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Organization of Credit Markets**

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# Costly Screening, Self-Selection, Fraud, and the Organization of Credit Markets

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## Abstract

This paper analyzes a credit market that includes a costly, universal and imperfect screening technology with both type I and type II errors and borrower self-selection. Universal screening is necessary because there are fraudulent borrowers. These characteristics, which have been omitted from other models, make it more representative of actual mortgage and consumer lending conditions. Contrary to the results in previous models with random screening, the combination of universal screening and type I screening error produces a pooling equilibrium as a non-trivial outcome. This result suggests that generalized lenders can sometimes compete with specialized lenders serving a single borrower type in credit markets that rely on costly lender screening. At other times pooling lenders will not be competitive and this, by itself, could lead to periodic waves of failure. These theoretical results appear to have implications for stability in markets for consumer credit.

JEL Classification: D82, D86, G20, L16

Keywords: costly screening, screening cost, self selection, pooling equilibrium, separating equilibrium

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# Costly Screening, Self-Selection, Fraud, and the Organization of Credit Markets

## I. Introduction

Since the seminal work by Rothschild and Stiglitz (1976), the literature on credit markets with adverse selection and screening<sup>1</sup> has held that a pooling equilibrium does not exist unless there is sequential interaction between lenders and borrowers in the equilibrium. This paper develops a simple credit market model without complex game-theoretic form, where risk-neutral borrowers self-select because lenders make use of a costly, imperfect and universal screening technology. The assumptions of this credit market model appear to match conditions in some credit markets, mortgage markets for example. In contrast to expectations based on previous literature, a pooling equilibrium appears as a non-trivial outcome simply because of high screening cost.

This model contributes to the literature by demonstrating that making assumptions that are both reasonable and realistic about screening technology and behavior can change the previous presumption that competitive markets relying on costly screening as a sorting device must be served by specialized lenders. This finding that a pooling equilibrium possible is important because different types of lending strategies have been tried over the past twenty years as credit has been extended to diverse borrower risk categories. Particularly in the mortgage market, it is clear that lenders successfully pool across different borrower types. The results presented here show why and when such pooling equilibria are likely to be stable.

The model in this paper embeds a general screening technology in a similar environment to that in Wang and Williamson (1998). The model can reproduce the classic separating equilibrium result as a special case of a more general world of lending possibilities.

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<sup>1</sup> Screening can be based on “hard” information like credit scores and employment records or on “soft” information that is collected by underwriters..

However, when screening cost for each contract in the separating equilibrium becomes sufficiently high, the market switches to a pooling equilibrium.

Failure to find a sustainable pooling equilibria in previous models with borrower self-selection and costly lender screening arises for two reasons. First, previous models assume that there is no need for universal screening because there are no fraudulent applicants. Second, there is no type I error in classifying applicants, i.e. good risks are never falsely rejected. Recent experience in mortgage lending has illustrated the importance of fraudulent applications, as documented in Jiang, Nelson and Vytlačil (2010). In such markets, it is necessary for a lender in the separating equilibrium to screen all applicants instead of random sampling. But the environment of universal screening, combined with type I screening error, falsely rejecting some good borrowers, imposes an extra cost on good borrowers at specialized lenders. When this cost is sufficiently high, a pooling equilibrium outperforms market separation by risk type.

The disturbing result of this analysis is that, some conditions produce a pooling equilibrium and others a separating equilibrium. As behavior of borrowers and screening technology changes, the character of market equilibrium may change. Obviously such change could trigger the failure of a particular lender type because their business model is outcompeted by an alternative. This may contribute to instability in markets for mortgage and consumer durables credit.

## **II. Relevant literature and general approach in this paper**

The seminal paper by Rothschild and Stiglitz (1976) launched the literature on screening in an environment of adverse selection where the uninformed party takes the lead to reduce information asymmetries, as opposed to the canonical signaling literature where the informed party takes the lead. So far in the literature, scholars have investigated three

different screening devices<sup>2</sup> used by the lender: first, a combination of price and quantity; second, a combination of price and collateral, and third, costly lender screening.

The literature about the first screening device, combinations of price and quantity, starts with Rothschild and Stiglitz (1976) who explore the lender's use of limited price and quantity contracts to promote self-selection of applicants in a competitive insurance market with risk-averse agents. They establish that a separating equilibrium is the only possible equilibrium as a seller can always separate good customers from bad by offering a range of contracts of different prices and quantities. Many papers have followed this pioneering work by adapting the model to various contexts. Dubey and Geanakoplos (2002) recast Rothschild and Stiglitz's model to study financial markets in the framework of competitive pooling<sup>3</sup>, where borrowers self-select into pools with different prices and quantity-limits as a screening device. They found that a separating equilibrium always exists and is unique. Martin (2007) changed the perturbations used in defining equilibria in Dubey and Geanakoplos's model and found that there are cases where pooling equilibrium can Pareto dominate a separating equilibrium.

The second screening device, interest rate and collateral, originates with Stiglitz and Weiss (1981) who study credit market screening and examine the lender's option of using interest rate or collateral as a device to induce different types of borrowers to self-select into different contracts. Bester (1985) establishes a separating equilibrium using interest rate and collateral simultaneously as a screening device. Hellwig (1986) finds a possible pooling equilibrium when extending Bester's (1985) screening environment to a three-

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<sup>2</sup> The term "screening device" first appeared in Stiglitz and Weiss (1981), in which they refer interest rate as one of the screening devices that a lender can use to distinguish different types of borrowers, in the environment where risky borrowers self-select loans with higher interest rates.

<sup>3</sup> Here the concept of "pooling" means that lenders and borrowers do not trade bilaterally, but through pools of different quantities (set exogenously) and prices (determined by the market), where a lender can purchase shares of a pool and borrowers can sell promises of deliveries into the pool. To the extent that borrowers of different risks can sell promises to the same pool, it is a pooling equilibrium; otherwise, it is a separating equilibrium. In their model, different pools exist before borrowers making a selection, so it is a screening model instead of signaling, although screening is regarded as part of signaling literature in general.

stage game. Dell'Ariccia and Marquez (2006) use the combination of interest rate and collateral as a screening device to the study of the dynamics of bank competition, and find that equilibrium can switch between separating and pooling depending on the changing distribution of borrowers. Martin (2006) embeds the interest rate and collateral screening device into a model of endogenous credit cycles, where collateral is linked to entrepreneurial wealth, and then the dynamic change of wealth is linked to the equilibrium regime switching between pooling and separating

For the first two screening devices, screening is costless to the lender. The lender offers different contracts as a screening device, and the borrowers self-select accordingly to produce a separating equilibrium. The device of price and quantity is free to both parties, while the device of price and collateral imposes a cost of collateral on the borrower, but not the lender.

The third screening device, costly underwriting by the lender, fits mortgage and major consumer goods lending where there is little ability to use collateral, in the form of large downpayments as a screening device. Lenders use a variety of underwriting techniques to screen borrowers, and borrowers self-select according to the expected cost of credit at different lenders. Underwriting applications is costly to the lender. Formal underwriting is used in debt contracts related to purchase of housing and automobiles because it is not feasible for the lender to design contracts with many different quantity or collateral requirements that are bundled with the interest rate in order to separate borrowers. For example, unlike insurance or revolving credit cards, in the mortgage market, the borrower has a specific home purchase goal and this leaves very little room to vary the loan amount, downpayment, or collateral value. Another important reason to use costly screening is that there are fraudulent borrowers in the market who have no intention to repay the loan or who may misreport the collateral value. Consequently a lender needs to assess the collateral value, to verify income, to check employment status, and to examine credit history in order to prevent fraud.

