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Abstract

Incomplete information makes trade more elastic. When firms face uncertainty about demand, the trade elasticity from entry is higher than under complete information. Unable to condition export decisions on demand, low productivity firms that would have otherwise profited from high demand choose not to enter export markets. Selection based on productivity alone is more stringent and amplifies the value of trade at the extensive margin. Using Brazilian export sales and quantity data, we quantify trade elasticities in models with and without uncertainty, and find that the amplification effect from demand uncertainty is large.

Keywords: Uncertainty, firm size distribution, extensive margin, trade elasticities.

JEL: F12, F13.

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1 Introduction

Welfare implications for standard trade theory, most recently developed in Arkolakis, Costinot, and Rodríguez-Clare (2012) and Melitz and Redding (2015), show that trade elasticities are key parameters for evaluating the welfare gains from trade. While these implications are derived from a broad class of models in which firms have complete information about their economic environment, a growing branch of the trade literature has demonstrated that models with uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior.[1] However, normative implications of this alternative information structure for measurements of trade elasticities, and therefore welfare gains from trade, are not yet well understood.[2] In this paper, we develop trade elasticity expressions for trade models with demand uncertainty and quantitatively show that demand uncertainty amplifies the size of these key trade elasticities that ultimately determine the welfare gains from trade.

The distinction between selection into exporting based on *productivity* versus *profitability* is central to measuring the effect of information structure on trade elasticities. In a stylized version of Jovanovic's (1982) learning model extended by Timoshenko (2015b) to a trade context, firms make export decisions after observing their productivity and before observing their demand. Hence, selection into exporting occurs based on *productivity*. In economic environments with complete information (Melitz (2003), Bernard, Redding, and Shott (2010), Arkolakis et al. (2012), Melitz and Redding (2015)) firms observe both productivity and product demand prior to choosing a quantity to export. Hence, selection into exporting occurs based on *profitability*, a combination of a firm's productivity and demand.

Selection into exporting with demand uncertainty implies a higher extensive margin trade elasticity than selection with complete information.[3] With com-

[1] These papers incorporate Jovanovic (1982) learning mechanism into the Melitz (2003) model, which features monopolistically competitive exporters that are heterogeneous in productivity and learn about their unobserved idiosyncratic demand in foreign markets. See Arkolakis, Papageorgiou, and Timoshenko (2015) for implications for firm growth as a function of age and size, Timoshenko (2015b) for implication for firm product switching behavior, and Bastos, Dias, and Timoshenko (2016) for implications for firm input and output pricing behavior.

[2] A notable exception is Arkolakis, Papageorgiou, and Timoshenko (2015), who characterize constrained efficiency of a model in which firms learn about demand, but do not engage in international trade.

[3] As defined in Chaney (2008), the extensive margin trade elasticity refers to changes in trade

plete information, a profitable firm does not necessarily possess high productivity. Some low productivity firms will self select into an export market provided they have observed sufficiently high demand. This does not occur in an environment with demand uncertainty, however, because for low productivity firms, the low probability of sufficiently high demand shock realizations after entry deters them from export participation altogether. Since selection based on productivity alone is more stringent, the size of a marginal exporter under uncertainty is larger than under complete information. Hence, demand uncertainty amplifies the extensive margin response of trade flows to changes in trade costs. We refer to demand uncertainty's effect on the extensive margin of trade flows as the *amplification effect*.

We quantify the magnitude of the amplification effect and find it is large. For quantification, we adapt the structural elasticity estimation approach in Bas, Mayer, and Thoenig (2015) to a stylized model with demand uncertainty along the lines of Jovanovic (1982) and productivity heterogeneity as in Melitz (2003). We demonstrate that, in contrast to a complete information environment where the extensive margin of trade elasticity can be inferred from the export sales data, with demand uncertainty the extensive margin is determined by the distribution of export *quantities*. Accordingly, we discipline the complete information model's profitability distribution with empirical export sales data and discipline the demand uncertainty model's productivity distribution with data on quantities exported. We apply our estimation strategy to Brazilian export data and find that demand uncertainty amplifies the extensive margin response by a factor of 100 relative to the model with complete information. We further find that the amplification effect is larger when an export destination exhibits larger demand uncertainty, as measured by the variance of ex post realizations of demand shocks.

This paper shows that the information structure faced by firms is crucially important for measuring the extensive margin response to a decline in trade costs. In countries or industries in which exporters face high demand uncertainty, by assuming away information asymmetries, trade elasticity estimates will likely understate the true magnitude of extensive margin adjustments, and therefore,

follows due to entry and exit in response to changes in variable trade costs.

the extent of welfare gains.

Our work contributes to the growing literature on decomposing trade elasticities. Chaney (2008) shows that the partial elasticity of trade can be decomposed into an intensive and an extensive margin of adjustment. Melitz and Redding (2015) further show that the extensive margin of adjustment crucially depends on the distributional assumptions with respect to the sources of firm-level heterogeneity. Sager and Timoshenko (2017) characterize a flexible distribution that well describes firm-level heterogeneity and find the extensive margin trade elasticity to be economically small. This paper demonstrates that selection into exporting (and hence the extensive margin of trade elasticity), depends on the information structure faced by firms.

Our work also contributes to a literature on measuring trade elasticities. Imbs and Mejean (Forthcoming) finds that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production. Furthermore, Imbs and Mejean (2015) document that elasticities computed using industry-level data are often larger than those using aggregated data. This paper demonstrates that firms' information sets affect trade elasticity measurement and documents an amplification effect on trade elasticities attributed to uncertainty faced by firms in foreign markets.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework, contrasts the elasticity implications between an environment with and without uncertainty, and describes a method to estimate trade elasticities based on a model with demand uncertainty. Section 3 presents elasticity estimation results. Section 4 concludes. Appendix A provides detailed characterization of a model with and without uncertainty.

2 Theoretical Framework

2.1 Economic Environment

In this section we consider a model with heterogeneous firms that export products in markets characterized by monopolistic competition. This environment is similar to that in Melitz (2003), and we assume exogenous entry as in Chaney (2008). We further introduce information asymmetries by constructing a stylized version of the learning model an in Jovanovic (1982) adopted by Timoshenko (2015b) to a trade context. All derivations are relegated to Appendix A.

