

# PRODUCTION STAGING: MEASUREMENT AND FACTS

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## Abstract

Has production become more vertically fragmented? To answer this question, this paper develops two measures of production staging using input-output tables. These measures exhibit significant variations across industries and time for the US. Interestingly, production has become less vertically fragmented over the past 50 years. An important part of this decrease reflects a shift of value-added towards downstream industries which now contribute to a larger fraction of final goods value. International trade has only marginally dampened the overall decrease in fragmentation. This paper also suggests alternative applications of these indexes to investigate patterns of specialization along production chains.

**Keywords:** fragmentation of production, vertical linkages, vertical specialization.

**JEL Classification:** F10, F14, L16, L23, O14.

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# 1 Introduction

Recent work has documented the increasing complexity of production chains, with the examples of iPods, airplanes or cars. In particular, production has become more fragmented across countries (Hummels, Ishii and Yi, 2001, Johnson and Noguera, 2012), associated with a large growth in intermediate goods trade. Yet, little is known about the fragmentation of production across plants within countries. How long are production chains on average? Is production more fragmented now than it was decades ago? Production staging and the number of plants sequentially involved in production chains (henceforth referred to as vertical fragmentation) matter for several key issues in trade and other economic phenomena. As trade costs decline, gains from trade are magnified when production is, or can be, fragmented: not only can consumers import goods at a lower price, but producers can reduce costs by importing inputs at lower prices as well. Similarly, vertical linkages and the possibility of fragmented production constitute one of the main sources of gains from agglomeration according to Marshall.<sup>1</sup> Economic development has also put a traditional emphasis on the role of vertical linkages (Hirshman, 1955), formalized more recently with the “O-ring” theory (Kremer 1993, Jones 2010).

In this paper, I provide new quantitative analyses of the length of production chains, the evolution of production staging over time, and its determinants. I develop two simple measures to reflect: i) the number of production stages embodied in each product;<sup>2</sup> ii) the average number of stages between production and final consumption. These two different indexes provide complementary information on the position of each product along value chains. In particular, the first index corresponds to a weighted average of the number of plants sequentially involved in the production of a certain good, where the weight is the value that has been added at each stage. I show that these indexes have simple structural interpretations and are closely linked to traditional concepts of backward and forward linkages. I also examine aggregation properties of these two indexes and to what extent industry-level data can provide information on fragmentation across plants within and between industries. Moreover, in a closed economy, I find that a weighted average of each of these two indexes across all sectors equals the ratio of total gross output to value added, thereby offering a novel interpretation of this ratio.

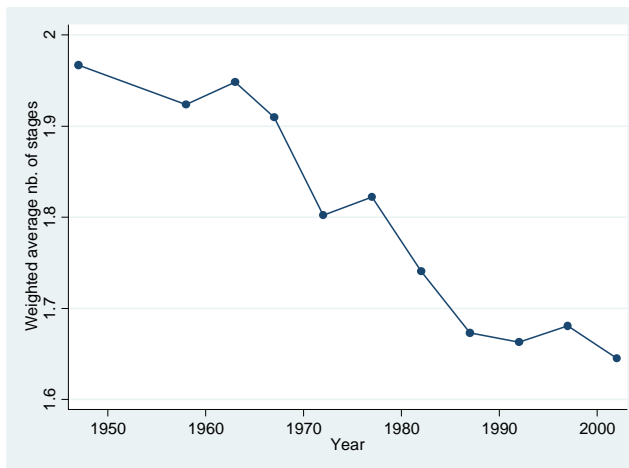
I calculate these measures of vertical fragmentation for the US using benchmark input-output tables from the Bureau of Economic Analysis for periods covering 1947 to 2002 (aggregate sectors) and 1967 to 1992 (6-digit product level). I find that production chains are short on average and that most of the value added comes from later stages: the weighted average of

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<sup>1</sup>Confirmed by Ellison, Glaeser and Kerr (2010).

<sup>2</sup>Here, stages correspond to plants. This definition may differ from a task-level approach where each stage could be associated with one task. It may also differ from a purely international perspective where “fragmentation” may refer to the fragmentation of production across countries (as in Johnson and Noguera, 2012).

Figure 1: Aggregate measure of vertical fragmentation (tradable goods excluding petroleum)



the number of production stages is smaller than 2 for the aggregate economy. Both indexes of fragmentation exhibit large variations across industries. In particular, I find that the number of embodied stages is negatively correlated with product specificity, R&D intensity, skill intensity and dependence on external finance, but does not seem to depend significantly on industry concentration (either proxied by the share of the largest firms in industry production or the Herfindahl Index).

The main and perhaps most surprising finding of the paper is that the weighted-average number of production stages has been decreasing by more than 10% over the past 50 years. While this decrease can be partly explained by the increasing share of services in total production, I find that the weighted-average number of production stages has also decreased for primary and manufacturing industries (“tradable” goods). Figure 1 plots this evolution, aggregating over all tradable goods excluding petroleum.<sup>3</sup>

Since the main measure of fragmentation captures an average weighted by value added, changes in relative price of intermediate goods versus final goods may potentially explain changes in this measure (holding quantities fixed). Indeed, swings in oil prices may explain short-term changes in observed fragmentation by magnifying the weight put on early stages. Over the long term, however, I show that changes in relative prices of commodities and intermediate goods do not explain the overall observed decline.

I also specifically investigate the role of trade. The decrease in the overall fragmentation of production remains puzzling since it coincides with the reorganization of supply chains across

<sup>3</sup>In appendix section C, I construct alternative indexes of vertical fragmentation to capture: i) an unweighted average of the number of production stages; ii) the dispersion of value added along production chains. These indexes also exhibit a decline over time.

borders.<sup>4</sup> We can expect that the large decline in transport costs over the past decades has provided new opportunities. I find indeed that increased import penetration induced an increase in vertical fragmentation, suggesting that foreign outsourcing might not just be a substitute to domestic outsourcing. This effect is small however, and not robust to instrumenting by transportation costs or tariff declines.

Perhaps the most intuitive way to understand the main finding of the paper is to look at the reallocation of value-added along production chains. I provide evidence of a large and significant shift over time of value-added towards industries that are closer to final demand (i.e. more downstream). In other words, early stages now contribute less to the final value of production, whereas more value is added at later stages. Industry characteristics can shed light on this shift of value-added. In particular, industries that are more intensive in advertising, in skilled labor and less intensive in capital have experienced a larger growth rate and are also relatively more downstream. Overall, such industry characteristics can explain about half of the shift of value-added towards downstream industries.

Furthermore, trade data suggest that this trend is global. I find evidence that the value of multi-lateral trade flows has grown faster in downstream industries relative to upstream industries (even if we omit trade to and from the US). This finding is similar to the shift of US value-added towards final stages and shows that this trend seems common to other countries.

This paper belongs both to the trade and industrial organization literature. Since the possibility to fragment production affects trade patterns and the gains from trade (Grossman and Rossi-Hansberg, 2008), it is important to measure the extent of the fragmentation of production. Empirical evidence provides various examples of global supply chains (e.g. Feenstra, 1998) and document large trade flows in intermediate goods (Yeats, 2001, Campa and Goldberg, 1997). In comparison, my paper aims at capturing the fragmentation of production across plants instead of fragmentation across borders. There is of course a strong connection: when production can be fragmented within borders it is also more likely to be fragmented across borders. The decision to fragment production within borders remains however largely underexplored.<sup>5</sup>

In this paper, I also discuss and provide alternative uses of the two measures of fragmentation to examine trade patterns. I show that developed and developing countries tend to specialize at different stages along the value chain. In particular, my results suggest that richer countries such as the US have a comparative advantage: i) in goods that involve fewer production stages

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<sup>4</sup>Note that the production staging index developed here accounts for both foreign and domestic sourcing. A pure substitution between foreign and domestic sources would not affect the index.