Few papers have examined the use of lender's costly screening as a sorting device an the

environment of borrower's self-selection<sup>4</sup>. Wang and Williamson (1998) is the most notable. It is plausible that screening should not be free. And the cost of screening needed to prevent fraud will be reflected in the interest rate, which induces borrower's self-selection. The Wang and Williamson model captures the information friction in the credit market caused by ex-ante screening cost, as opposed to the ex post monitoring cost that was examined in earlier models by Townsend (1979) and Bernanke and Gertler (1989).<sup>5</sup>

Wang and Williamson (1998) derive the same no-pooling result as in Rothschild and Stiglitz (1976) although in a different model environment. In their model, there are two types of borrowers, good and bad, in a production economy. Lenders have a screening technology that allows them to identify borrower type perfectly, given a fixed expenditure for underwriting. Because applicants are aware of lender underwriting, it is only necessary to examine a proportion of applicants to deter bad risks from applying to lenders attempting to serve good borrowers. Thus screening is only applied to a fraction of applicants. Lenders in a pooling equilibrium accept all applicants and do not attempt to screen. In the separating equilibrium, the lender for the good borrowers conducts random and perfect screening sufficient to deter bad borrowers from applying. Wang and Williamson demonstrate that, if a pooling equilibrium does exist, one can always find a separating contract that is strictly preferred by the good borrowers and earns a non-negative profit to the lender. Hence the pooling equilibrium will be broken.

In addition to the general literature on organization of credit markets, mortgage markets

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<sup>4</sup> There is a relatively recent line of literature about the lender's use of active and costly screening without borrower's self selection. A primary aim of this literature is to show the negative externality in the quality of applicant's pool caused by the competing bank's screening activities (Broecker 1990, Cao and Shi 2001, Direr 2008). Gehrig and Stenbacka (2004) examined this screening externality in a dynamic setting, in which pooling equilibrium becomes possible when there is a large share of good applicants in the market. Separately, Bubb and Kaufman (2010) provided a theory of lender's cutoff rule in the mortgage market due to costly screening. These models do not have self-selection. Borrowers are passive agents who are subject to active screening by the lender.

<sup>5</sup> Ex post monitoring cost refers to the cost a lender spends to verify the outcome of a project, particularly in the case when the borrower declare bankruptcy, so that a borrower will honestly report the outcome.

have received special attention. Fixed rate mortgage products subject lenders to prepayment risk. Screening is not an effective mechanism for revealing the borrower's private information regarding the probability of prepayment. A variety of papers, including particularly Brueckner (1994) and LeRoy (1996), established that lenders optimally screen applicants for future mobility by offering contracts in which there is a tradeoff between interest rate and mortgage points. Another branch of the mortgage literature deals with the consequences of the empirical evidence that many borrowers do not default ruthlessly. This has been explained by Kau, et al (1992) by appealing to the existence of differences in default cost that are not observed by lenders. This led Brueckner (2000) to conclude that the range of mortgage products offered may be limited. Such results are a form of pooling equilibrium in that a single contract is offered to borrowers who differ default cost and hence in credit risk. However, both the literatures on points versus interest rates and ruthless default assume that lenders do not have a screening technology capable of detecting either differences in borrower mobility or default costs. The fact that the former literature concludes that points can be used to achieve a separating equilibrium and the latter that a single pooling contract will be offered, demonstrates the potential importance of asymmetric information for the organization of the mortgage market. This paper analyzes a situation in which private information may be revealed by underwriting but it assumes that lenders are offering a single type of credit contract differentiated only by interest rate.

This paper adds two features not found in previous models. First, the possibility of fraud makes it necessary for lenders to underwrite all applicants, i.e. there is no random screening strategy.<sup>6</sup> Fraudulent borrowers always have an incentive to apply as the cost of application is trivial compared to the potential gain. To a lender, the entire amount of loan approved to a fraudulent applicant becomes a loss. Second is the relaxation of the

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<sup>6</sup> The experience with low and no documentation mortgage loans illustrates the importance of fraudulent applications. Initially loss rates on these loans were actually lower than full documentation mortgages but, over time, it appears that fraudulent applicants discovered this type of credit and default losses soared above those for other types of mortgage credit.



assumption of perfect identification of all applicants screened. Even with costly underwriting, it is possible to have type I error, i.e. rejection of good risks, and type II error, failure to reject poor risks. These additional features are common in credit markets, particularly in mortgage and large consumer durable finance.

Once the screening technology is changed to be universal and imperfect, i.e. every borrower is screened and both type I and type II errors can take place, a pooling equilibrium becomes a possible outcome when the screening cost is high. The intuition is similar to the case of using collateral as a screening device. In that environment, the use of collateral imposes a cost on the good borrower. When this cost is higher than the implicit subsidy that a good borrower has to pay to the bad borrower in the pooling equilibrium, good borrowers will self-select the pooling contract and the equilibrium will be pooling. In the case of lender's costly screening, it is the lender who bears the cost. When the cost of screening is high, the specialized lender has to raise the cost of credit in the separating contract and or subject good applicants to costs of type I error. Both these effects cause good borrowers to select pooling contracts producing a pooling equilibrium.

It is well known in the literature of screening (and signaling), as pointed out in Hellwig (1985), that the set of equilibria generally depends on the equilibrium defined in the model, dubbed as refinement of equilibrium in the game theory literature. For example, in the case of financial markets, Dubey and Geanakoplos (2002) found that the equilibrium is separating in the framework of competitive pooling. Subsequently Martin (2007) refined the definition of equilibria by changing the perturbation methods in the same framework and found that a pooling equilibrium is possible. In the case of financial intermediaries, Hellwig (1986) interpretes models that produce a separating equilibrium such as Rothschild and Stiglitz (1976), Wilson (1977) and Bester (1985) as a two-stage game. At the first stage, the uninformed agents offer contracts; at the second stage, the informed agents choose among the offers. Hellwig further proposes a three-stage game based on Bester's (1985) model environment, where at the third stage, the uninformed agents have the right to

reject any contract applications from the informed agents. This, of course, changes the expectations and strategies of the informed agents at the second stage. Hellwig argues that in that three stage setting, the only stable equilibrium is pooling.

The costly screening model developed here can be regarded as a three-stage game. Lenders offer contracts of with combinations of screening intensity and interest rate first. Borrowers choose among the offers and submit loan applications. Finally, lenders conduct costly screening and have the right to reject the unqualified applicants. This paper elaborates the screening technology used in the previous costly screening models and makes it more consistent with observed market behavior. As a result of these changes, the model demonstrates that, in addition to the usual separating equilibrium, it is also possible to have pooling. This is consistent with previous literature that establishes the existence of a pooling equilibrium using the first two screening devices, the combination of price and quantity, and the combination of price and collateral. Thus it appears that a pooling equilibrium is possible in an environment where all three types of screening devices are used.