2.1.1 Demand

There are N countries and K sectors, such that each country is indexed by j and each sector is indexed by k. Each country is populated by a mass of L_j identical consumers whose preferences are represented by a nested constant elasticity of substitution utility function given by

$$U_j \;=\; \prod_{k=1}^{K}\left[\left(\int_{\omega\in\Omega_{ijk}}\left(e^{\theta_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k-1}{\epsilon_k}}\,d\omega\right)^{\frac{\epsilon_k}{\epsilon_k-1}}\right]^{\mu_k}, \tag{1}$$

where Ω_{ijk} is the set of varieties in sector k consumed in country j originating from country i, $c_{ijk}(\omega)$ is the consumption of variety $\omega\in\Omega_{ijk}$, ϵ_k is the elasticity of substitution across varieties within sector k, $\theta_{ijk}(\omega)$ is the demand parameter for variety $\omega\in\Omega_{ijk}$, and μ_k is the Cobb-Douglas utility parameter for goods in sector k such that $\sum_{k=1}^{K}\mu_k = 1$.

Each consumer owns a share of domestic firms and is endowed with one unit of labor that is inelastically supplied to the market. Cost minimization yields a standard expression for the optimal demand for variety $\omega\in\Omega_{ijk}$, given by

$$c_{ijk}(\omega) = e^{\theta_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{kj} P_{jk}^{\epsilon_k-1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega\in\Omega_{ijk}$, Y_{jk} is total expenditures in country j on varieties from sector k, and P_{jk} is the aggregate price index in country j in section k.[4]

2.1.2 Supply

Each variety $\omega\in\Omega_{ijk}$ is supplied by a monopolistically competitive firm. Each firm can potentially supply one variety of a product from each sector. Upon entry, a firm is endowed with an idiosyncratic labor productivity level e^{φ} and a product-destination specific demand parameter θ_{ijk}. Productivity and demand parameters

[4] Note that $Y_{jk} = \mu_k Y_j$, where Y_j is aggregate income in country j.

are drawn from independent distributions. Denote by $g_{ijk}(.)$ the distribution from which firms draw productivity, φ, and by $h_{ijk}(.)$ the distribution from which firms draw demand parameter, θ_{ijk}. Firms from country i face fixed costs, f_{ijk}, and variable costs, τ_{ij}, of selling output to country j. Fixed and variable costs are denominated in units of labor.

Firms choose a quantity to export to each destination market before knowing destination-specific demands for their product. As a result, firms choose a quantity to export in order to maximize *expected* profits, subject to consumer demand (2) and prior beliefs about demand, $E(\exp(\theta_{ijk}/\epsilon_k))$. We assume that prior beliefs are the same across firms and equal the population mean.[5] The firm's decision problem yields an expression for the optimal quantity exported, given by

$$q_{ijk}(\varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k \varphi} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{3}$$

Once all goods are supplied to markets, demand shocks are realized and prices clear the goods markets for each variety. A firm's realized revenue is

$$r_{ijk}(\theta_{ijk}, \varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} e^{(\epsilon_k - 1)\varphi + \frac{\theta_{ijk}}{\epsilon_k}} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k - 1} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tag{4}$$

2.1.3 Entry

There is a mass of potential entrants, J_i, and each entrant draws a productivity level. By the assumption of exogenous entry into export markets, firms will enter as long as expected profits are at least zero. This "zero expected-profit condition" yields the following export participation threshold

$$e^{(\epsilon_k - 1)\varphi^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}. \tag{5}$$

The entry and exit of firms into export markets is determined by the *productivity* cutoff rule, φ^*_{ijk}. Since firms make entry and exit decisions before realizations

[5] While this paper considers a static model to enable a comparison with the standard Melitz (2003) framework, more general formulations of the learning model are considered in Arkolakis et al. (2015) and Timoshenko (2015b).

of demand shocks are observed, this cutoff rule depends on the expected demand level, $E\left(\exp\left(\theta_{ijk}/\epsilon_k\right)\right)$, common across firms. Hence, the market participation thresholds is common across firms of various ex-post demand realizations.

2.1.4 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} = M_{ijk} \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\theta,\varphi) h_{ijk}(\theta) \frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi > \varphi^*_{ijk})} d\theta d\varphi,$$

where M_{ijk} is the mass of firms exporting from country i to country j in industry k. The partial elasticity of trade flows with respect to the variable trade costs is given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = \underbrace{(1-\epsilon_k)}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left(\underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\frac{g_{ijk}(\varphi^*_{ijk}) e^{(\epsilon_k-1)\varphi^*_{ijk}}}{(\epsilon_k - 1)\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi} g_{ijk}(\varphi) d\varphi}}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right) \quad (6)$$

From equation (6), the extensive margin of the partial trade elasticity is governed by the productivity entry threshold, φ^*_{ijk}, the distribution of productivity, $g_{ijk}(.)$, and the elasticity of substitution across varieties, ϵ_k.

2.2 Comparison to Complete Information

The difference in the partial trade elasticities between the complete and incomplete information environments arises from differences in selection. In the economy with uncertain product demand, equation (6) shows that the extensive margin of the trade elasticity depends on the *productivity* entry threshold, φ^*_{ijk}. In this section, we show that the *profitability* threshold determines selection in the economy with complete information.

In a model with complete information, firms make export participation decisions after observing both the productivity and demand shocks.[6] Therefore,

[6] Appendix A.2 contains a formal description of the complete information economy.

firms' decisions depend on a single profitability parameter defined by $z_{ijk} \equiv (\epsilon_k - 1)\varphi + \theta_{ijk}$, and the "zero-profit" condition pins down the *profitability* entry threshold:

$$\left(e^{z^*_{ijk}}\right)^{\text{CI}} = \frac{\epsilon_k w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1}}, \tag{7}$$

where the superscript 'CI' stands for the 'Complete Information' environment. In such environment the two shocks simultaneously determine selection into exporting and the entry threshold. The trade elasticity is further given by

$$(\eta_{ijk})^{\text{CI}} = (1 - \epsilon_k)\left(1 + \frac{g^z_{ijk}(z^*_{ijk})}{Prob^z_{ijk}(z > z^*_{ijk})}\left(\frac{\tilde{r}_{ijk}}{r^{\min}_{ijk}}\right)^{-1}\right). \tag{8}$$

In contrast to equation (6), under complete information the partial trade elasticity is determined by the distribution of firm profitability, $g^z_{ijk}(.)$, the profitability entry threshold, z^*_{ijk}, and the average-to-minimum ratio of export sales, $\tilde{r}_{ijk}/r^{\min}_{ijk}$. As shown by Bas et al. (2015), all three components can be recovered from the empirical distribution of export sales.