<sup>5</sup>A notable exception is Fort (2011) who examines the decision to fragment production (domestically and internationally) in a cross section of US plants in 2007. In all industries, she finds that most firms *do not* fragment their production, even domestically. This supports my results that production is not highly fragmented vertically. The data however do not allow her to examine the evolution of fragmentation over time.

and ii) in goods that are closer to final demand. Previous indexes on vertical specialization describe the use of imported inputs in exported goods or the value-added content in trade (e.g. Hummels, Ishii and Yi, 2001, Johnson and Noguera, 2010), but are not informative about the position of traded goods along the value chain and their sorting across countries.<sup>6</sup>

This paper also relates to several trends within the industrial organization literature. Firstly, it contributes to analyses of input-output tables pioneered by Leontief (1941). This literature has traditionally examined inter-industry “linkages” and the propagation of shocks across industries and regions (Miller and Blair 2009). Instead, I show how input-output matrices can provide very interesting information on the number of plants involved sequentially in production chains and quantify the relative position industries along production chains.<sup>7</sup> To my knowledge, this is the first paper to document a decrease in vertical fragmentation and a shift of value added towards downstream stages.

Secondly, it relates to an extensive amount of work in industrial organization on the make-or-buy decision and the determinants of vertical integration (see Lafontaine and Slade, 2007, for a survey of previous empirical works). Within this literature, many studies take as given the decision to source from a supplier, and focus on the ownership structure, i.e. on the decision to integrate this supplier or not. However, as documented by Hortacsu and Syverson (2011), most domestic shipments occur between two independent firms while plants within the same firm do not trade much among themselves. Hence it may be just as important to examine the decision to source inputs from within the same plant vs. from another plant, as reflected by the index developed here.<sup>8</sup>

The remainder of the paper contains four sections. Section 2 defines the key indexes and describes their properties. Section 3 describes the data. Section 4 presents descriptive statistics and the main empirical results. Section 5 presents another application and Section 6 concludes.

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<sup>6</sup>Additional findings on the role of institutions and focusing on the second index (distance to final demand, also referred to as “upstreamness”) are further described in Antràs, Chor, Fally and Hillberry (2012). Note that Fally (2012) precedes Antràs, Chor, Fally and Hillberry (2012) as well as Antràs and Chor (2012).

<sup>7</sup>The work on “average propagation length” (Dietzenbacher and Romero 2007, Bosma, Dietzenbacher and Romero 2005) also provides a step in this direction. See also Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2011), who examine the network structure of intersectoral linkages and its role in the propagation of shocks.

<sup>8</sup>Various indexes have been used to measure the extent of vertical integration such as firm size or the ratio of value added to gross output (Adelman, 1955). The closest index related to this paper is the “Vertical Industry Connection Index” and similar indexes of vertical integration that take higher values when a firm owns a plant producing goods in an industry having strong make-buy relationship according to the input-output table (e.g., automobile manufacturing and steel) as in Maddigan (1981), Hitt (1999), Fang and Lan (2000), Acemoglu, Johnson and Mitton (2007), Acemoglu, Aghion, Griffith and Zilibotti (forthcoming). The later approach has several caveats however. The first is that it requires detailed plant-level data which makes it difficult to study the evolution of an entire economy over an extended period of time. A second caveat is that it is sensitive to the product classification, especially if inputs and outputs are classified in the same category. Another caveat is that this index is based on ownership structure rather than actual shipments of intermediate goods (this index can take a high value even if these plants do not actually trade).

## 2 Definitions and properties

### 2.1 Embodied production stages: index $N$

In this section, I define two measures  $N_i$  and  $D_i$  defined by industry or product<sup>9</sup> to characterize the position along production chains. For each product  $i$ , I define:

- i)  $N_i$  to reflect how many stages on average enter the production of  $i$  (average number of stages embodied in good  $i$ ). This corresponds to a weighted-average number of plants involved sequentially in the production of  $i$ .
- ii)  $D_i$  to measure how many plants on average this product will go through (e.g. by being assembled with other products) before reaching final demand. In other words, it captures the distance to final demand in terms of production stages.<sup>10</sup>

To construct  $N_i$ , I rely on information provided by input-output tables. In particular, I use data on the amount (value) of product  $j$  used to produce one dollar of goods  $i$ , which I denote by  $\mu_{ij}$ . I define this index recursively: the average number of production stages embodied in a good depends on how many stages are embodied in each intermediate good. Using these  $\mu$ 's, I implicitly define  $N_i$  for each industry  $i$  by:

$$N_i = 1 + \sum_j \mu_{ij} N_j \quad (1)$$

This provides one equation for each industry. This system of linear equations generally has a unique solution that characterizes  $N_i$ .<sup>11</sup>

If production does not require any intermediate goods, the measure of fragmentation  $N$  equals one. If production relies on a particular intermediate good, the measure of production stages  $N$  depends on how important intermediate goods are in the production process and on how many production stages are needed to produce these intermediate goods.

Note that, in a special case where  $N_j = N_i$  for all inputs  $j$  entering the production of good  $i$ , the index  $N_i$  would be equal to the gross-output-to-value-added ratio. This GO-VA ratio has previously been used as a measure of vertical fragmentation at the industry level.<sup>12</sup> In

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<sup>9</sup>While the US input-output classification after 1967 is precise enough to name each category as a “product”, I will henceforth refer to  $i$  indifferently as an industry or as a product. For convenience, time subscripts are dropped in this section and will be added in the empirical section.

<sup>10</sup>In Antras *et al.* (2012) we refer to this index as a measure of “upstreamness”.

<sup>11</sup>This measure of production stages corresponds to the sum of “total requirement” coefficients for a given industry. As a corollary of the Perron-Frobenius theorems for non-negative matrices, this system has a unique solution if  $\sum_j \mu_{ij} < 1$  for all  $i$  (this condition is always satisfied in practice). By inverting this system of equations, we obtain the (transposed) matrix of total requirements.

<sup>12</sup>See for instance Adelman (1995), Woodrow (1979), Macchiavello (2009).

general, though,  $N_i$  differs from the gross-output-to-value-added ratio and better accounts for inter-industry linkages when  $N_j \neq N_i$  for a significant fraction of intermediate goods  $j$ .

Another way to understand the intuition behind this index is to decompose output into slices of value-added. Let us denote by  $V_i$  the total value-added of industry  $i$  (gross output minus intermediate goods purchase). By construction, we have the following accounting equality:  $\frac{V_i}{Y_i} + \sum_j \mu_{ij} = 1$ . We can then see that a fraction  $v_i^{(1)} = \frac{V_i}{Y_i}$  of the value of output has “gone through” only one stage since it has been added within the plant.

Then, looking more closely at intermediate goods, a fraction  $v_i^{(2)} = \sum_j \mu_{ij} \frac{V_j}{Y_j}$  of output value comes from first-tier suppliers and has gone through 2 stages (including the value added by first-tier suppliers within the same industry  $i$ ). Similarly, a fraction  $v_i^{(3)} = \sum_{j,k} \mu_{ij} \mu_{jk} \frac{V_k}{Y_k}$  of the value has gone through 3 stages (i.e. comes from suppliers of first-tier suppliers), and so forth. We can thus decompose each dollar of output  $i$  into different slices of value-added corresponding to different stages along the production chain:

$$1 = \frac{V_i}{Y_i} + \sum_j \mu_{ij} \frac{V_j}{Y_j} + \sum_{j,k} \mu_{ij} \mu_{jk} \frac{V_k}{Y_k} + \dots = \sum_{n=1}^{\infty} v_i^{(n)}$$

where  $v_i^{(n)}$  denotes the fraction of output value going through  $n$  stages. This fraction  $v_i^{(n)}$  can be defined recursively by  $v_i^{(n+1)} = \sum_j \mu_{ij} v_j^{(n)}$ , with  $v_i^{(1)} = \frac{V_i}{Y_i}$ . Based on this decomposition, we obtain the following result:

**Proposition 1** *If  $N_i$  is defined recursively as in equation (1) and  $v_i^{(n)}$  is defined as above, then:*

$$N_i = \sum_{n=1}^{\infty} n v_i^{(n)}$$

*In other words,  $N_i$  is the average number of stages to produce good  $i$  weighted by the share  $v_i^{(n)}$  of value added at each stage  $n$  ( $n = 1$  being most downstream).*

The proof is provided in the appendix section. Hence the index  $N_i$  can be reinterpreted as the average number of stages involved in the production chain, weighted by the value added at each stage.<sup>13</sup> Note that an input coming from a different plant (a supplier) counts as a different stage even if this input is classified in the same industry as the output.