### **III. A model with fraud and costly, imperfect screening**

The model environment<sup>7</sup> has two periods. Investment takes place in period 1 and agents consume in period 2. There are four types of agents: lenders, type  $g$  borrowers, type  $b$  borrowers and fraudulent borrowers. Fraudulent borrowers have no intention to repay. They can be detected with certainty with a modest level of screening effort. In the credit market, there is a continuum of borrowers and lenders, with the measure of borrowers being strictly less than the measure of lenders, so competition drives the profit of lenders down to zero. Among the non-fraudulent borrowers, a fraction  $\alpha$  is type  $g$ , and the remaining proportion  $(1 - \alpha)$  is type  $b$ . Both lenders and borrowers are risk-neutral.

Each lender invests one unit of an investment good in period 1, either lending to a

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<sup>7</sup>As the model environment in this paper is similar to that of Wang and Williamson (1998), notation in this paper is kept the same as in their paper insofar as possible.

borrower in exchange for payment in period 2, or investing in an alternative risk-free investment project with a certain return of  $r$  units of consumption good in period 2. Borrowers have no endowment in period 1 but each has access to an investment project that can generate  $x$  units of return of consumption good for every unit of investment good in period 1. The return,  $x$ , of type  $i$  is randomly distributed along its support on  $[0,1]$  according to the cumulative probability distribution function  $F_i(x)$ , with corresponding probability density function  $f_i(x)$ . Assume  $f_i(x) > 0$ , and that  $F_g(x)$  stochastically dominates  $F_b(x)$  because that is what it means to be a good versus bad borrower.

Borrower type is private information. But each lender has access to a screening technology that allows her to observe a borrower's type. It is assumed that a borrower can contact at most one lender in period 1, but a lender may be contacted by many borrowers. As a result, there is no negative screening externality as modeled in the banking competition literature.<sup>8</sup>

Screening is costly and exhibits decreasing returns. If a lender spends  $C_o$  on each application, she can perfectly screen out fraudulent borrowers. It is likely that  $C_o$  is fairly low, because fraudulent borrowers lack valid documentation and can be more easily detected.<sup>9</sup> The cost of identifying fraudulent borrowers is the same for lenders specializing in good borrowers as it is for a pooling lender. The cost to separate good,  $g$ , from bad,  $b$ , borrowers with probability  $p_g$  is given by:

$$C = \beta(p_g - p_f)^i, \quad i \in [1, \infty], \quad p_g \in [p_f, 1] \quad (1)$$

where screening cost  $C$  is a quasi-convex function of the identification probability  $p_g$  (or screening accuracy).  $\beta$  is the parameter that influences the marginal cost of screening accuracy, or the slope of the cost curve.  $p_f$  is the identification probability that can be achieved based on the free information available in the market. Therefore, when  $p_g = p_f$   $C = 0$ . However, the presence of fraudulent applicants means that the lender will need to

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<sup>8</sup> Negative screening externality refers to the case where good borrowers receive credit and leave the market, while bad borrowers are rejected by a lenders as a result of screening and choose to stay in the market seeking credit from other lenders, the whole market will have growing share of bad borrowers. This is an important element in models by Broecker 1990, Cao and Shi 2001, Gehrig and Stenbacka 2004, Direr 2008.

<sup>9</sup> If fraudulent borrowers become organized, as was evident in substantial number of zero payment defaults (ZPDs) in 2007 and 2008, the size of  $C_o$  may become significant. All the parameters characterizing screening technology are subject to change over time.

expend some minimal screening effort to identify fraud. Let  $C_o$  be the cost of eliminating fraud, which can be done with certainty. Therefore the structure of the unit screening cost,  $C$ , experienced a lender who chooses an identification probability,  $p$ , sufficient to deter fraudulent applicants and designed to identify type  $g$  borrowers is:

$$C = \begin{cases} C_o & \text{if } p \in [p_f, p_o] \\ \beta(p_g - p_f)^i & \text{if } p \in (p_o, 1] \end{cases} \quad (2)$$

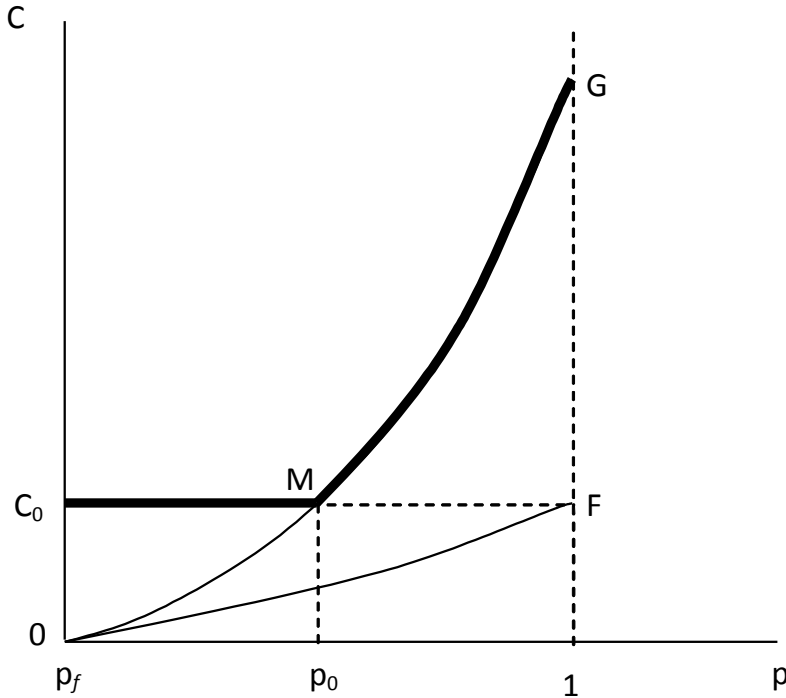
Where:  $p_o$  is the identification probability of a type  $g$  borrower when the screening cost is  $C_o$ , i.e.  $C_o = \beta(p_o - p_f)^i$ . This assumes that the screening technology used to identify fraudulent applicants also has some utility in separating good from bad applicants.

The screening cost function is illustrated in the Figure 1. The curve OG represents the cost curve of identifying type  $g$  borrowers while OF represents the cost curve of identifying fraudulent borrowers. Curve OG is much steeper than OF as the marginal cost of accuracy is much higher in differentiating between good and bad borrowers than it is between good or bad borrowers and fraudulent borrowers. Before screening, all borrowers appear identical to a lender. The cost function is nonlinear in identification probability. Clearly, once a lender identified an application as fraudulent, she would not expend further effort underwriting the application. However, any equilibrium requires that fraudulent borrowers are deterred from applying so screening effort is always greater than or equal to  $C_o$ . Given that the cost of fraudulent applications is negligible, the probability of rejection must be unity in order to deter them. Lenders recognize this and engage in sufficient screening to deter perfectly all fraudulent borrowers. Therefore, the curve  $C_o$ MG represents the screening cost of a specialized lender for type  $g$  borrowers.<sup>10</sup> The literature on screening cost functions is limited. However, Agarwal et al (2011) find that lenders initially use relatively inexpensive “hard” information and follow this with more expensive “soft” information which implies the convex cost function displayed below.

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<sup>10</sup>This definition of unit screening cost is different from Wang and Williamson's model, where unit screening cost is a fixed.. If screening is imperfect, identification probability  $p$  becomes a lender choice variable. This fits the reality of mortgage and consumer durable lending well. There is substantial evidence that screening effort varied significantly across lenders in these markets.