In a model with demand uncertainty, however, equation (8) is not well defined since the average-to-minimum ratio of export sales, $\tilde{r}_{ijk}/r^{\min}_{ijk}$, is a random variable. This can be seen by substituting the market participation threshold from equation (5) into equation (4) and evaluating the sales for a marginal entrant with productivity φ^*_{ijk} yielding

$$r^{\min}_{ijk} = e^{\frac{\theta_{ijk}}{\epsilon_k}} \frac{\epsilon_k w_i f_{ijk}}{E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)}. \tag{9}$$

Hence, if the true data generating process is a model with demand uncertainty, the export sales distribution data cannot be used to measure trade elasticities because the sales of a marginal exporter, $r^{\min}_{ijk}$, is a random variable, as it is determined by a random demand shock, θ_{ijk}.

In the next section we adopt an estimation approach suggested by Bas, Mayer, and Thoenig (2015) and extended by Sager and Timoshenko (2017). Relative to these papers, we extend the approach to an environment with demand uncertainty.

2.3 Estimation Approach

In this section we detail our approach to estimating partial trade elasticities in the presence of demand uncertainty. As shown in equation (6), in an environment with uncertainty, selection occurs based on the productivity alone. Hence the extensive margin of trade elasticity depends on the productivity entry threshold and the distribution of the productivity draws. Both can be recovered using the data on the distribution of export quantity as we now describe.

Consider the following change of notation. Let $\phi_{ijk} \equiv \epsilon_k \varphi$ and denote by $g^{\phi}_{ijk}(.)$ the probability distribution function of ϕ_{ijk}. Given the change in notation, $g^{\phi}_{ijk}(.)$ is just the distribution of φ, $g_{ijk}(.)$ scaled by the elasticity of substitution, ϵ_k.

With the change in notation, the partial trade elasticity can be expressed as

$$\eta_{ijk} \;=\; (1-\epsilon_k)\left(1+\frac{\epsilon_k}{\epsilon_k-1}\frac{g^{\phi}_{ijk}(\phi^*_{ijk})e^{\frac{\epsilon_k-1}{\epsilon_k}\phi^*_{ijk}}}{\int_{\phi^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k-1}{\epsilon_k}\phi} g^{\phi}_{ijk}(\phi)d\phi}\right). \qquad (10)$$

The distribution $g^{\phi}_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. Taking the logarithm of equation (3) yields

$$\log q_{ijk} \;=\; C_{ijk} + \phi_{ijk}, \qquad (11)$$

where $C_{ijk} = \log\left(\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}\right)$. Observe that the distribution of log-export quantity is given by the distribution of ϕ_{ijk} scaled by a constant. Hence, parameters of $g^{\phi}_{ijk}(.)$ can be recovered from fitting the distribution to the empirical distribution of log-export quantity.

Given the distribution of $g^{\phi}_{ijk}(.)$, the scaled productivity entry threshold, ϕ^*_{ijk}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities given by

$$\text{Average-to-Minimum Ratio} \;=\; e^{-\phi^*_{ijk}} \int_{\phi^*_{ijk}}^{+\infty} \frac{e^{\phi} g^{\phi}_{ijk}(\phi)}{Prob^{\phi}_{ijk}(\phi > z\phi*_{ijk})} d\phi. \quad (12)$$

Finally, to compute the partial trade elasticity, η_{ijk}, one needs an estimate of the elasticity of substitution across varieties, ϵ_k. We will use the structure of shocks in our model to recover the ϵ_k. Substituting equation (3) into equation (4) and taking the logarithm we obtain

$$\log r_{ijk} \;=\; \frac{\epsilon_k - 1}{\epsilon_k} \log q_{ijk} + FE_{jk} + \frac{\theta_{ijk}}{\epsilon_k}, \quad (13)$$

where $FE_{jk} = \log\left(Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k - 1}{\epsilon_k}}\right)$. Notice that in the context of our model, the demand shock, θ_{ijk}, is uncorrelated with the quantity decision, q_{ijk}. Hence, one can recover the elasticity of substitution, ϵ_k, from a coefficient on log-quantity by running an OLS regression for equation (13) separately for each industry k using destination fixed effects.

In the Section 3 we apply the described elasticity estimation approach to quantify the partial trade elasticity.

3 Quantifying Trade Elasticities

3.1 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[7] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

We proxy the theoretical notion of export quantity with an empirical measure

[7] For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

of export weight.[8] The properties of export weight differ substantially across industries. Hence, we further conduct our analysis at the destination-year-industry level. We define an industry as a 2-digit HS code, and hence aggregate the sales and quantity data to the 2-digit HS level.

We define an observation to be a distribution of export quantity across firms for a given destination-year-industry triplet, and focus on observations where at least 100 firms export.[9]

The final sample consists of 714 destination-year-industry observations, and covers 32 destinations and 35 industries. Table 1 provides summary statistics of log-export quantities and log-sales distributions in our sample.

3.2 Parameter Estimates

3.2.1 The Export Quantity Distribution

To recover the partial trade elasticity we proceed by, first, assuming that the productivity is drawn from a Double EMG distribution, $DEMG(m, \upsilon^2, \xi_L, \xi_R)$. The resulting log-export quantity distribution, $g^{\phi}_{ijk}(.)$, then also follows a Double EMG distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$ with parameters scaled by the elasticity of substitution, ϵ_k, and described by the following cumulative distribution function:[10]

$$G(\phi) = \Phi\left(\frac{\phi-\mu}{\sigma}\right) - \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(\phi-\mu)+\frac{\sigma^2}{2}\lambda_R^2} \Phi\left(\frac{\phi-\mu}{\sigma} - \lambda_R\sigma\right) + \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(\phi-\mu)+\frac{\sigma^2}{2}\lambda_L^2} \Phi\left(-\frac{\phi-\mu}{\sigma} - \lambda_L\sigma\right), \quad (14)$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.