To better grasp what  $N_i$  is measuring with respect to trade, firm ownership and the type of integration, several comments are in order:

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<sup>13</sup>In appendix section C, I examine alternative indexes to try to capture: i) the unweighted number of production stages; ii) the dispersion of value added along the chain, inspired from the Herfindahl Index.

Snakes or spiders?<sup>14</sup> While this measure aims at capturing the sequential nature of production, it obviously does not reflect all dimensions of complexity of production chains. In particular,  $N$  and  $D$  depend on the number of plants involved *sequentially* (production stages) but do not depend on the number of plants involved *in parallel*. Suppose that, among other intermediate goods, the production of good  $i$  relies on inputs  $j$  and  $j'$  with  $N_j = N_{j'}$ . Index  $N_i$  for good  $i$  would depend on the sum  $\mu_{ij}N_j + \mu_{ij'}N_{j'} = (\mu_{ij} + \mu_{ij'})N_j$  and would not depend on whether inputs  $j$  and  $j'$  are sourced from two different plants or from the same plant as long as the total use  $(\mu_{ij} + \mu_{ij'})$  remains the same. This point is further detailed and illustrated in the appendix.

Plants or firms? When the input-output table is constructed from plant-level data (as is the BEA I-O table for the US), this index reflects the fragmentation of production across plants independently from the ownership structure (i.e. does not depend on whether suppliers are affiliated or not).<sup>15</sup> Note that, according to Hortacsu and Syverson (2011), shipments across plants belonging to the same firm account for only a very small fraction of total shipments. It suggests that similar results would be obtained if within-firm transactions were excluded.

Foreign or domestic sourcing? Index  $N_i$  does not depend on the share of imported inputs in intermediate goods purchases as long as products of the same classification require the same number of production stages abroad as domestically.<sup>16</sup> Here I implicitly assume that production of input  $j$  is associated with the same measure  $N_j$  whether it is imported or produced domestically, taking the US as the benchmark. Formally, if we differentiate input usage into domestic  $\mu_{ij}^D$  vs. foreign purchases  $\mu_{ij}^F$ , the sum of these two coefficients correspond to the observed input-output coefficient  $\mu_{ij} = \mu_{ij}^D + \mu_{ij}^F$ . Ideally, if we denote by  $N_i^D$  and  $N_i^F$  the weighted average number of production stages required to produce goods  $i$  from domestic and foreign sources respectively, we would like to define  $N_i^D$  by the following recursive equation:

$$N_i^D = 1 + \sum_j \mu_{ij}^D N_j^D + \sum_j \mu_{ij}^F N_j^F$$

Assuming that  $N_j^F = N_j^D = N_j$ , we obtain the same equality as in equation (1):

$$N_i = 1 + \sum_j (\mu_{ij}^D + \mu_{ij}^F) N_j = 1 + \sum_j \mu_{ij} N_j$$

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<sup>14</sup>Baldwin and Venables (2010) classify production chains into “snakes” and “spiders”; my index captures the length of snakes and is indifferent to the number of a spider’s legs.

<sup>15</sup>A similar point has been made by Woodrow (1979) about the value-added-to-gross-output ratio: transactions are recorded in the input-output table even if it involves two plants owned by the same firm. It is however difficult to track intra-firm transactions between plants.

<sup>16</sup>Input-output tables generally account for both imported and domestically produced inputs. The BEA tables incorporate the use of imports. However, they do not provide information on the share of imports.



This also means that this index does not differentiate between foreign sourcing (offshoring) and domestic sourcing, as long as both types of transactions occur across plants. If there is only a substitution between domestic and foreign sourcing, there is no effect of trade on index  $N_i$ . There is an effect only if foreign sourcing is a substitute to in-house (within-plant) production.

## 2.2 Distance to final demand: Index $D$

Whereas  $N_i$  reflects the number of stages *before* obtaining good  $i$ , an alternative measure  $D_i$  can be constructed to reflect the number of production stages between production of good  $i$  and final demand. For each product  $i$ , now we need to know the share of its production that is used as intermediate goods in industry  $j$ . We denote this coefficient by  $\varphi_{ij}$ . In a closed economy, this coefficient  $\varphi$  satisfies:

$$\varphi_{ij} = \frac{Y_j \mu_{ji}}{Y_i}$$

where  $Y_i$  stands for both the demand for good  $i$  and the supply of good  $i$ . In an open economy, part of the local demand is met by imports while a fraction of the local production is exported. Assuming that the share of production that is purchased by industry  $j$  is the same whether the good is internationally traded or not, then  $\varphi_{ij}$  should satisfy:

$$\varphi_{ij} = \frac{Y_j \mu_{ji}}{Y_i + M_i - X_i}$$

where  $Y_i$  stands for the value of production of good  $i$ ,  $M_i$  for imports and  $X_i$  for exports. The denominator  $Y_i + M_i - X_i$  is total demand (absorption) of good  $i$  in the country, and thus  $\varphi_{ij}$  is the fraction of this demand that corresponds to intermediate input demand from industry  $j$ .<sup>17</sup>

We can now use these coefficient  $\varphi_{ij}$  in the same way as for input-output coefficients  $\mu_{ij}$ . For each product  $i$ , we define the “distance to final demand”  $D_i$  by:

$$D_i = 1 + \sum_j \varphi_{ij} D_j \tag{2}$$

Again, it defines one equation for each industry. This system of linear equations generally has a unique solution.

The intuition behind this index  $D$  mirrors the intuition for  $N$ . While  $N$  reflects the number of production stages embodied in production,  $D$  reflects the number of stages that have yet to be achieved before reaching final demand. In the extreme case where the entire production of this good is used as final consumption, this measure of distance to final demand is one. If part

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<sup>17</sup>Note that this open-economy adjustment is consistent with situations where countries specialize at different stages of production. More details on open-economy adjustments are provided in Antràs, Chor, Fally and Hillberry (2012) where we further examine specialization patterns across a broad range of countries.

of the production is used as an intermediate good, this index is greater than 1 and depends on the share of production used as intermediate good and as well as the number of stages separating the corresponding downstream industry from final demand.

We should also note that it improves on a simple classification of parts versus final goods. As noted by Hummels *et al.* (2001), goods such as tires can be used as both intermediate goods and final goods. Index  $D_i$  does not suffer from this drawback since it more precisely account for the share of output being purchased by final consumers and producers.

A simple example to grasp the intuition behind index  $D$  is the one of a purely sequential production chain (“snake”). Suppose that a number  $S$  of plants produce sequentially, where each plant is indexed by  $n = 1, \dots, S$  from the most downstream plant ( $n = 1$ ) to the most upstream plant ( $n = S$ ). In this example, plant  $n$  only sells to plant  $n - 1$ . Simple calculations show that index  $D$  corresponds to the position of plant  $n$  on the chain:  $D_n = n$ .

As we show in Antràs, Chor, Fally and Hillberry (2012), this intuition can be generalized to more complicated cases where the whole production in a particular stage is not necessarily sold to a unique plant. In particular, we can decompose output in a similar way as for Proposition 1 above. The above definition of  $D_i$  is equivalent to constructing a weighted average of the number of stages between an industry’s output and final demand:

$$D_i = \sum_{n=1}^{\infty} n s_i^{(n)}$$

weighted by the share of output  $s_i^{(n)}$  of industry  $i$  that goes through  $n$  stages before reaching final demand. In particular,  $s_i^{(1)}$  corresponds to  $1 - \sum_j \varphi_{ij}$ , the fraction of output of industry  $i$  that goes to final demand.  $s_i^{(2)}$  corresponds to the  $\sum_j \varphi_{ij}(1 - \sum_k \varphi_{jk})$ , i.e. the fraction of output of industry  $i$  that is purchased as inputs for the production of goods that are then sold to final consumers, etc. The fraction  $s_i^{(n)}$  can be formally defined by  $s_i^{(n+1)} = \sum_j \varphi_{ij} s_j^{(n)}$ , with  $s_i^{(1)} = 1 - \sum_j \varphi_{ij}$ .