**Figure 1. Individual Screening Cost as a Function of Identification Probability**



Imperfect screening suggests that the effects of both type I and type II screening errors be considered. For example, if the lender for type  $g$  borrowers adopts identification probability  $p_g$  in screening, and if the identification probability is symmetric between good borrowers and bad borrowers, then the two-way screening error is described as:

$$\begin{aligned} \text{Prob}(\text{good} / \text{good}) &= p_g & \text{Prob}(\text{bad}/\text{good}) &= (1 - p_g) \\ \text{Prob}(\text{good}/\text{bad}) &= (1 - p_g) & \text{Prob}(\text{bad}/\text{bad}) &= p_g \end{aligned}$$

Universal screening is the only way that a lender can exclude fraudulent borrowers from receiving credit. In the market where all lenders spent at least  $C_0$  on screening each application, fraudulent borrowers will be deterred from applying as they know they will be screened out with certainty. If any lender spends less than  $C_0$  on screening, she will be flooded with applications from fraudulent borrowers, so lender no has incentive to deviate from this screening scheme.

Equilibrium contracts consist of a payment schedule and identification probability pairs  $[R_i(x), p_i]$ ,  $i = g, b, , x \in [0,1]$ , where  $R_i(x)$  denotes the payment made by the borrower to the lender when the return on the borrower's investment is  $x$ , and  $p_i$  denotes the

identification probability associated with the lender screening effort to reveal a borrower's type. There are two types of contracts: pooling and separating. A pooling contract is offered by a lender to all borrower types except for fraudulent borrowers. A separating contract is offered by a lender to a particular borrower type, i.e. good borrower lenders only offer contracts to those screened as good but bad borrower lenders offer contracts to anyone who is not a fraud. A pooling equilibrium is an equilibrium where both  $b$  and  $g$  borrowers are served by the same lender. Pooling of good and poor risks can be accomplished without serving fraudulent applicants but the lender must conduct sufficient screening to detect these applications.

### III.1 Pooling equilibrium

In a pooling contract with no price rationing, a lender lends to both type  $g$  and type  $b$  borrowers with the same payment schedule,  $\bar{R}(x)$ . The lender still needs to engage in a minimum level of screening activity that costs  $C_0$  to screen out fraudulent borrowers. When a lender spends  $C_0$  on screening, all the fraudulent borrowers will be deterred from applying as they know they will be rejected with certainty. A pooling equilibrium is characterized by the payment schedule  $\bar{R}(x)$  that satisfies the following properties:

$$\begin{aligned}
 &0 \leq \bar{R}(x) \leq x; x \in [0,1] \\
 &x \leq y \Rightarrow \bar{R}(x) \leq \bar{R}(y); x, y \in [0,1] \\
 &\alpha \int_0^1 \bar{R}(x) dF_g(x) + (1 - \alpha) \int_0^1 \bar{R}(x) dF_b(x) \geq r + C_0 \quad (3)
 \end{aligned}$$

where  $\alpha$  is the probability of lending to a good borrower in a pooling contract.

Condition (3) is the individual rationality (IR) constraint for the lender. It states that the expected return from the equilibrium pooling contract for a lender must be no less than the return on the alternative risk-free investment plus the minimum screening cost for excluding frauds, so that the lender can make a non-negative profit.  $\alpha$  is the fraction of type  $g$  borrowers in the credit market after excluding fraudulent borrowers. Therefore  $\alpha$  is the probability of lending to a good borrower in a pooling contract. In a competitive market with zero economic profit, this constraint is binding and the equality holds.

### III.2 Separating equilibrium

A separating equilibrium is characterized by a pair of contracts  $[R_i(x), p_i]$ ,  $i = g, b$  for each borrower type. In a separating equilibrium, the lender specialized in type  $b$  borrowers will never reject a type  $g$  borrower if a type  $g$  applies. By including type  $g$  borrowers, the  $b$  lender can lower the risk in the pool of accepted borrowers (and receive higher payments). In equilibrium, the lender specializing in type  $b$  borrowers only spends  $C_0$  on screening each application to deter fraudulent borrowers. The lender in effect grants credit to everybody who submits a loan application (as fraudulent borrowers will not apply). In other words, the identification probability  $p_b$  that the lender for type  $b$  borrowers adopts is 0. Therefore, in a separating equilibrium, equilibrium the contracts are  $[R_g(x), p_g]$  offered by the lender specialized in type  $g$  borrowers and  $[R_b(x), 0]$  offered by the lender specializing in type  $b$  borrowers. These contracts must satisfy the following conditions:

$$0 \leq R_i(x) \leq x; x \in [0,1]; i = g, b$$

$$x \leq y \Rightarrow R_i(x) \leq R_i(y); x, y \in [0,1]; i = g, b \quad (4)$$

$$p_g \int_0^1 R_g(x) dF_g(x) + (1 - p_g)\mu_g \leq \int_0^1 R_b(x) dF_g(x) \quad (4.1)$$

$$\int_0^1 R_b(x) dF_b(x) \leq (1 - p_g) \int_0^1 R_g(x) dF_b(x) + p_g\mu_b \quad (4.2)$$

$$\int_0^1 R_g(x) dF_g(x) = r + \frac{c}{p_g} \quad (4.3)$$

$$\int_0^1 R_b(x) dF_b(x) = r + C_0. \quad (4.4)$$

Conditions (4.1) and (4.2) are incentive compatibility (IC) constraints for borrowers to self-select their corresponding lenders. The left side of (4.1) is the expected cost of credit for borrower  $g$  if she seeks credit at her corresponding lender. At this lender, she has probability  $p_g$  of being correctly identified and being accepted for credit with the correct payment schedule  $R_g(x)$ , and has probability  $(1 - p_g)$  of being denied credit due to type I error. If a loan is denied, borrower  $g$  cannot fund her project because a borrower can contact only one lender per time period, thus she consumes zero, and the loss is  $\mu_g$ . This loss is a very influential parameter and varies with the nature of the transaction. If

investment opportunities cannot be stockpiled, then  $\mu_g$  is at least the loss in net return from being able to exploit the current investment opportunity.<sup>11</sup> The right side of (4.1) is the expected cost of credit if a  $g$  borrower applies to a  $b$  lender, where type  $g$  borrowers are always be accepted as the lender only engages in minimum screening to deter fraudulent borrowers. The left side of (4.1) characterizes the effect of type I screening error. Condition (4.1) is satisfied when  $R_g(x) < R_b(x)$  so that type  $g$  borrowers will never want to seek credit from lenders specialized in type  $b$  borrowers.

The left side of (4.2) is the expected cost of credit if a  $b$  borrower seeks credit at her corresponding lender, where she will always be accepted. The right side of (4.2) is the expected cost of credit if a  $b$  borrower misrepresents her type and seeks credit at the lender specializing in type  $g$  borrowers. If the borrower  $b$  is correctly identified, which happens with probability  $p_g$ , she is rejected, and her loss from not being able to borrow is  $\mu_b$ .<sup>12</sup> But with probability  $(1 - p_g)$ , she is accepted and the payment schedule of her loan is  $R_g(x) < R_b(x)$ . The right side of (4.2) thus reflects the effects of the type II screening error. Condition (4.2) determines the level of screening accuracy,  $p_g$ , such that the lender specialized in type  $g$  borrowers is able to deter type  $b$  applicants.