The Double EMG distribution provides a very flexible generalization of com-

[8]Export weight is used as a measure of export quantity in a number of studies including Bastos et al. (2016).

[9]The thresholds of 100 firms ensures that an empirical distribution can be accurately described by percentiles. This threshold is also consistent with the literate. See Fernandes et al. (2015), Sager and Timoshenko (2017).

[10]The parameters of the productivity versus log-export quantity are related as follows: $\mu = \epsilon_k m$, $\sigma^2 = \epsilon_k^2 \upsilon^2$, $\lambda_L = \xi_L/\epsilon_k$, and $\lambda_R = \xi_R/\epsilon_k$.

mon distributional assumptions used in the literature. From equation (14), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2015) and Fernandes et al. (2015). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution. By assuming the Double EMG distribution we, therefore, allow the data to recover the best fit of distribution between the Exponential, Normal, Double Exponential or the corresponding convolutions.[11]

For each destination-year-industry observation, we choose distribution parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$ so that the percentiles of the theoretical log-quantity distribution match the percentiles of the empirical log-quantity distribution. We follow Sager and Timoshenko (2017) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(q_i^{data} - q_i(\mu, \sigma^2, \lambda_L, \lambda_R)\right)^2,$$

where q_i^{data} is the i-th percentile of the empirical quantity distribution for a given destination-year-industry, $q_i(\mu, \sigma^2, \lambda_L, \lambda_R)$ is the model implied i-th quantity percentile for given parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$, and N_P is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical quantity distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Panel A in Table 2 summaries distribution parameter estimates across 714 observations. As can be seen from the Table, the average sample value of σ is 2.42, rejecting a common assumption of Exponentially or Double Exponentially

[11]See Sager and Timoshenko (2017) for a more thorough characterization of the Double EMG distribution.

distributed productivity shocks.[12] Furthermore, as can be inferred from the values of the left and right tail parameters, λ_L and λ_R, distributions exhibit substantial heterogeneity in the fatness of both tails. The value of the right tail parameter, λ_R varies between 0.22 and 18.76, with about 63 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. This finding is consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[13] Furthermore, we also find that majority of distributions exhibit fatness in the left tail: $\lambda_L < 2$ in approximately 59 percent of observations. [14]

3.2.2 Entry Threshold

Next, we use the fitted distribution to recover the productivity entry thresholds. For each destination-year-industry observation we solve equation (12) for the productivity entry threshold using the data on the corresponding average-to-minimum ratio of export quantity and the distribution parameter estimates.

Figure 1 provides a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of log-export quantity. Each dot in the Figure corresponds to a destination-year-industry observation. The values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution. Hence, the values represent deviations from the mean of the distribution. Figure 1 shows that the greater is the deviation, i.e. the more negative is the value on the y-axis, the lower is the entry threshold. A lower entry threshold relative to the mean implies a smaller size of a marginal exporter relative to an average exporter.

3.2.3 Elasticity of Substitution Across Varieties

Next, we recover the elasticity of substitution across varieties, ϵ_k, by running an OLS regression for each of the 35 2-digit HS industries in our sample. We estimate

$$\log r_{fjtk} = \beta_k \log q_{fjtk} + FE_{jt} + u_{fjtk},$$

[12] Out of 714 distributions, only 10 have a value of σ close to zero.

[13] See Axtell (2001) and di Giovanni et al. (2011).

[14] Sager and Timoshenko (2017) document fat left tails in the context of export sales distributions.

where f indexes firms, j indexes destinations, t is a year, k indexes industries, FE_{jt} is destination-year fixed effect, and u_{fjtk} is an error term. As argued in Section 2.3, the error term is uncorrelated with the regressor. Given an estimated value for β_k, equation (13) shows us that the elasticity of substitution across varieties can be computed as $\epsilon_k = 1/(1 - \hat{\beta}_k)$.

The resulting elasticity estimates are presented in Panel B of Table 2. As shown in the Table, the average value of the elasticity of substitution across industries is 4.68, and varies between 1.38 for an industry with relatively homogeneous products (Pearls, Precious and Semi-Precious Stones and Metals, HS code 71) and 13.07 for an industry with a relatively larger degree of product differentiation (Footwear, HS code 64).

The elasticity estimates we obtain lie within the range estimated in the literature. Broda and Weistein (2006) estimate the average elasticity of substitution to range from 4 to 6 when estimated at a level of aggregation comparable to 2-digit HS level used in this paper. In a dynamic version of a learning environment we consider in the paper, Berman, Rebeyrol, and Vicard (2015) estimate industry-level elasticities at the 6-digit HS level of aggregation to be on average 11.15 with a median value of 8.10. Since we estimate elasticities at a higher level of aggregation, it is expected that their values are smaller. Bas, Mayer, and Thoenig (2015) use similar export data to estimates the elasticity of substitution, and obtain magnitudes ranging from 1.8 to 6.

3.3 Trade Elasticities

3.3.1 Elasticity Estimates

Using the components from the previous section, we compute the partial trade elasticity, η_{ijk}, and the extensive margin contribution to the trade elasticity from equation (10). Table 3 summarizes average values for and heterogeneity in elasticity estimates.

Observe from Table 3, that an average contribution of the extensive margin to trade elasticity appears to be small: an average order of magnitude 10^{-4}.[15]

[15] Sager and Timoshenko (2017) show that this magnitude is a result of an abundance of small exporters in export sales distributions. Other frequently used trade data sets exclude these small firms and, hence, generate much higher extensive margin elasticities.

In the context of the volume of aggregate trade flows, this magnitude can be understood as follows. Suppose, for example, that a decline in trade costs leads to an increase in trade flows by a million dollars. For an average observation, the new exporters would account for approximately $660 out of a million dollars of the newly created trade.