## 2.3 Structural interpretations

While the index  $N_i$  has an intuitive interpretation, the link with previous models and more structural interpretations is not straightforward and depends on the structure of production. This section motivates this index from a more structural standpoint, linking  $N_i$  to: i) cumulative trade costs along production chains; ii) the elasticity of prices to productivity; iii) the elasticity of output to productivity; iv) the gains from trade in a Ricardian framework.

**i) Cumulative transport costs:** As shown by Yi (2010), vertical specialization and multiple border crossings along production chains magnify the effect of transport costs on trade. A

similar effect can apply to domestic trade between plants.

To illustrate the relevance of index  $N_i$ , let us examine cumulative trade costs  $T_i$  that are being paid for the production of good  $i$ , assuming that there is a transport cost  $\tau$  to ship one dollar of production between any two plants. While  $\tau$  is paid for transporting product  $i$ , a transport cost  $\mu_{ij}\tau$  has been incurred for intermediate goods  $j$  used to produce one dollar of good  $i$ . If we also account for transport costs in more upstream industries, we find:

$$T_i = \tau + \sum_j \mu_{ij} T_j$$

$\frac{T_j}{\tau}$  satisfies the same recursive definition as  $N_i$  (equation 1) and thus  $T_i = \tau N_i$ . This implies, for instance, that an increase in transport costs  $\tau$  has a larger effect on high- $N$  industries.

**ii) Price multiplier:** Now, let us consider an economy with  $J$  goods, characterized by the following production functions:

$$Q_i = Z F_i(Q_{i1}, Q_{i2}, \dots, Q_{iJ}, L_i)$$

where  $Z$  is a economy-wide productivity term, and  $F_i$  is a good-specific production function with constant returns to scale,  $Q_{ij}$  the quantity of good  $j$  used in the production of good  $i$ , and  $L_i$  the amount of labor used for  $i$ . In this general setting, after normalizing wages to unity, we obtain that the elasticity of prices to economy-wide productivity shocks corresponds to the fragmentation index  $N_i$  (see proof in appendix):

$$\frac{\partial \log P_i}{\partial \log Z} = -N_i$$

In the spirit of the O-ring theory (Kremer, 1993) and a more recent model of Costinot, Vogel and Wang (2012), we could further assume that mistakes are made at each stage of production and that mistakes destroy both production and inputs used in production, so that productivity is determined as  $Z = e^{-\lambda}$  where  $\lambda$  is the Poisson rate of arrival of mistakes. In this setting, the semi-elasticity of prices to the rate of mistakes  $\lambda$  equals  $-N_i$ .

We can further examine how a change in productivity affects welfare. In this general framework, we obtain that the effect of productivity on welfare depends on the average of index  $N_i$  weighted by the share of each good in final consumption:

$$\frac{\partial \log e}{\partial \log Z} = -\frac{\sum_i C_i N_i}{\sum_i C_i} \tag{3}$$

where  $e$  denotes the expenditure function for a given level of utility.<sup>18</sup>

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<sup>18</sup>Since wages are normalized to unity, a decrease in  $e$  reflects a increase in welfare.

**iii) Output multiplier:** While the role of index  $N$  as a multiplier for prices holds in a general framework, the link between productivity and output depends on the structure of the economy and the shape of production functions. In the appendix, I illustrate the role of  $N_i$  and  $D_i$  in two cases: with Cobb-Douglas and with Leontief production functions.

In a first simple case where production functions and preferences are Cobb-Douglas functions of goods  $i$ , the elasticity of output in industry  $i$  to economy-wide productivity shocks  $Z$  correspond to the index  $N_i$ :

$$\frac{\partial \log Q_i}{\partial \log Z} = N_i$$

This simple case formalizes the link with “total output multipliers” that are well-known in the input-output literature (Chenery and Watanabe 1958, Rasmusen 1956).

In a second case where production functions and preferences are Leontief, the elasticity of output in industry  $i$  to economy-wide productivity shocks  $Z$  now depends on the index  $D_i$ :

$$\frac{\partial \log Q_i}{\partial \log Z} - \frac{\partial \log Q_j}{\partial \log Z} = D_i - D_j$$

In other words, a change in productivity (or in the rate of mistakes as in Costinot et al 2012), the effect on output is the largest for the more upstream goods, i.e. goods that are the “furthest” from final demand. In both cases, we can see that the position on the production chain (measured by  $N_i$  or  $D_i$ ) determines the sensitivity of output to productivity shocks.

**iv) Welfare gains multiplier:** As motivated in the introduction, the fragmentation of production magnifies the gains from trade and economic integration. This intuition can be formalized by taking the same approach as in Arkolakis, Costinot and Rodriguez-Clare (2012).

For simplicity, let us assume that we have several industries  $i$  and that production in each industry is as in Eaton and Kortum (2002): markets are perfectly competitive, productivity draws for each variety follow a Frechet distribution, labor is the only factor of production, trade flows satisfy a gravity equation, and demand is CES (see Arkolakis et al 2012, for more details on the underlying assumptions of the competitive case). If there is only one production stage, and if the wage at home is normalized to unity, Arkolakis et al (2012) show that the change in the price index is given by:

$$\hat{P}_i = \frac{\hat{\lambda}_i^{dom}}{\theta_i}$$

where  $\lambda_i^{dom}$  refers to the fraction of goods that are *not* imported (in the consumption of goods in industry  $i$ ) and where  $\theta_i$  is both the coefficient of dispersion of the Frechet distribution of production in industry  $i$  and the elasticity of trade to trade costs in this industry.

If we extend their model by allowing for inter-industry linkages, assuming Cobb-Douglas

production functions with coefficients  $\mu_{ij}$  for the share of input  $j$  in the production of good  $i$ , the expression above becomes:

$$\hat{P}_i = \frac{\hat{\lambda}_i^{dom}}{\theta_i} + \sum_j \mu_{ij} \hat{P}_j$$

If we further assume that the change in import penetration is the same in all industries ( $\hat{\lambda}_i^{dom} = \hat{\lambda}^{dom}$ ) and that  $\theta_i = \theta$  is also constant across industries, then we obtain that the change in the price index  $P_i$  is proportional to the average number of production stages as measured by  $N_i$ :

$$\hat{P}_i = \frac{\hat{\lambda}^{dom}}{\theta} \cdot N_i$$

The intuition is simple. When a country opens to trade, not only consumers can have access to cheaper foreign goods but domestic producers can also reduce their costs by importing cheaper inputs. This magnifies the gains from trade in industries with multiple production stages.

## 2.4 Index for the aggregate economy

Before turning to the data and computing these indexes, I show that these two indexes satisfy interesting and useful aggregation properties.

While both measures  $N_i$  and  $D_i$  are defined by product, we need to characterize the aggregate economy. For aggregation purposes, the key is to consider the appropriate weights to compute averages. With these two indexes at hand, we can compute:

- i) The number of production stages embodied in final goods (using index  $N_i$ ), averaged across all goods purchased by final consumers. For this purpose, a natural weight is the total value of good  $i$  used for final consumption. As shown in equation (3), this would be also a natural weight to examine welfare implications.
- ii) The average number of stages between production and final consumption (distance to final demand), making use of index  $D_i$ . For this purpose, a natural weight is the value added by industry  $i$ .

I denote by  $C_i$  the value of final consumption of good  $i$ . It satisfies:  $C_i = Y_i - \sum_j \mu_{ji} Y_j + M_i - X_i$ . It corresponds to total production minus the amount used as intermediate goods by domestic plants, plus net imports. Similarly, I denote by  $V_i$  the value added by industry  $i$ , which equals production of good  $i$  minus intermediate goods use:  $V_i = (1 - \sum_j \mu_{ij}) Y_i$ .

**Closed economy.** In a closed economy, net imports equal zero and  $C_i = Y_i - \sum_j \mu_{ji} Y_j$ . Using accounting equalities and the definition of the index (see proof in the appendix), it turns out

that the weighted average of both measures of fragmentation equal the ratio of gross output to value added:

**Proposition 2** *For a closed economy, the average of the number of production stages  $N_i$  across all industries weighted by their contribution to final demand  $C_i$  equals the average distance to final demand  $D_i$  weighted by value added  $V_i$ , and both equal the ratio of total gross output over GDP:*

$$\frac{\sum_i C_i N_i}{\sum_i C_i} = \frac{\sum_i V_i D_i}{\sum_i V_i} = \frac{\sum_i Y_i}{\sum_i V_i}$$

This result provides an interesting interpretation of the gross-output-to-value-added ratio in an economy: it equals the average number of production stages and reflects the fragmentation of production in the economy (note that this is not the case at the industry level).