Conditions (4.3) and (4.4) are the individual rationality (IR) constraints for lenders. They state that the expected return to a lender from each separating contract must be equal to the return from the alternative risk-free investment opportunity  $r$  plus the average screening cost spent on each funded borrower,  $\frac{C}{p_g}$  for the type  $g$  lender or  $C_0$  for type  $b$ . Note that the imperfect screening raises the screening cost of a type  $g$  lender even if no type  $b$  applicants apply because some type  $g$ 's will be wrongly classified as  $b$ 's and rejected. This is a distinguishing characteristic of the model setup adopted here. These conditions are binding because competition drives the lender's profit to zero. Condition (4.3) is the IR constraint for the lender specialized in type  $g$  borrowers. Condition (4.4) is the IR constraint for the lender specialized in type  $b$  borrowers.

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<sup>11</sup> The loss of return is at least as large as the amount that the applicant could have expected to earn if she had sought credit from an alternative source to have the project funded but may be larger if there are costs to being unable to proceed with the project in a timely manner. In the case of mortgage finance, this could be rejection of a bid on a particularly desirable house. Clearly there are many possibilities for calibrating loss in cases where an applicant is rejected.

<sup>12</sup> As was the case with  $\mu_g$ , the loss to a bad applicant from being unable to borrow in the current period,  $\mu_b$ , is at least as large as her expected net return from borrowing at an alternative type of lender.



## IV. Existence of a pooling equilibrium?

Market equilibrium, if it exists, is either separating or pooling. If an equilibrium exists, it must be the case that each lender is making a non-negative profit and the borrowers have no incentive to deviate from the existing equilibrium contract. In previous literature, particularly in Rothschild and Stiglitz (1976) and Wang and Williamson (1998), a pooling equilibrium never exists, because if it does, a separating lender can always offer a non-negative-profit contract that makes the type  $g$  borrowers better off but does not benefit type  $b$ . In Wang and Williamson's model, in the environment of random screening with the absence of type I error, such a separating contract is achieved by lowering the probability of screening so as to lower the average screening cost of each application and make the interest rate to type  $g$  borrowers lower than the interest rate in the pooling equilibrium.

The intuition for existence of a pooling equilibrium in the model developed here is easily explained. When screening is both universal and imperfect with two-way errors, a pooling equilibrium becomes a possible outcome. In this case, the choice variable that a lender can use to adjust average screening cost of each contract is screening accuracy  $p_g$ . However,  $p_g$  becomes a very consequential variable for good applicants in the presence of type I error. When a separating good lender tries to lower  $p_g$ , the chance that a good borrower is falsely rejected increases, which imposes a cost on good borrowers seeking credit at the type  $g$  lender. As this cost becomes sufficiently high, it drives good borrowers away from the separating type  $g$  lender to a pooling lender. Accordingly  $p_g$  is bounded from below. With this lower bound on  $p_g$ , there will be a threshold for the parameter  $\beta$  in the unit screening cost function above which the screening cost will be too high for a separating contract to be profitable. Furthermore, the type  $g$  borrowers cost of rejection,  $\mu_g$ , also becomes consequential as it raises the cost of type I error. In this case, pooling becomes a possible outcome. This argument will become the basis for the proof of Proposition 1 below.<sup>13</sup>

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<sup>13</sup> There is a key difference between the probability of random screening and the probability of correct identification  $p_g$  (or accuracy), which is why random and perfect screening in Wang and Williamson's model is not equivalent to universal and imperfect screening with two-way errors, even though it is appealing to think the two as the same for modeling purposes. A low screening probability lowers the chance of a good

The remainder of the paper first presents the proof for the existence of a pooling equilibrium in the environment of universal screening with both type I and type II screening errors. Next it demonstrates that random screening with both type I and type II errors does not produce a pooling equilibrium. The proof for the non-existence of a pooling equilibrium in the environment of universal screening with only type II error illustrates the importance of type I error for the possible existence of a pooling equilibrium, formalizing the intuitive argument made above.

#### IV.1 Universal and imperfect screening with both type I and type II errors

**Proposition 1:** *If screening is universal and imperfect (including both type I and II errors), then a pooling equilibrium exists.*

Proof of the existence of a pooling equilibrium, requires the demonstration that, there exists a pooling equilibrium such that no separating contract can outperform the pooling contract; and that, when the existing equilibrium is separating, there is a case in which a pooling contract can outperform separating contracts.<sup>14</sup>

The imperfect screening technology has identification probability  $p_g$ , symmetric between good type borrowers and bad type borrowers, and allows both type I and type II error.

$$\begin{aligned} \text{Prob}(\text{good} | \text{good}) &= p_g & \text{Prob}(\text{bad} | \text{good}) &= (1 - p_g) \\ \text{Prob}(\text{good} | \text{bad}) &= (1 - p_g) & \text{Prob}(\text{bad} | \text{bad}) &= p_g \end{aligned}$$

By having unit screening cost a function of identification probability, the model allows identification probability  $p_g$  to be a choice variable in the separating contract  $[R_g, p_g]$  so that a lender can lower the average screening cost by lowering  $p_g$ .

First, suppose there exists a pooling contract in the market characterized by the

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borrower being falsely rejected, while a low screening identification probability  $p_g$  raises the chance of a good borrower being falsely rejected. That is why type I error can only take effect in a model with universal screening but not random screening

<sup>14</sup>In this environment, every borrower in the separating contract needs to be screened. Universal screening is necessary because there are fraudulent borrowers in the market whose opportunity cost or penalty of being caught cheating is very low, so only universal screening can deter them from applying. With universal screening, the optimal loan contract is still a debt contract. The proof for Proposition 2 in Wang and Williamson's paper is not affected. The debt contract arises from the lender's objective to minimize the screening identification probability  $p_g$  subject to the binding incentive compatibility constraint for bad borrowers and the zero expected profit constraint for the lender.

relations in (3) above.

In order for a separating contract  $[R_g, p_g]$  for good borrowers to outperform pooling, it must satisfy the following three conditions:

1. A zero expected profit constraint for the lender specializing in good borrowers:

$$\int_0^1 R_g(x) dF_g(x) = r + \frac{C_g}{p_g} \quad (5)$$

where the unit screening cost for the type  $g$  lender is,  $C_g = \frac{\beta(p_g - p_f)^t}{p_g}$ .

2. An incentive compatibility constraint for bad borrowers

$$r_b \leq (1 - p_g) \int_0^1 R_g(x) dF_b(x) + p_g \mu_b, \quad (6)$$

where the left side,  $r_b \equiv \int_0^1 \bar{R}(x) dF_b(x) < r + C_0$ , is the expected payment by a bad borrower in the pooling contract; the right side is the expected cost of credit of a bad borrower in the separating contract. This condition must hold so that a bad borrower does not have an incentive to deviate from the existing pooling equilibrium and she self selects to stay out of the separating contract for good borrowers. She does not have an incentive to deviate from pooling because her expected benefit from the type II error of false acceptance of bad borrowers,  $(1 - p_g) \int_0^1 R_g(x) dF_b(x)$ , is not large enough.

3. The incentive compatibility constraint for the good borrower is

$$r_g \geq p_g \int_0^1 R_g(x) dF_g(x) + (1 - p_g) \mu_g, \quad (7)$$

here the left side,  $r_g \equiv \int_0^1 \bar{R}(x) dF_g(x) > r + C_0$ , is the expected payment by a good borrower in the pooling contract; and the right side is the expected cost of credit of a good borrower in a separating contract. This constraint states that a good borrower should weakly prefer the separating contract over pooling. It also characterizes type I error as it indicates that a good borrower considers the cost of being denied credit weighed by the probability  $(1 - p_g)$  of being falsely rejected.