3.3.2 Amplification Effect

To compare the estimates of trade elasticity between the two information environments, we first re-estimate the partial trade elasticity under the assumption of complete information. As discussed in Section 2.2, in a model with complete information the partial trade elasticity depends on the distribution of export sales. Hence, we re-fit the Double EMG distribution to match the distribution of log-export sales, and further use the average-to-minimum ratio of export sales to impute the value of the profitability entry threshold. Notice that in the complete information environment, the contribution of the extensive margin to trade elasticity is independent of the elasticity of substitution. To compute the overall magnitudes of trade elasticity, however, an elasticity of substitution parameter is needed. For comparability, we use the elasticity of substitution parameter estimates from Section 3.2.3 for computing both complete and incomplete information trade elasticities. Panel A in Table 3 provides summary statistics of the elasticity estimates.

Result 1: *Demand uncertainty amplifies the extensive margin contribution to the trade elasticity by an average order of magnitude of 100.*

As can be seen from Panel A in Table 3, the complete information economy yields lower values for the extensive margin elasticity.[16] In a complete information environment, the average contribution of the extensive margin is smaller by two orders of magnitude relative to a model with demand uncertainty. Panel B in Table 3 compares the elasticity estimates across the same observations. In particular, it reports summary statistics of the ratio of the quantity implied trade elasticity relative to the sales implied trade elasticity. We call this ratio the *am-*

[16] In both models, however, the average partial trade elasticity is around 3.6 as a result of the overall small contribution of the extensive margin to that elasticity.

plification effect because information uncertainty implies a higher contribution of the extensive margin to trade, an order of magnitude of 100.

To motivate this magnitude, consider the following example. Suppose trade increases by a million dollars due to a decline in trade costs. Then, an trade elasticity estimate from a complete information model would attribute approximately \$9 out of a million dollars of new trade to trade generated by entrants. In a model with incomplete information, \$660 out of a million dollars can be attributed to trade by entrants. Hence, complete information dampens the (already small) contribution of new exporters to trade. Conversely a model with uncertainty amplifies the contribution of the extensive margin to trade.

3.3.3 Role of Demand Uncertainty

The magnitude of the uncertainty amplification effect is tightly linked to the extent of variation arising from the demand shocks. Notice from equation (13) that the distribution of the demand shocks generates a wedge between the distributions of log-export sales and log-export quantity. This wedge is larger when the variance of demand shocks is higher. If the variance of the demand shocks is zero, then the distributions of log-export sales and log-export quantity would coincide, yielding a no amplification effect. As the variance of the demand shock rises, the distributions of log-export sales and log-export quantities are more dissimilar. Hence we would expect a larger amplification effect.

Given equation (13), we measure the extent of demand variation in a given destination-year-industry as the difference between the variance of log-export sales and the variance of log-export quantities. As before, we assume that the demand shocks are uncorrelated with log-export quantities. Applying the variance operator to both side of equation (13) and rearranging the terms yields:

$$V\left(\frac{\theta_{jk}}{\epsilon_k}\right) = V(\log r_{jk}) - \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^2 V(\log q_{jk}). \tag{15}$$

We first compute the variance of log-export sales, $V(\log r_{jk})$, and the variance of log-export quantity, $V(\log q_{jk})$, across firms within a given destination-year-industry observation. Using the elasticity of substitution estimates from Section 3.2.3, we then use equation (15) to back out the value of the variance of the demand shocks, $V(\theta_{jk}/\epsilon_k)$ for each destination-year-industry observation.

Result 2: *The magnitude of the uncertainty amplification effect is larger in more uncertain economies.*

Figure 2 depicts a relationship between the variance of the demand shocks and the amplification effect. The x-axis measures the variance of the demand shocks, while the y-axis measures the ratio of the export quantity implied relative to the export sales implies estimate of the extensive margin elasticity. The figure confirms that the amplification effect increases with an increase in demand uncertainty. Hence, the contribution of the extensive margin to the trade elasticity from the model with complete information should be thought of as a lower bound. In the data, exporters do not have full information about product demand in destination markets and introducing uncertainty into the model leads to a larger extensive margin adjustment.

4 Conclusion

Recently, models of learning along the lines of Jovanovic (1982) have been extensively applied to analyze firm behavior such as growth (Arkolakis et al., 2015), export participation (Timoshenko, 2015a), product switching (Timoshenko, 2015b), and pricing decisions (Bastos et al., 2016). The role of information structure in measuring the trade elasticity has so far been omitted in this literature.

In this paper, we study the implications of information uncertainty for the partial trade elasticity. We introduce uncertainty with respect to product demand to an otherwise standard new trade model with heterogeneous firms, as in Melitz (2003). With demand uncertainty, firms must choose how much of their product to export prior to observing the destination specific demand shock. As a result, firms make export decisions based on their productivity and, hence, selection into exporting and the extensive margin of adjustment are driven by firms' productivity.

In a model with complete information, firms know their product demand in destination markets. Firms can choose how much of their product to export with complete information about their profitability. Profitability is a measure of productivity *and* demand that characterizes idiosyncratic profit across firms. Profitable firms are not always high productivity firms, because a low productivity firm may face high demand for its product. On the margin, a low productivity firm may export as a result of observing a high demand shock. Low productivity firms dampen the effect of selection relative to an incomplete information environment.

We quantify the effect of uncertainty by comparing the trade elasticity in model environments with and without product demand uncertainty. To compute the trade elasticity, we reformulate the structural estimation approach in Bas, Mayer, and Thoenig (2015) for use in an environment with incomplete information. We discipline the distribution of productivity separately from that of profitability by using Brazilian microdata on export quantities and export sales. Upon measuring trade elasticities, we find that demand uncertainty amplifies extensive margin adjustments relative to the complete information economy. This amplification effect is large, implying an extensive margin trade elasticity that is 100 times larger with demand uncertainty than with complete information. Furthermore, the effect is stronger in economies with higher demand uncertainty (e.g., higher variance in sales distributions relative to variance in quantity distributions).

This paper shows that the information structure faced by firms is crucially important for measuring the extensive margin response to a decline in trade costs. In countries or industries in which exporters face high demand uncertainty, by assuming away information asymmetries, trade elasticity estimates will likely understate the true magnitude of extensive margin adjustments.

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Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across destination-year-industry observations over 1997-2000.