**Open economy.** In an open economy, there is no longer equality between supply and demand for intermediate goods by domestic industries (net imports  $M_i - X_i$  no longer equal zero). In this case, the weighted average of the number of production stages is no longer equal to the ratio of gross output to GDP, and no longer equal to the average distance to final demand weighted by value added. Interestingly, the differences between each index and the GO/VA ratio can be expressed as a correlation term between net imports and each index across products:

**Proposition 3** *For the aggregate economy, the average of the number of production stages  $N_i$  across all products  $i$  weighted by final consumption  $C_i$  and the average number of stages between production and final demand  $D_i$  weighted by value added  $V_i$  satisfy:*

$$\frac{\sum_i C_i N_i}{\sum_i C_i} = \bar{N} + \frac{\sum_i (M_i - X_i)(N_i - \bar{N})}{\sum_i C_i} \quad (4)$$

$$\frac{\sum_i V_i D_i}{\sum_i V_i} = \bar{N} - \frac{\sum_i (M_i - X_i)(D_i - 1)}{\sum_i V_i} \quad (5)$$

where  $\bar{N}$  denotes the gross-output-to-value-added ratio.

When net trade  $(M_i - X_i)$  is not correlated with either fragmentation index  $N_i$  or  $D_i$ , then the equality to the gross-output-to-value-added ratio continues to hold even in an open economy. When net imports are positively correlated to the number of production stages  $N_i$ , the gross-output-to-value-added ratio underestimates the weighted average number of production stages as it does not account for the larger number of production stages embodied in imports. Conversely, the gross-output-to-value-added ratio underestimates the average number of stages to final demand when a country tends to export goods that are relatively further from final demand.

## 2.5 Additional results on aggregation

### From varieties to industries

For the calculation of  $N_i$  and  $D_i$ , the unit of observation would be ideally the plant or the product variety. Unfortunately, calculating these indexes at the plant or variety level would require plant-level input-output matrices (with data on transactions matched between buyers and suppliers) that are not available. In appendix A (propositions 4 and 5), I derive conditions under which each index measured using industry-level matrices equals a weighted average of ideal indexes  $N(\omega)$  and  $D(\omega)$  defined at the plant/variety level.<sup>19</sup> Under reasonable assumptions, index  $N_i$  derived at the industry level is equal to the average of  $N(\omega)$  across varieties  $\omega$  classified in industry  $i$ , weighted by final consumption for each variety. Similarly, index  $D_i$  derived from an industry-level input-output matrix should equal the average of  $D(\omega)$  across varieties classified in industry  $i$ , weighted by the value added by the plant producing variety  $\omega$ .

I also provide an empirical validation of these aggregation properties. In appendix section C, I show that aggregation yields very little bias when I use an artificially aggregated input-output matrix (aggregating the US input-output matrix at the 2-digit instead of 6-digit level). Index  $N_i$  constructed with the aggregated matrix is very close to the weighted average of indexes  $N_{m,i}$  constructed with the disaggregated matrix for each sub-product  $m$  in industry  $i$  (with less than a 1% error on average). This suggests that the measurement of  $N_i$  based on equation (1) is robust to the use of aggregated data.

### Cross-border production sharing and the VAX ratio

In this analysis, the measure of fragmentation captures the number of plants (or stages) involved sequentially in production whether these stages occur within the same country or not. Johnson and Noguera (2012) instead define fragmentation as cross-border production sharing. Their main measure of fragmentation for the world economy is based on the ratio of total value-added content of exports to the total gross value of exports (“VAX\_world”).

As one could expect, there is a close link between these two measures of fragmentation, the VAX ratio and the gross-output-to-value-added ratio. To see the correspondence, one could consider each country as the equivalent of one plant. In line with this interpretation, the counterpart of gross output would be total exports for the world in Johnson and Noguera’s case (within-country transactions are not counted in the measure of total exports, such as within-plant transactions in the measure of gross output) and the counterpart of value added

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<sup>19</sup>Here varieties do not just refer to final goods but also to specific varieties of intermediate goods for each plant (and for each production stage). Final consumption of a variety can be zero if it is purely an intermediate good.

would be total value-added content of trade.<sup>20</sup> Using Propositions 1 and 2, we can conclude that the inverse of the VAX ratio corresponds to the number of embedded border crossings in each dollar of imported final good, weighted by the contribution of each country to total value-added content of trade (a formal proof is provided in the appendix).

### 3 Data

The main data sources are the US input-output matrices developed by the Bureau of Economic Analysis (see Horowitz and Planting, 2009, for a description of the methodology). The US input-output matrices are unique among all countries: they cover the longest time span (since 1947) and are available at a very detailed level (6-digit classification since 1967). Input-output tables for other countries are generally not available at such disaggregated level or only for a much shorter time span.<sup>21</sup>

I use the BEA input-output tables for benchmark years, which are available online.<sup>22</sup> Unfortunately, industry classifications are not always homogenous across periods. The 1997 and 2002 IO tables follow the NAICS classification (430 product categories); the 1967, 72, 77, 82, 87 and 92 IO tables are based on the SIC classification (6-digit level, up to 540 product categories); the 1963 table also follows the SIC classification but is defined at the 4-digit level; previous tables (1947 and 1958) are aggregated to 85 industries.

When I construct the vertical fragmentation index for the aggregate economy I can thus cover 55 years. When more disaggregated data are required for cross-industry comparisons, I rather focus on the period 1967 to 1992 which provides a panel of 377 harmonized product categories.<sup>23</sup> No very precise concordance table is available for NAICS to SIC and so I do not consider the 1997 and 2002 IO tables in my regressions by industry.<sup>24</sup>

Note that the industry classification is more precise for manufacturing goods and commodities, with 330 disaggregated categories in these industries. Some services sectors (such as retail and wholesale trade) are not described at a detailed level. Also, I complete these data with a

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<sup>20</sup>The part of value-added that corresponds to final consumption within the same country is not counted in the total value-added content of trade.

<sup>21</sup>This is particularly the case for input-output tables that have been homogenized across several countries, e.g. OECD IO Tables (constructed for 40 industries since 1992), IDE-JETRO IO Tables and GTAP IO Tables (about 80 industries). Among specific countries, Denmark probably has the best coverage (about 200 industries since 1966), which still does not compare to the coverage provided by US IO tables.

<sup>22</sup>[http://www.bea.gov/industry/io\\_benchmark.htm](http://www.bea.gov/industry/io_benchmark.htm)

<sup>23</sup>Some sectors are more disaggregated for certain years but I consolidate these industry classifications to obtain a homogenous classification across all years (the final one is close to 1987 SIC). The consolidated classification is made available on the following webpage: <http://spot.colorado.edu/~fally/data.html>

<sup>24</sup>See Pierce and Schott (2009) for a discussion. My attempts to include these two years generally confirm my results for 1967-1992.



set of various covariates that are used throughout Section 4. The source and construction of these variables are described in the appendix section B. Given the greater availability of data for manufacturing industries, regressions performed at the industry level mostly focus on the manufacturing sector. The manufacturing sector is composed of 305 consolidated input-output industries, 266 of which having information on all variables.

Several remarks are in order about the construction of these data. First, the US input-output matrices are based on data on establishments, or plants. As defined by the Census Bureau, an establishment is “a business or industrial unit at a single physical location that produces or distributes goods or that performs services.” (Horowitz and Planting, 2009, p39). Hence, each input-output matrix should reflect transactions between plants even if these plants are classified in the same industry. In the construction of indexes  $N_i$  and  $D_i$ , these within-industry transactions do matter in order to measure the degree of vertical linkages not just across industries but also across plants within industries. Specifically, these within-industry transactions are reflected in the diagonal terms  $\mu_{ii}$  of the IO matrix. Finally, we should note that, given the level of disaggregation of the US tables, these diagonal terms are not large: only 10% of intermediate goods purchases are recorded from within the same industry (between 9.8% and 10.9% each year). This fraction is typically much larger in other input-output tables where product classifications are much more aggregated.