Solving for  $p_g$  in the incentive compatibility constraint for bad borrowers in (6) yields

$$p_g \geq \frac{r_b - \int_0^1 R_g(x) dF_b(x)}{\mu_b - \int_0^1 R_g(x) dF_b(x)} \equiv P_{DB} \quad (8)$$

Here  $P_{DB}$  is the minimum identification probability at type  $g$  lenders required to deter bad borrowers from applying, i.e. to cause them to select pooling over a separating contract. As might be expected,  $P_{DB}$  varies directly with the rate charged  $b$  borrowers in a pooling contract because higher rates at pooling lenders make application to type  $g$  lenders more attractive. It varies inversely with the cost of rejection because that is a potential cost of applying to type  $g$  lender. The relation between  $R_g(x)$  and  $P_{DB}$  follows from the condition that, for any borrowing by bad risks, it must be true that  $r_b < \mu_b$ . Given this condition  $P_{DB}$  varies inversely with  $R_g(x)$  because raising the required payment at lenders specializing in good borrowers deters applications from bad borrowers and this lowers the  $p_g$  needed to deter bad borrowers from applying to type  $g$  lenders in the hope of being misclassified.

It is informative to consider the relation among  $p_g$ ,  $R_g(x)$ , and  $P_{DB}$ . From equation (4.3) and the convexity of the screening function, we know that  $R_g(x)$  is increasing in  $p_g$ . Therefore, the lowest supply price from type  $g$  lenders, noted  $R_g(x) = \underline{R}_g(x)$ , occurs when the screening cost is at its minimum at  $[C_o, p_o]$ , i.e.  $R_g(x) = \{\underline{R}_g(x) : \arg \int_0^1 \underline{R}_g(x) dF_g(x) = r + \frac{C_o}{p_o}\}$ . Of course, this is also the lowest supply price of credit at type  $g$  lenders and hence will be associated with a large value of  $P_{PB} = P_{PB}(\underline{R}_g(x))$ . Clearly this is unlikely to satisfy equation (6) because  $p_o = p_g$  is most likely less than  $P_{DB}$  and type  $g$  lenders will need to screen more to produce a separating equilibrium, i.e. a type  $g$  lender will need to set  $p_g > p_o$  in order to deter type  $b$  applicants. As  $p_g$  rises,  $R_g(x)$  rises and  $P_{DB}$  falls. In order to have a separating equilibrium, there must be a point that satisfies equation (8), i.e. where  $p_g \geq P_{DB}$ .

Solving for  $p_g$  in the incentive compatibility constraint for good borrowers in (7) yields

$$p_g \geq \frac{\mu_g - r_g}{\mu_g - \int_0^1 R_g(x) dF_g(x)} \equiv P_{IG} \quad (9)$$

Here  $P_{IG}$  is the minimum identification probability at type  $g$  lenders required to induce good borrowers to choose a specialized lender over pooling. Obviously raising the rate available in a pooling equilibrium,  $r_g$ , lowers the minimum required probability of acceptance because it makes the pooling lender less attractive. An increase in  $R_g(x)$  raises  $P_{IG}$  because raising  $R_g(x)$  makes the separating contract less attractive compared to pooling.

As  $r_g \rightarrow \int_0^1 R_g(x) dF_g(x)$ ,  $P_{IG} \rightarrow 1$  because if the cost of credit in the pooling equilibrium approaches the price at the specialized type  $g$  lender, the probability of type I error must go to zero. In general, raising  $p_g$  reduces type I error and attracts good borrowers to the separating contract. Note the difference between the effects of higher  $p_g$  on the attractiveness of type  $g$  lenders. There is an inevitable increase in cost of credit but the improved classification accuracy attracts good borrowers who benefit from lower type I error and deters bad borrowers who experience lower gains from type II error. Finally,  $dP_{IG}/d\mu_g > 0$  because raising the loss associated with rejection makes application at specialized  $g$  lenders less attractive to type  $g$  applicants.

The lower limit of screening at  $[C_o, p_o]$  set by the need to deter fraudulent applicants determines a lower limit for  $p_g$  based on cost of supplying credit. This determines the minimum  $P_{IG} = P_{IG}(\underline{R}_g(x))$ . At the other extreme, when good applicants are identified with certainty,  $p_g = 1$ , the payment schedule in the pooling contract must equal that in the separating contract,  $R_g(x) = \bar{R}_g(x)$ .

Both constraints (8) on  $P_{DB}$  and (9) on  $P_{IG}$  must be satisfied for a separating equilibrium. Therefore the minimum  $p_g$  needed to support a separating equilibrium has to be the greater of these two constraints.

$$p_g \geq \text{Max}[ P_{DB}, P_{IG} ] \quad (10)$$

The first condition under which a separating equilibrium fails to outperform a pooling equilibrium is evident from observing that  $dP_{IG}/d\mu_g > 0$  in (9). It follows that  $P_{IG}$  increases monotonically to a limit at unity as the loss associated with rejection at a specialized type  $g$  lender rises. Consider, for example, applicants seeking mortgage credit to purchase a particular home in a market where there are other potential buyers. Rejection can easily mean loss of the ability to purchase the home to another buyer. Of course, bad borrowers always prefer a pooling equilibrium and this preference increases as  $p_g$  rises. Accordingly, for sufficiently high  $u_g$  a pooling equilibrium is preferred to a separating equilibrium by both  $b$  and  $g$  borrowers.

The second condition under which a separating equilibrium fails to outperform a pooling equilibrium involves the parameters of the screening cost function. Recall that unit screening cost  $C = \beta(p_g - p_f)^i$ . For the pooling lender, screening cost is  $C_o = \beta(p_o -$

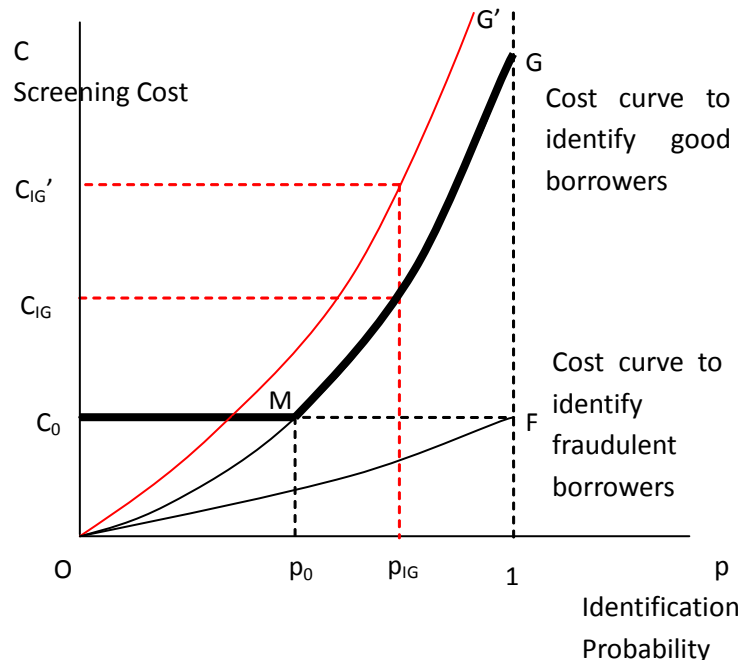
$p_f)^i$  while, for the lender specializing in type  $g$  borrowers, it is  $C_g = \frac{\beta(p_g - p_f)^i}{p_g}$ . Given  $p_g > p_o$ , it follows that  $dC_g/d\beta > dC_o/d\beta$  and  $dC_g/di > dC_o/di$  and, more importantly, that  $d(C_o/C_g)/d\beta < 0$  and  $d(C_o/C_g)/di < 0$ . Given that  $r_g/\int_0^1 R_g(x)dF_g(x)$  varies directly with  $C_o/C_g$ , it follows from equation (9) that  $P_{IG}$  varies directly with both  $\beta$  and  $i$ . This means that, for given  $p_g$ ,  $P_{IG}$  rises without limit as either  $\beta$  or  $i$  increase so that a type  $g$  borrower prefers a pooling equilibrium over a separating equilibrium.