Statistic	Mean	Standard Deviation	Min	Max
Panel A: Properties of log-quantity				
Standard Deviation	3.03	0.56	1.35	4.79
Skewness	-0.07	0.46	-1.63	1.03
Interquartile Range	4.19	1.04	1.81	7.75
Kelly Skew	-0.02	0.13	-0.44	0.43
Panel B: Properties of log-sales				
Standard Deviation	2.57	0.43	1.38	3.92
Skewness	-0.15	0.33	-1.61	0.84
Interquartile Range	3.38	0.67	1.96	6.11
Kelly Skew	-0.02	0.11	-0.44	0.41

Note: the statistics are reported across 714 destination-year-industry observations where at least 100 firms export. An industry is defined as a 2-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Parameter estimates.

Statistic	Mean	Standard Deviation	Min	Max
Panel A: Double EMG distribution parameter estimates[a]				
σ	2.42	0.88	$1.0 \cdot 10^{-8}$	4.47
λ_L	4.24	4.63	0.19	25.13
λ_R	4.10	4.77	0.22	18.76
Panel B: The elasticity of substitution across varieties[b]				
ϵ_k	4.68	2.42	1.38	13.07

[a] The summary statistics are reported across 714 destination-year-industry observations.

[b] The summary statitics are reported across 35 2-digit HS codes observed in the sample.

Table 3: Trade elasticity estimates.

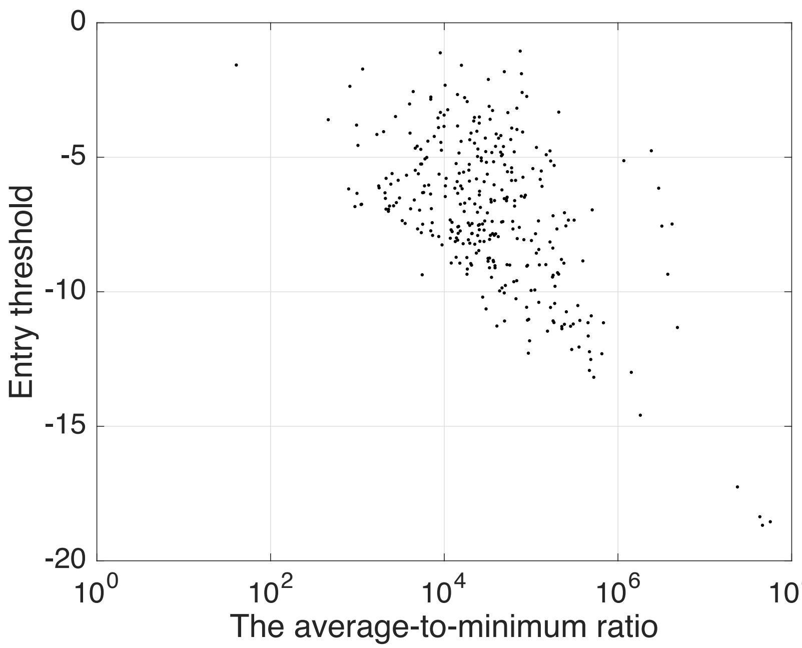
Measure	Extensive Margin Elasticity		Partial Trade Elasticity, η_{ijk}	
	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: Estimates of trade elasticity				
Quantity based[a]	$6.6 \cdot 10^{-4}$	0.005	3.60	2.35
Sales based[b]	$9.2 \cdot 10^{-6}$	$5.1 \cdot 10^{-5}$	3.63	2.51
Panel B: Amplification effect				
Amplification effect[c]	$8.4 \cdot 10^{2}$	$5.0 \cdot 10^{3}$	1.0002	$7.8 \cdot 10^{-4}$

[a] The quanity based measure of trade elasticity is based on a model with demand uncertainty. The summary statistics are reported across 330 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[b] The sales based measure of the trade elasticity is based on a model with complete information. The summary statistics are reported across 304 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

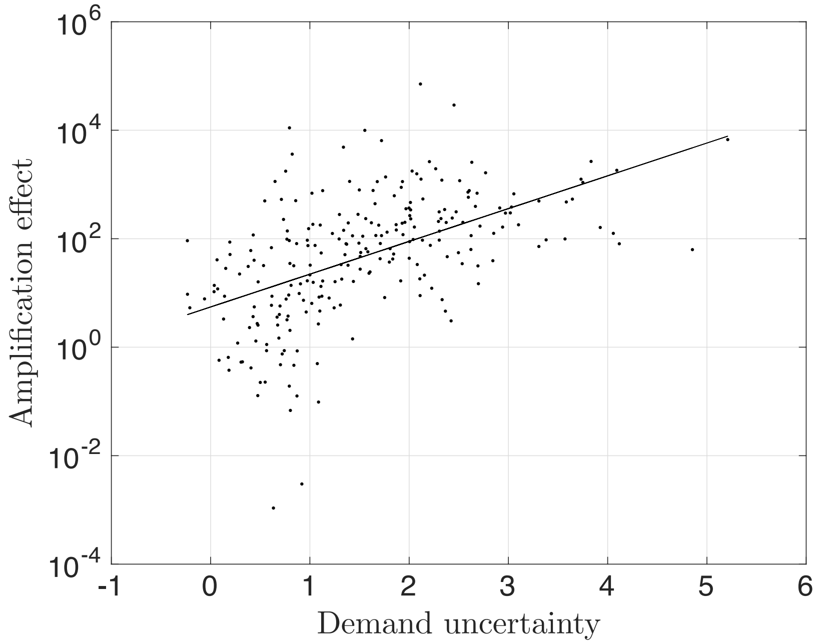
[c] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 240 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures. The presented results exclude an outlier for which the amplification effect equals $3.1 \cdot 10^{9}$.

Figure 1: The entry threshold and average-to-minimum ratio.



Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of export quantity for observation with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a destination-year-industry observation. Values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution.

Figure 2: Amplification effect and demand uncertainty.



Notes: The figure depicts a scatter plot of the amplification effect and demand uncertainty. The amplification effect is defined as the ratio of the extensive margin elasticity estimates between the quantity based and the sales based measures. Demand uncertainty is defined as the variance of the demand shocks estimated using equation (15). Each dot corresponds to a destination-year-industry observation. The solid line is an OLS best fit line.