## 4 Empirical Findings

I first present descriptive statistics on variations of these indexes over time and across products (section 4.1). I then examine compositional effects (4.2) based on within-between decompositions of aggregate changes. Section 4.3 investigates the determinants of changes by industry and section 4.4 finally examines the reallocation of value added along production chains.

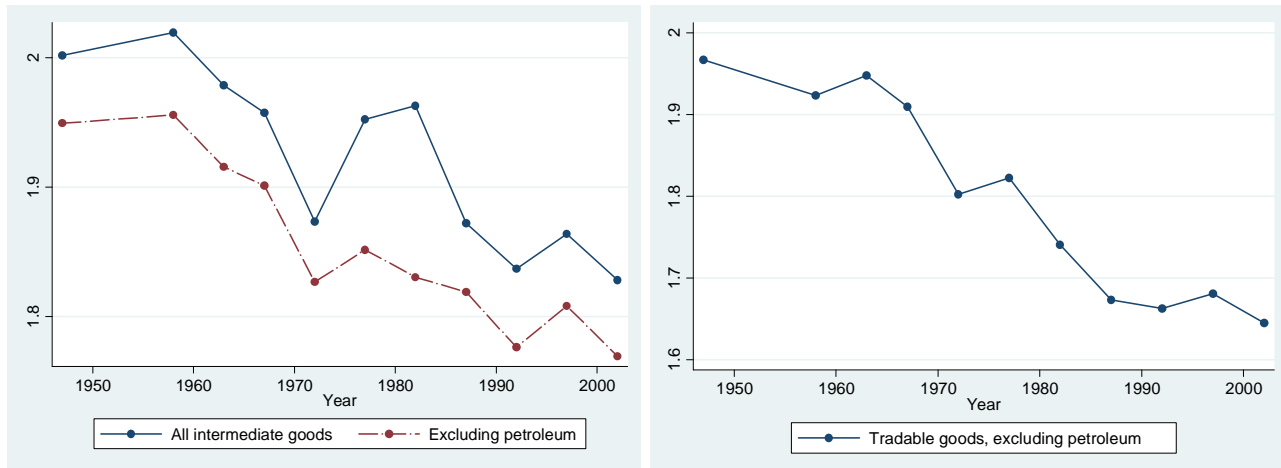
### 4.1 Descriptive statistics

#### Evolution of production staging, 1947-2002

The first striking fact is that the weighted-average number of production stages for the US is below 2 except for 1947 and 1958. This is shown in Figure 2 with the average index of production staging proxied by the gross-output-to-value-added ratio for all products. Production is not as disintegrated as we could expect. In other words, value added embodied in production goes through less than two plants (two stages) on average before reaching final demand.

Moreover, production in the US has become less vertically fragmented over time. This decrease in the fragmentation of production has been quite smooth over time except for years

Figure 2: Weighted average number of production stages



1977 and 1982. An obvious candidate explanation for the peak in 1977 and 1982 is the increase in oil prices.<sup>25</sup> When I thus reconstruct my index by excluding petroleum-related industries (crude petroleum and refining), the 1977 and 1982 peak almost disappears and the overall decline in the fragmentation of production is confirmed.<sup>26</sup>

One simple potential explanation is the increasing role played by services in the US economy. Services now account for more than two thirds of GDP but generally require fewer production stages. Moreover, we need to carefully interpret the fragmentation measure using services as the input-output matrix is much more aggregated for these sectors.<sup>27</sup> In comparison, data on manufacturing sectors are more finely detailed.

In the right panel of Figure 2, I compute the aggregate index of fragmentation using only tradable goods and tradable inputs (manufacturing goods and commodities, excluding services and petroleum-related industries). Even if we exclude services, the downward trend is confirmed. The average number of embodied stages for tradable goods declined from 2 to 1.6 over the past 50 years. We can further restrict our attention to manufacturing industries but the picture remains similar.

The figures above are based on the gross-output-to-value-added ratio, adjusting value added for the use of excluded industries such as petroleum. This amounts at considering the US as a closed economy. In an open economy, aggregate measures of fragmentation may differ, as shown in Proposition 3. In particular, the aggregate number production stages  $\frac{\sum_i N_i C_i}{\sum_i C_i}$  (weighted by final consumption) can differ from the aggregate number of stages to final demand  $\frac{\sum_i D_i V_i}{\sum_i V_i}$

<sup>25</sup>See sections 4.3 and 4.4 for more precise analyses of the role of prices.

<sup>26</sup>The negative trend is statistically significant even after correcting for auto-correlation.

<sup>27</sup>For instance, wholesale trade and retail correspond to only two industries in the input-output table.

Table 1: Aggregation biases in open economy

Year	Import Penetration	GO/VA Ratio	$\Delta$ Number of stages	$\Delta$ Distance to final demand
1967	0.033	1.937	0.006	-0.002
1972	0.064	1.805	0.012	-0.005
1977	0.073	1.814	0.013	0.011
1982	0.094	1.728	0.020	0.023
1987	0.140	1.665	0.011	-0.021
1992	0.157	1.658	0.012	0.005

*Notes:* GO/VA is the ratio of gross output to value added calculated for tradable goods (excluding petroleum). The terms  $\Delta N$  and  $\Delta D$  corresponds to the difference between each aggregate index and the GO/VA ratio.

(weighted by value added). Using product-level trade data from 1967 to 1992, I compute the difference between each of these aggregated indexes and the GO/VA ratio, as described in Proposition 3.

Results are shown in Table 1. While trade has grown very rapidly during this period (import penetration rose from 3.3% in 1967 to 15.7% in 1992), not adjusting for trade only generates a small bias in the computation of these aggregate measures of fragmentation. Deviations are smaller than 0.02, i.e. a 1% error at most. Figure 2 would thus remain identical after correcting the fragmentation index for international trade. As shown in Proposition 3, this implies that net trade volumes for the US are not systematically related to the position on the value chain. This issue is further discussed in Section 4.3.<sup>28</sup>

### Indexes of production staging by industry in 1992

I now turn to cross-industry variations in the production staging indexes and describe industries with the largest values of embodied production stages  $N_i$ . I find that food industries typically involve long production chains with little value added at each stage (see Table 2a). Among the top-5 industries with the largest values for  $N_i$ , we find meat packing, sausages, cheese and butter industries (poultry is next). Among the top 25 industries, 17 are related to food. Non-food industries in the top 25 are metal-intensive industries (e.g. cans), leather tanning, petroleum refining, video and audio equipment, wood preserving and the car industry.<sup>29</sup>

If we only look at tradable intermediate goods (manufacturing goods and commodities, excluding services and petroleum-related industries), the ranking among top industries is almost the same. In line with case studies (e.g. Helper, 1991), the car industry appears to be quite

<sup>28</sup>In Section 4.3 I confirm that import penetration is not significantly correlated with the number of production stages. I find, however, that fragmentation has increased relatively more in sector with larger import penetration.

<sup>29</sup>The full dataset is posted on this webpage: <http://spot.colorado.edu/~fally/data.html>

Table 2a: **Largest values in index  $N_i$  (embodied production stages)**

Production stages	All inputs	Tradables	GO/VA
<b>Top-5 industries:</b>			
Meat packing plants	3.49	2.67	8.74
Sausages and other prepared meat products	3.39	2.65	4.88
Leather tanning and finishing	3.15	2.43	3.93
Natural, processed, and imitation cheese	3.15	2.35	5.55
Creamery butter	3.13	2.35	5.12
<b>Motor vehicle industries:</b>			
Motor vehicles and passenger car bodies	2.79	2.03	6.09
Motor vehicle parts and accessories	2.40	1.78	3.15
Truck and bus bodies	2.41	1.82	2.83
Truck trailers	2.59	1.91	3.75

Table 2b: **Largest values in index  $D_i$  (distance to final demand)**

Stages to final demand	All inputs	Tradables
Nonferrous metal ores, except copper	7.17	6.48
Copper ore	5.10	4.37
Oil and gas field machinery and equipment	4.45	3.22
Primary smelting and refining of copper	4.39	3.65
Iron and ferroalloy ores mining	4.32	3.59

disintegrated, though not as disintegrated as the food industry. The weighted average number of stages is 2.8, and it is 2.4 for auto parts.<sup>30</sup>

In turn, if we look at index  $D_i$  on distance to final demand, primary goods exhibit the largest values. The largest is obtained for basic metal products (Table 2b).