Now suppose the existing equilibrium in the market is separating, characterized by a contract of interest rate and screening accuracy pair  $[R_g(x), p_g]$  by the lender specialized for the good borrower and  $[R_b(x), 0]$  by the lender for the bad type borrower. Is there a possible pooling equilibrium that can outperform separation?

Comparing the zero-profit constraint (5) for the lender specialized in bad borrowers in the separating equilibrium and the zero-profit constraint (3) for the pooling lender, it is clear that  $R_b(x) > \bar{R}(x)$ , as in the pooling equilibrium there is an implicit subsidy from good borrowers to bad borrowers. Consequently, bad borrowers always strictly prefer a pooling equilibrium. Therefore, for sufficiently large  $\beta$  or  $i$ , both borrower types prefer a pooling equilibrium over a separating equilibrium. QED.

The intuition behind the above proof can be illustrated in Figure 2, where the cost curve spins from  $OG$  to  $OG'$  as either  $\beta$  or  $i$  increase, and  $p_g$  is constrained by its lower bound  $p_{IG}$  as the minimum identification probability required to induce good borrowers to apply. Correspondingly, the screening cost of the lender specialized in good borrowers rises from  $C_{IG}$  to  $C'_{IG}$ . When  $C'_{IG}$  is sufficiently high, no payment schedule  $R_g(x)$  can make the specialized lender profitable or breakeven. As a result, the feasible market equilibrium is pooling.

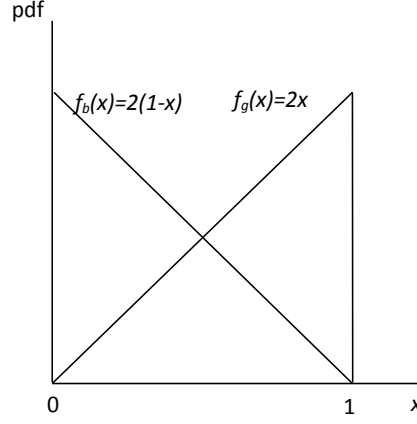
**Figure 2: Effects of Increasing Screening Cost**



### IV.2 Numerical example illustrating a feasible pooling equilibrium

It is possible to illustrate the relation between the unit screening cost function and the condition for a feasible pooling equilibrium by considering the effect of changing  $\beta$ ,  $i$ , and  $\mu_g$ , on the ability of specialized lenders to compete against a pooling equilibrium. Assume that the probability density function of project return for the good borrower is  $f_g(x) = 2x$ , and the probability density function of the project return for the bad borrower is  $f_b(x) = 2(1 - x)$ . Therefore  $f_g(x)$  first order stochastically dominates  $f_b(x)$ , and  $\int_0^1 f_i(x) dx = 1$ , as shown on Figure 3.

**Figure 3. Probability density functions of project returns**



The pooling contract characterizing the condition in equation (3) when the zero profit constraint is binding, can be rewritten as

$$\alpha \int_0^1 \bar{R}(x) f_g(x) dx + (1 - \alpha) \int_0^1 \bar{R}(x) f_b(x) dx = r + C_0 \quad (11)$$

In this example, let the payment schedule  $\bar{R}(x) = \bar{R}x$ , where  $\bar{R}$  is the fixed percentage rate of return  $x$  to be paid to the lender in the pooling contract. Substituting  $\bar{R}(x) = \bar{R}x$ ,  $f_g(x) = 2x$  and  $f_b(x) = 2(1 - x)$  into (11) yields an expression for the price of a pooling lender.

$$\bar{R} = \frac{3(r + C_0)}{(1 + \alpha)} = \frac{3(r + \beta(p_o - p_f)^i)}{(1 + \alpha)} \quad (12)$$

Assume that the real risk free interest rate,  $r = 0.02$ ,  $\beta = 0.01$ ,  $p_o = 0.6$ ,  $p_f = 0.5$ ,  $i = 2$ , the minimum screening cost  $C_0 = \beta(p_o - p_f)^i = 0.001$ , and the proportion of good borrowers  $\alpha = 0.5$ , then  $\bar{R} = 0.042$ .

After substituting  $f_g(x)$  and  $f_b(x)$  into constraints (8) and (9), the minimum  $p_g$  in (10) becomes:

$$p_g \geq \text{Max} \left[ 1 - \left( \frac{\mu_b - \frac{1}{3}\bar{R}}{\mu_b - \frac{1}{3}R_g} \right), \left( \frac{\mu_g - \frac{2}{3}\bar{R}}{\mu_g - \frac{2}{3}R_g} \right) \right] \quad (13)$$

Given  $\bar{R} = 0.042$ , and assuming  $\mu_g = 1$ , and  $\mu_b = 0.5$ , then for any non-negative  $R_g$ ,  $1 - \left( \frac{\mu_b - \frac{1}{3}\bar{R}}{\mu_b - \frac{1}{3}R_g} \right) < \left( \frac{\mu_g - \frac{2}{3}\bar{R}}{\mu_g - \frac{2}{3}R_g} \right)$ , therefore  $P_{IG}$  is the binding constraint on a separating



equilibrium and, in this numerical example:

$$p_g \geq \left( \frac{\mu_g - \frac{2}{3}\bar{R}}{\mu_g - \frac{2}{3}R_g} \right) = \frac{0.972}{1 - \frac{2}{3}R_g} = P_{IG} \quad (14)$$

Of course the zero-profit constraint of lenders specialized in good borrowers given by equation (5) determines the relation between  $R_g$  and  $p_g$  as a function of  $\beta$  and  $i$  that satisfies the zero profit constraint. Rewriting (5) and substituting in the parameter values selected for this example yields:

$$\int_0^1 R_g(x) dF_g(x) = r + \frac{\beta(p_g - p_f)^i}{p_g} \quad (5)$$

$$\frac{2}{3}R_g = 0.02 + \frac{0.01(p_g - 0.5)^2}{p_g} \quad (15)$$

Substituting (15) for  $R_g$  into (14), it is possible to solve for the value of  $p_g$  that satisfies the incentive compatibility constraint given the choice of  $\beta = 0.01$ ,  $p_o = 0.6$ ,  $p_f = 0.5$ ,  $i = 2$ , and  $\mu_g = 1$  in this numerical example. This value is  $p_g = 0.995$  and reflects the fact that this example was chosen to reflect conditions near the boundary at which the pooling equilibrium becomes feasible. Indeed, any significant increase in the chosen values of  $\beta$ ,  $i$ , or  $\mu_g$  raises  $P_{IG}$  above the zero profit  $p_g$  and type  $g$  borrowers choose the pooling equilibrium over borrowing from a specialized lender.<sup>15</sup> Of course type  $b$  borrowers also choose pooling because their rates are lower than those available from either specialized lender.