A Theoretical Appendix

A.1 A Model with Information Uncertainty

In this section we provide derivations for the theoretical results in Section 2. We consider a monopolistically competitive environment as in Melitz (2003) with exogenous entry as in Chaney (2008). We further introduce information asymmetries by constructing a stylized version of the learning model in Timoshenko (2015b).

A.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E[\pi(\varphi)] = \max_{q_{ijk}} E_{\theta_{ijk}} \left(p_{ijk} q_{ijk} - \frac{w_i \tau_{ij}}{\varphi} q_{ijk} \right) - w_i f_{ijk}, \tag{16}$$

subject to the demand equation (2). The expectation over the demand draw, θ_{ijk}, is given by the distribution from which the demand parameter is drawn, $h_{ijk}(.)$. Substituting equation equation (2) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{q_{ijk}(\varphi)} q_{ijk}(\varphi)^{\frac{\epsilon_k - 1}{\epsilon_k}} E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right) Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{1-\epsilon_k}{\epsilon_k}} - \frac{w_i \tau_{ij}}{\varphi} q_{ijk}(\varphi) - w_i f_{ijk}. \tag{17}$$

The first order conditions with respect to quantity yield the optimal quantity,

$$q_{ijk}(\varphi) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k \varphi} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{18}$$

A.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, φ^*_{ijk}, is a solution to the zero-expected profit condition given by

$$E[\pi(\varphi^*_{ijk})] = 0. \tag{19}$$

Substituting equation (18) into equation (17) and solving equation (19) for φ^*_{ijk} yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}. \tag{20}$$

A.1.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} = M_{ijk}\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} r_{ijk}(\theta,\varphi)h_{ijk}(\theta)\frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi>\varphi^*_{ijk})}d\theta d\varphi, \tag{21}$$

where M_{ijk} is the mass of firms exporting from country i to country j in industry k. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} = J_i \times Prob_{ijk}(\varphi>\varphi^*_{ijk}), \tag{22}$$

where J_i is the exogenous mass of entrants. Equation (21) can then be simplified as follows:

$$\begin{aligned}
X_{ijk} &= J_i\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} q_{ijk}(\varphi)p_{ijk}(\theta,\varphi)h_{ijk}(\theta)g_{ijk}(\varphi)d\theta d\varphi \\
&= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{\varphi E\left(e^{\frac{\theta}{\epsilon_k}}\right)}\left(\int_{-\infty}^{+\infty} e^{\frac{\theta_{ijk}}{\epsilon_k}}h_{ijk}(\theta)d\theta\right)g_{ijk}(\varphi)d\varphi \\
&= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{\varphi}g_{ijk}(\varphi)d\varphi \\
&= J_i\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi}g_{ijk}(\varphi)d\varphi
\end{aligned} \tag{23}$$

Differentiate equation (23) with respect to τ_{ij} to obtain

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = (1-\epsilon_k)\frac{X_{ijk}}{\tau_{ijk}} - \frac{X_{ijk}}{\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi}g_{ijk}(\varphi)d\varphi}e^{(\epsilon_k-1)\varphi^*_{ijk}}g_{ijk}(\varphi^*_{ijk})\frac{\partial \varphi^*_{ijk}}{\partial \tau_{ijk}} \tag{24}$$

Differentiate equation (20) with respect to τ_{ij} to obtain

$$\frac{\partial \varphi^*_{ijk}}{\partial \tau_{ij}} = \frac{1}{\tau_{ij}}. \tag{25}$$

Combine equations (24) and (25) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = (1-\epsilon_k)\left(1 + \frac{g_{ijk}(\varphi^*_{ijk})e^{(\epsilon_k-1)\varphi^*_{ijk}}}{(\epsilon_k-1)\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi} g_{ijk}(\varphi)d\varphi}\right). \tag{26}$$

A.1.4 Estimation Approach

Consider the following change of notation: let $\phi_{ijk} \equiv \epsilon_k \varphi$. Denote by $g^{\phi}_{ijk}(.)$ the probability distribution function of ϕ_{ijk}. Given the change in notation, $g^{\phi}_{ijk}(.)$ is the distribution of φ, $g_{ijk}(.)$, scaled by the elasticity of substitution, ϵ_k.

With the change in notation, equations (20) and (23) can be written as

$$e^{\frac{\epsilon_k-1}{\epsilon_k}\phi^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}} \tag{27}$$

$$X_{ijk} = J_i\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}\int_{\phi^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k-1}{\epsilon_k}\phi} g^{\phi}_{ijk}(\phi)d\phi \tag{28}$$

Differentiating equation (27) and (28) with respect to τ_{ij}, the partial trade elasticity can be expressed as

$$\eta_{ijk} = (1-\epsilon_k)\left(1 + \frac{\epsilon_k}{\epsilon_k-1}\frac{g^{\phi}_{ijk}(\phi^*_{ijk})e^{\frac{\epsilon_k-1}{\epsilon_k}\phi^*_{ijk}}}{\int_{\phi^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k-1}{\epsilon_k}\phi} g_{ijk}(\phi)d\phi}\right).$$

The distribution $g^{\phi}_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. From equation (18), the optimal quantity can we written as

$$q_{ijk}(\phi_{ijk}) = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} e^{\phi_{ijk}}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{29}$$

Hence, the distribution of log-export quantity is given by the distribution of ϕ_{ijk}. Given the distribution of $g^{\phi}_{ijk}(.)$, the scaled productivity entry threshold, ϕ^{*}_{ijk}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (29) the average export quantity, $\tilde{q}_{ijk}$, and the minimum export quantity, $q^{\min}_{ijk}$, are given by

$$
\begin{aligned}
\tilde{q}_{ijk} &= \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\phi^{*}_{ijk}}^{+\infty} \frac{e^{\phi} g^{\phi}_{ijk}(\phi)}{Prob^{\phi}_{ijk}(\phi > \phi^{*}_{ijk})} d\phi \\
q^{\min}_{ijk} &= \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} \left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{\phi^{*}_{ijk}}.
\end{aligned}
$$

Hence, the average-to-minimum ratio, $\tilde{q}_{ijk}/q^{\min}_{ijk}$, is given by

$$
\text{Average-to-Minimum Ratio} = e^{-\phi^{*}_{ijk}} \int_{\phi^{*}_{ijk}}^{+\infty} \frac{e^{\phi} g^{\phi}_{ijk}(\phi)}{Prob^{\phi}_{ijk}(\phi > \phi^{*}_{ijk})} d\phi. \tag{30}
$$

A.2 A Model with Complete Information

In this section, for comparison purposes, we develop theoretical results in a model with complete information. The information structure only affects the supply side of the economy. Hence, on the demand side, the utility of a representative consumers is still given by equation (1), and the demand for a given variety is given by equation (2).