Industries with the smallest index of production stages  $N_i$  are generally services industries (see Table 3). If we only consider tradable goods, industries with the smallest number of production stages  $N_i$  correspond to primary goods. Similarly, industries that are closest to final demand are generally services industries. In 1992, eight products are not used as intermediate goods: “Residential care”, “Hospitals”, “Cigarettes”, “House slippers”, “Doctors and dentists”, “Owner-occupied dwellings”, “Child day care services”, “Ordnance and accessories, n.e.c”.

An important point to note that these two indexes are only weakly correlated across all commodities and manufacturing industries. The correlation between  $N_i$  and  $D_i$  is negative until 1982: -7.5% in 1967, -4.3% in 1972, -1.9% in 1977. Then it lies between -1% and 1% after 1982. This small correlation shows that these two indexes capture different dimensions

<sup>30</sup>Note that the fragmentation index  $N_i$  differs from the gross-output-to-value-added ratio  $Y_i/V_i$  at the industry level. Fragmented industries generally exhibit a large GO/VA ratio but the difference between the two indexes can also be large (first vs. last column) and the ranking is not preserved.

Table 3: Industries with the smallest average number of production stages

Production stages	All inputs	Production stages	Tradables
Owner-occupied dwellings	1.23	Carbon black	1.03
Other Federal Government enterprises	1.25	Greenhouse and nursery products	1.08
Greenhouse and nursery products	1.33	Manufactured ice	1.14
U.S. Postal Service	1.34	Miscellaneous crops	1.15
Real estate	1.44	Forestry and fishery products	1.16

of the fragmentation of production and can be both informative to characterize the position of an industry along supply chains.

An overall comparison between commodities, manufacturing goods and services confirms the intuition above (Table 4). Manufacturing industries embody more production stages than commodities and commodities more than services. Commodities are further from final demand than manufacturing industries, while services are closer to final demand than manufacturing industries on average. The comparison between manufacturing goods and commodities carries over if we only consider tradable inputs and exclude petroleum-related products.

Table 4: Averages for groups of industries

Inputs from:	All industries		Tradables excl. oil	
Index:	Production stages $N_i$	Stages to final demand $D_i$	Production stages $N_i$	Stages to final demand $D_i$
Manufacturing	2.19	2.11	1.60	1.53
Commodities	2.06	3.01	1.38	2.45
Services	1.75	1.79	/	/
Petroleum	2.33	3.48	/	/

Now I show that, among manufacturing industries, there are systematic differences between industries and these differences depend on various industry characteristics. The choice of these characteristics is primarily motivated by the literature on firm boundaries: the property-right approach (Grossman and Hart, 1986), the transaction cost approach, and the literature on strategic vertical interactions.<sup>31</sup> Even if these measures of fragmentation only capture within-plant integration (boundaries of the plant), it may well be influenced by factors determining ownership (boundaries of the firm). Hortacsu and Syverson (2011) show that shipments that occur within the firm account for a very small portion of all shipments across plants. This

<sup>31</sup>See Gibbons (2005) for a survey of existing theories, Rey and Vergé (2008) for a survey on the economics of vertical restraints, including the double-marginalization problem.

result implies that the decision to integrate supply chains within the same firm often goes along within-plant production instead of involving several plants within the same firm.

The literature on the boundaries of the firm has identified various determinants of vertical integration.<sup>32</sup> First, innovative industries rely less intensively on outsourcing whereas mature industries are more likely to outsource components (Acemoglu, Aghion and Zilibotti, 2007). We can thus expect a negative correlation between R&D intensity and vertical fragmentation. Skill intensity and the complexity of tasks may also affect externalization decisions, with more complex tasks more likely to be performed within the firm (Costinot, Oldenski and Rauch, 2009).<sup>33</sup> Antràs (2003) model based on the property-right approach shows that the internalization decision also depends on capital intensity. Other factors affecting integration include competition among suppliers and market thickness (de Fontenay and Gans, 2005) and financial constraints (Acemoglu, Johnson and Mitton, 2007, Carluccio and Fally, 2012). I proxy competition by the fraction of output produced by the 4 largest companies in the industry<sup>34</sup> and financial constraints by an index of external finance dependence (Rajan and Zingales, 1998). Another factor to be considered is product specificity: sourcing is more difficult and costly for specific products, especially when contracts are difficult to enforce. As in Nunn (2007), I use Rauch (1999) classification to identify specific products (goods sold on thin markets). We may also expect goods that are closer to final demand to be more specific and customized.

Pairwise correlations between each index and these industry characteristics are shown in Table 5 (see appendix for details on data and variable definitions).<sup>35</sup> The first column shows that high-tech industries generally embody a smaller average number of production stages. These results are in line with the literature on vertical integration. In particular, there is a negative and significant correlation for  $N_i$  with product specificity, R&D intensity, skill intensity and dependence in external finance. We may expect high-tech industries to be more complex and thus involve more production stages, but complex inputs are more difficult to source from other plants.<sup>36</sup> Finally, there is no significant correlation between  $N_i$  and either capital intensity, productivity or industry concentration.

Turning to the second column (index  $D_i$ ), industries that are further from final demand have lower values of skill intensity and product specificity. In particular, these industries are

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<sup>32</sup>See Lafontaine and Slade (2007) for a survey of empirical work. As described by Lafontaine and Slade (2007), the property-right and the transaction-cost approaches often generate similar predictions.

<sup>33</sup>Here I focus on a measure skill intensity. I obtain similar results with the measure of non-routine vs. routine task developed by Costinot, Oldenski and Rauch (2009). The latter is however initially defined following the NAICS classification, which is difficult to match with the SIC classification.

<sup>34</sup>Alternatively, we can use the Herfindahl-Hirschman Index. Results are qualitatively the same.

<sup>35</sup>Very similar results are obtained with multivariate regressions.

<sup>36</sup>Also, as mentioned before, the measure of vertical fragmentation  $N_i$  depends does not depend on how many different inputs are produced in parallel and assembled, conditional on the value share of outsourced inputs.

Table 5: Pairwise correlations with industry characteristics

Variable:	Production Stages ( $N_i$ )	Distance to final demand ( $D_i$ )
Specificity	-0.266*	-0.498*
R&D	-0.259*	-0.038
Capital intensity	0.091	0.524*
Skill intensity	-0.219*	-0.167*
Advertising intensity	-0.083	-0.267*
Productivity	-0.025	0.030
Financial Dep	-0.185*	0.322*
Share of top 4 firms	-0.040	0.075

Notes: Variables for year 1992. A star denotes significance at 1%

less intensive in the use of advertisements, which is quite intuitive (advertising industries are those that are closer to final consumers). These upstream industries are also more intensive in capital and rely more heavily on external finance. In particular, the latter is consistent with the predictions of Kim and Shin (2012).

## 4.2 Within-between decompositions of aggregate changes

Since the degree of vertical fragmentation varies sensibly across industries, I now examine whether the overall decrease in the fragmentation of production can be explained by composition effects. Is there a continuous shift towards industries with fewer production stages? Or can we only explain the overall decrease by changes within each industry?

Composition effects can occur along two dimensions. First, consumption may be shifting towards goods that require fewer production stages. Second, value added can *shift* towards industries that are closer to final demand, meaning that downstream industries contribute to a larger fraction of final goods value. Following Proposition 2, both shifts can contribute to the aggregate decrease in fragmentation. Hence, the combined use of these two indexes provides two different angles to look at these composition effects.

