AD_i(\alpha)
\]
A general form of the Adjusted Headcount Ratio

\[ AD_i(\alpha) = \frac{1}{DT} \sum_{d=1}^{D} w_d \sum_{t=1}^{T} [g(x_{itd}, z_d)]^\alpha = \frac{1}{DT} \sum_{t=1}^{T} w_d \sum_{d=1}^{D} w_d [g(x_{itd}, z_d)]^\alpha \]

Where \( AD_i(\alpha) \) is the average deprivation of the individual \( i \)

- Multidimensional Chronic Poverty
- Chronic Multidimensional Deprivation

\[ g(x_{itd}, z_d) = \begin{cases} 
1 - x_{itd}/z_d, & x_{itd} < z_d \\
0, & Otherwise
\end{cases} \]
A general form of the Adjusted Headcount Ratio

Double Union Approach

\[ M^{\text{cmp}}(k_d = \min\{w_d\}, k_t = \min(w_t), \alpha, z_d) = M^{\text{mcp}}(k_d = \min\{w_d\}, \min(w_t), \alpha, z_d) \]

Double Intersection Approach

\[ M^{\text{cmp}}(k_d = D, k_t = T, \alpha, z_d) = M^{\text{mcp}}(k_d = D, k_t = T, \alpha, z_d) \]

Decomposition techniques (dimension, time and subgroup) & measures of transient poverty are also presented in the paper.
Longitudinal Multidimensional Poverty

\(k_d=20\%

<table>
<thead>
<tr>
<th>Chronic Multidimensional Poverty</th>
<th>Never Poor</th>
<th>Once</th>
<th>Twice</th>
<th>Always</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Never Poor</td>
<td>4,050</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4,050</td>
</tr>
<tr>
<td>Once</td>
<td>2,020</td>
<td>1,781</td>
<td>253</td>
<td>10</td>
<td>4,064</td>
</tr>
<tr>
<td>Twice</td>
<td>0</td>
<td>20</td>
<td>851</td>
<td>94</td>
<td>965</td>
</tr>
<tr>
<td>Always</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>383</td>
<td>383</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6,070</strong></td>
<td><strong>1,800</strong></td>
<td><strong>1,104</strong></td>
<td><strong>487</strong></td>
<td><strong>9,461</strong></td>
</tr>
</tbody>
</table>

Income Poverty   Chronic Multidimensional Poverty
----------------------------------|------------------------|------------------------|
Income Poverty      1          |                         |
Chronic Multidimensional Poverty | 0.44*                  | 1                      |
Multidimensional Chronic Deprivation | 0.49*                  | 0.85*                 |

OPHI Oxford Poverty & Human Development Initiative
Longitudinal Multidimensional Poverty

\( (k_d=20\%, \, k_t=3) \)
Longitudinal Multidimensional Poverty

- “Only” difference in the identification

<table>
<thead>
<tr>
<th>Chronic Multidimensional Poverty</th>
<th>Multidimensional Chronic Deprivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Identification: the multidimensionally poor in every period</td>
<td>First identification: the chronically poor in every dimension</td>
</tr>
<tr>
<td>Second identification: the chronically multidimensionally poor</td>
<td>Second identification: the multidimensionally chronically poor</td>
</tr>
<tr>
<td>Less poverty with dynamically stable dimensions</td>
<td>Less poverty with more volatile dimensions</td>
</tr>
</tbody>
</table>

**KEY:** Related to probability to remain multidimensionally poor

**KEY:** Probability to remain deprived in a dimension
Thanks