A.2.1 Supply

In contrast to a model with uncertainty, in a model with complete information firms make their market participation and quantity decisions after observing their productivity and demand shocks.

For each destination and industry firms maximize profits given by

$$
\max_{q_{ijk}} p_{ijk} q_{ijk} - \frac{w_i \tau_{ij}}{\varphi} q_{ijk} - w_i f_{ijk}, \tag{31}
$$

subject to the demand equation (2). The first order conditions with respect to quantity yield the optimal quantity being given by

$$q_{ijk}(\theta_{ijk},\varphi) \;=\; \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{32}$$

Notice that in contrast to equation (18), in a complete information environment the quantity choice is determined by a combination of a supply and a demand shocks, i.e. by a firm's profitability. Using equations (2) and (32), a firm's optimal sales are further given by

$$r_{ijk}(\theta_{ijk},\varphi) \;=\; \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} e^{(\epsilon_k-1)\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{33}$$

A.2.2 Entry

Given the optimal profits, firms enter the market as long as the profit is positive. Hence, the optimal any demand draw, θ_{ijk}, productivity entry threshold, $\varphi^*_{ijk}(\theta_{ijk})$, is a solution to the zero-profit condition given by

$$\pi(\varphi^*_{ijk}(\theta_{ijk})) \;=\; 0. \tag{34}$$

Substituting equation (32) into equation (31) and solving equation (34) for $\varphi^*_{ijk}(\theta_{ijk})$ yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}(\theta_{ijk})} \;=\; \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}Y_{jk}P_{jk}^{\epsilon_k-1}e^{\theta_{ijk}}}. \tag{35}$$

Notice that in contrast to the incomplete information environment discussed in Section A.1.1 and equation (20), the productivity entry threshold depends on the realized value of demand parameter, θ_{ijk}. Firms with a higher demand parameter have a lower productivity entry threshold.

Equation (35) can be viewed as defining an entry boundary in the space of (θ_{ijk},φ) or as defining the profitability entry threshold z^*_{ijk}. The profitability entry thresholds is given by the sum of $(\epsilon_k-1)\varphi_{ijk}$ and θ_{ijk} such that equation

(35) holds:

$$e^{z^*_{ijk}} \quad = \quad \frac{\epsilon_k w_i f_{ijk}(w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1}}. \tag{36}$$

Hence, in a model with complete information, selection into exporting occurs based on profitability rather than productivity as is the case in a model with uncertainty.

A.2.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is given by

$$\begin{aligned} X_{ijk} &= J_i \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} q_{ijk}(\theta, \varphi) p_{ijk}(\theta, \varphi) h_{ijk}(\theta) g_{ijk}(\varphi) d\theta d\varphi \qquad (37) \\ &= J_i \left(\frac{\epsilon_k}{\epsilon_k - 1} \frac{\tau_{ij} w_i}{P_{jk}} \right)^{1 - \epsilon_k} Y_{jk} \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} e^{(\epsilon_k - 1)\varphi + \theta_{ijk}} h_{ijk}(\theta) g_{ijk}(\varphi) d\theta d\varphi \end{aligned}$$

Define $z_{ijk} = (\epsilon_k - 1)\varphi + \theta_{ijk}$. From equation (36) the entry into exporting occurs when $z_{ijk} > z^*_{ijk}$. Using this change of variables, equation (37) can be be written as

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k - 1} (\tau_{ij} w_i)^{1 - \epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz, \tag{38}$$

where $g^z_{ijk}(.)$ is the distribution of profitability z_{ijk}.

Compare the expressions for the aggregate trade flow between the two information environments, equation (23) versus equation (38). Notice, that in the incomplete information environment, the aggregate trade flows are determined by the distribution of *productivity*, $g_{ijk}(\varphi)$, while in the complete information environment the aggregate trade flows are determined by the distribution of *profitability*, $g^z_{ijk}(z)$.

Following the same differentiation steps as in Section A.2.3, the partial elasticity of trade flows with respect to the variable trade costs is given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = (1-\epsilon_k)\left(1 + \frac{g^z_{ijk}(z^*_{ijk})e^{z^*_{ijk}}}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z)dz}\right) =$$

$$= (1-\epsilon_k)\left(1 + \frac{g^z_{ijk}(z^*_{ijk})}{Prob^z_{ijk}(z > z^*_{ijk})}\left(\frac{\tilde{r}_{ijk}}{r^{\min}_{ijk}}\right)^{-1}\right).$$

The last equality hold due to equation (40) below.

A.2.4 Estimation Approach

The distribution $g^z_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export sales. From equation (33), the optimal sales can we written as

$$r_{ijk}(z_{ijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} e^{z_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{39}$$

Hence, the distribution of log-export sales is given by the distribution of z_{ijk}. Given the distribution of $g^z_{ijk}(.)$, the profitability entry threshold, z^*_{ijk}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (29) the average export sales, $\tilde{r}_{ijk}$, and the minimum export sales, $r^{\min}_{ijk}$, are given by

$$\tilde{r}_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^*_{ijk}}^{+\infty} e^z \frac{g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz$$

$$r^{\min}_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{z^*_{ijk}}.$$

Hence, the average-to-minimum ratio, $\tilde{r}_{ijk}/r^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-z^*_{ijk}} \int_{z^*_{ijk}}^{+\infty} \frac{e^z g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz. \tag{40}$$

To contrast the two information environments, notice that while equations for estimating the entry thresholds are similar, equation (30) versus (40), different data are used for estimation. In the environment with information uncertainty the relevant distributions and entry thresholds are identified from the empirical export quantity distributions, while in the complete information framework, log export sales identify the necessary parameters.