### IV.3 Effect of fraud and type I error on existence of a pooling equilibrium

It is now possible to demonstrate the importance of the fraudulent borrowers, whose detection requires universal screening, and type I error on the existence of the pooling equilibrium identified in proposition 1. This is done by showing that when there is no fraud so that lenders can employ random screening, and only type II errors are present, a pooling equilibrium does not exist. In this environment, for any given level of screening accuracy  $p_g$ , the lender specializing in good risks can vary the screening probability,  $\pi_g$ , to

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<sup>15</sup> This result follows when (15) is solved for  $R_g$ , substituted into (14) and the possibility of a value of  $1 > p_g \geq P_{IG}$  evaluated numerically.

produce a separating contract that outperforms pooling.

The intuition for the effect of random screening is straightforward. When the screening probability,  $\pi$ , is lowered for any given screening precision,  $p$ , the chance of a good borrower being falsely screened out is also lowered. Therefore, unlike the decision by the type  $g$  lender to lower costs by lowering  $p$ , lowering  $\pi$  does not impose an additional cost on good borrowers. Furthermore, the removal of type I screening error means that the potential cost of a false rejection does not deter good risks from applying to a specialized lender. Note that the subscripts on  $p$  and  $\pi$  have been dropped because only type  $g$  lenders need engage in screening.

**Proposition 2:** *In the absence of fraudulent borrowers, if screening can be random but only includes type II errors, a pooling equilibrium does not exist.*

In this environment, there are two types of probabilities: In the separating contract, the lender for good borrowers randomly conducts screening with probability  $\pi_g$ ; and each time the lender conducts screening, the screening has the potential to fail to reject a bad risk with probability  $p$ .

$$Prob(good | bad) = (1 - p) \quad Prob(bad | bad) = p$$

As defined in (2), the unit screening cost  $C$  is the same function of  $p$ . Suppose there exists a pooling contract in the market characterized by (3) except that  $C_0 = 0$ , because with no fraudulent applicants in the market there is no need to maintain a minimum level of screening.

The separating contract for good borrowers is characterized by  $[R_g(x), \pi, p]$ . In order to outperform pooling, it must satisfy three conditions:

1. the zero expected profit constraint for lenders specializing in good borrowers

$$\int_0^1 R_g(x) dF_g(x) = r + \pi C \quad (16)$$

where  $C$  is defined in (2), and  $\pi$  is the fraction of applications that are screened.

2. the incentive compatibility constraint for bad borrowers

$$r_b \leq (1 - \pi p) \int_0^1 R_g(x) dF_b(x) + \pi p \mu_b \quad (17)$$

where cost of credit to bad borrowers under pooling is  $r_b \equiv \int_0^1 \bar{R}(x) dF_b(x) < r$ .

3. the incentive compatibility constraint for the good borrower

$$R_g(x) \leq \bar{R}(x) \quad (18)$$

here it is understood that  $\mu_g > r + \pi C$  so that the good risks have an incentive to participate in the credit market.

The incentive compatibility constraint for the bad borrower is never binding because, letting  $R_g(x) = \bar{R}(x)$  so that the incentive compatibility constraint for good borrowers is just satisfied and equation (17) becomes:

$$r_b \leq (1 - \pi p) \int_0^1 \bar{R}(x) dF_b(x) + \pi p \mu_b \quad \text{or}$$

$$\int_0^1 \bar{R}(x) dF_b(x) \leq \int_0^1 \bar{R}(x) dF_b(x) + \pi p [\mu_b - \int_0^1 \bar{R}(x) dF_b(x)] \quad (19)$$

Given that  $\mu_b - \int_0^1 \bar{R}(x) dF_b(x) > 0$  or the type  $b$  applicant will not borrow, it follows that the inequality in (19) holds. Therefore, if the incentive compatible constraint in (18) for good risks is satisfied, it is also satisfied for the bad risks.

It is therefore only necessary to demonstrate the existence of a contract,  $[R_g^*, p^*, \pi^*]$ , that satisfies the constraints in (16) and (18). This is also easily established in a fashion that demonstrates the importance of the absence of fraudulent borrowers so that universal screening is not necessary. From (16) it follows that  $\{ R_g(x) : \arg \int_0^1 R_g(x) dF_g(x) = r + \pi C \}$  while the pooling equilibrium prices credit such that  $\{ \bar{R}(x) : \arg \int_0^1 \bar{R}(x) dF_g(x) > r \}$ . It follows that, for sufficiently small  $\pi$ , (16) is satisfied and  $R_g(x) \leq \bar{R}(x)$ , so that (18) is also satisfied. Therefore all three conditions for the existence of a separating contract,  $[R_g^*, p^*, \pi^*]$ , that is superior to the pooling equilibrium are satisfied and Proposition 2 is proved. **QED.**

Put another way, if a pooling equilibrium exists, specialized lenders can always find a separating contract characterized by  $[R_g^*, p^*, \pi^*]$  which can make the good borrower better off, deter bad borrowers from applying for good contracts, and earn zero expected profit for the lender. Therefore, a pooling equilibrium does not exist.

## V. Conclusions

Previous literature on insurance and credit markets has suggested that, when there is significant diversity in applicant risk that is private information which can be revealed through costly screening, the expectation is that the market will be characterized by a

separating rather than a pooling equilibrium. The result is a diversity of contract types and/or specialized lenders conducting different levels of screening. This paper demonstrates that, in a credit market model where borrowers self-select and lender's screening is costly, a pooling equilibrium can arise when lenders must use costly, universal and imperfect screening technology. The finding that pooling equilibria can exist does not require sequential interaction between lenders and borrowers as modeled in the screening literature based on other types of screening devices. The pooling equilibrium is simply a result of the high cost of screening, the possibility of both type I and II errors in classifying applicants, and the threat of fraudulent applicants which prompts universal screening. All of these possibilities are actually certainties in many credit markets, particularly the market for residential mortgages and financing for consumer durables.

The insight that pooling equilibria are possible has important implications for the organization of credit markets. It provides another explanation for cases in which credit markets pool borrowers with different risks. For example, it can shed light on the long-standing debate in the mortgage market regarding whether the mortgage credit should be supplied by pooling all qualified borrowers pooled in a conventional loan market or borrowers should be separated into A, Alt-A and subprime types that all served by different lenders.

The model also suggests that, when either screening technology or the activities of fraudulent applicants change, market equilibrium may switch from pooling to separating or back again. This challenges the presumption that the organization and business models found in credit markets should be stable. Over the past twenty years, there have been dramatic changes in underwriting methods in consumer credit, particularly mortgage markets. Business models, such as low documentation lending, were provided in a separating equilibrium. Subsequently, it appears that a rise in fraudulent applications changed the viability of the low documentation loan market. It now appears that lenders are making all applicants provide substantial documentation, even if that is costly for them. Hopefully models in which screening costs are substantial but variable, the likelihood of falsely rejecting good risks is not zero, and the supply of fraudulent borrowers varies will allow a more complete model of possibilities for organizing the market for credit.

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