To examine these questions quantitatively, I decompose the changes in the fragmentation of production into “between” and “within effects”. Between two periods, the change in the aggregate index can be expressed as (Decomposition 1):

$$\Delta \bar{N}_t = \underbrace{\left[ \sum_i \frac{(N_{i,t} + N_{i,t-1})}{2} \cdot \Delta c_{i,t} \right]}_{\text{Between 1}} + \underbrace{\left[ \sum_i \Delta N_{i,t} \cdot \frac{(c_{i,t} + c_{i,t-1})}{2} \right]}_{\text{Within 1}}$$

with  $\Delta$  denoting simple differences between periods  $t$  and  $t - 1$ , and  $c_{i,t} \equiv C_{i,t}/[\sum_j C_{j,t}]$  the share of consumption in sector  $i$  at time  $t$ . Decomposition 1 is based on the number of production stages. Alternatively, we can use the distance to final demand weighted by value added (Decomposition 2):

$$\Delta \bar{D}_t = \underbrace{\left[ \sum_i \frac{(D_{i,t} + D_{i,t-1})}{2} \cdot \Delta v_{i,t} \right]}_{\text{Between 2}} + \underbrace{\left[ \sum_i \Delta D_{i,t} \cdot \frac{(v_{i,t} + v_{i,t-1})}{2} \right]}_{\text{Within 2}}$$

where  $v_{i,t} \equiv V_{i,t}/[\sum_j V_{j,t}]$  denotes the share of value added in sector  $i$  at time  $t$ . In each decomposition, the first term reflects a change in the composition (between effect) whereas the second term reflects changes within industries. As documented in Table 1, aggregate indexes  $\bar{N}_t$  and  $\bar{D}_t$  are almost equal to each other, and very close to the ratio of gross output to value added.<sup>37</sup> Hence, these two approaches can be seen as two alternative decompositions of the evolution of the aggregate average number of production stages.

I first decompose the change in the index calculated for all industries, including all inputs (Table 6, Panel A). Panel A shows similar results for both decompositions. In both decompositions, the within and between effects are equally large. Summing across all years, the between effect actually dominates. This negative trend for both indexes can be explained by a shift of demand and production towards services. Services require fewer stages and are also closer to final demand. While the between effect is consistently negative in Decomposition 1, the between effect in Decomposition 2 is positive for the transition period between 1972 and 1977. This can be explained by the increase in basic commodity prices such as petroleum, which increases the share of industries that are further from final demand. For other years, the between effect is negative though. Similarly, increases in commodity prices can explain the positive within effect in the first decomposition (see Table 8 in the next section).

Then, I decompose the change in fragmentation by considering tradable goods only (manufacturing and commodities excluding petroleum). Panel B shows that the between effect in Decomposition 1 is much smaller for tradable goods, and a large part of the evolution across years is explained by the “within” effect. This confirms that results from Panel A are partially driven by the shift towards services and shows that, among tradable goods, there has been no shift of consumption towards less fragmented goods. Hence, changes in consumption patterns across tradable goods do not explain the decline in production staging.

The “between” effect in Decomposition 2 remains large compared to Decomposition 1.

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<sup>37</sup>In theory, the weighted average of the number of production stages may differ from the weighted average of the distance to final demand in an open economy. However Table 1 show that, in practice, these two measures are almost equal to each other for the US.



Table 6: Within-between decompositions

Year	Average number of production stages: (decomposition 1 along $N_i$ )			Average distance to final demand: (decomposition 2 along $D_i$ )		
	Aggregate change	Between effect	Within effect	Aggregate change	Between effect	Within effect
<b>Panel A: All industries</b>						
67-72	-0.087	-0.028	-0.059	-0.100	-0.023	-0.078
72-77	0.070	-0.009	0.078	0.049	0.032	0.016
77-82	0.013	-0.033	0.045	0.026	-0.016	0.042
82-87	-0.086	-0.007	-0.079	-0.097	-0.041	-0.055
87-92	-0.031	-0.030	-0.001	-0.014	-0.010	-0.004
<b>Panel B: Tradeable goods</b>						
67-72	-0.127	0.022	-0.148	-0.136	-0.002	-0.134
72-77	0.011	-0.024	0.035	0.025	0.042	-0.017
77-82	-0.079	-0.030	-0.049	-0.074	-0.055	-0.019
82-87	-0.072	-0.002	-0.070	-0.107	-0.055	-0.052
87-92	-0.006	-0.006	0.001	0.019	0.009	0.010

*Notes:* Panel A: all industries are included except petroleum; Panel B: primary and secondary industries are included except petroleum. See text for within and between decomposition. It is applied to the number of production stages in columns 3 and 4 and to the number of stages to final demand in columns 5 and 6. The values in column 2 (difference in aggregate GO/VA between two years) equal the sum of columns 3 and 4 and also the sum of columns 5 and 6.

Except for 1967, the variations in aggregate distance to final demand are mostly driven by the between effect. Except for 1972-1977 period, value-added has been shifting towards manufacturing industries that are closer to final demand.

In what follows, I will first examine the evolution of the number of production stages  $N$  focusing on the “within” effect of decomposition 1 (section 4.3). Then I will turn to distance to final demand by providing additional evidence on the shift of value-added towards final stages (section 4.4). The latter provides simple and intuitive insights on the aggregate decrease in the weighted number of production stages.

### 4.3 Determinants of “Within” changes

As shown previously (decomposition 1), the aggregate decrease in fragmentation in the manufacturing sector mostly corresponds to “within” effects rather than a shift of consumption towards goods that require fewer production stages. As motivated previously, whether production chains are more vertically fragmented across plants may depend on the complexity of tasks, on the need for capital, on the thickness of upstream markets, etc. Now, are there empirical

Table 7: Within-industry changes

Dependent variable:	$\Delta N$	
	coef.	s.e.
Specificity	2.121***	[0.665]
R&D intensity	0.565***	[0.153]
Capital intensity	1.490***	[0.475]
Skill intensity	-6.285**	[3.052]
Advertising intensity	-0.018	[0.079]
Productivity growth	-2.666	[2.052]
Financial Dependence	-0.293*	[0.168]
Top 4 share	-0.014	[0.013]
Number of industries	266	
R-squared	0.12	

*Notes:* OLS regressions with robust standard errors in brackets;  
\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%.

regularities that could explain the *change* in vertical fragmentation by industry?

Table 7 explores the change in fragmentation by industry depending on various industry characteristics. The dependent variable is increase in the index of fragmentation:  $\Delta N_i = N_{i,1992} - N_{i,1967}$ . Results show that the change in fragmentation is positively related to product specificity (measured by Rauch 1999 index), R&D intensity and capital intensity, and negatively related to skill intensity and financial dependence. All-in-all, these industry characteristics can account for about 12% of the variance in the change in fragmentation (R-squared). The positive correlation with variables characterizing high-tech industries (such as R&D intensity) is consistent with a product-cycle interpretation: innovative industries become more fragmented as they mature (see e.g. Antràs 2005). However, while this interpretation might help understand why some industries are becoming more fragmented than others, it does not shed light on the overall decline fragmentation.

In the working paper version (Fally 2012), I further examine whether the decline in fragmentation can be explained by *changes* in these industry characteristics. As R&D, skill and capital intensity are important determinants of differences vertical fragmentation across industries, one might suspect that *changes* in R&D, capital and skill intensities might be driving the decrease in fragmentation. However, I do not find any significant relationship between the change in these industry characteristics and the change in fragmentation  $\Delta N_i$ . I also examine the role of upstream industry characteristics using averages of industry characteristics weighted by direct input-output coefficients as in Nunn (2007). Results are very similar to the ones presented in Table 7, with even stronger correlations with capital and R&D intensity.

## Adjusting for prices

Since the measure of fragmentation developed here is based on value-added weights, a natural question is whether changes in these weights (and changes in the overall measure) are not simply reflecting changes in relative prices along value chains. For instance, if competition among suppliers has eroded their bargaining power compared to final goods producers, we could expect the relative price of intermediate goods to decrease, thus reducing the share (in value) of intermediate goods in final goods production. Such an effect would be reflected in the index  $N$  as a decrease. To disentangle such price effects, I propose a further decomposition of the “within” effect computed in Table 6 (decomposition 1).

Let  $Q_{ij,t}$  denote the quantity of intermediate good  $j$  used in the production of good  $i$  at time  $t$ . The input-output coefficient could then be rewritten as:  $\mu_{ij,t} = \frac{P_{j,t}Q_{ij,t}}{P_{i,t}Q_{i,t}}$  where  $P_{i,t}$  denotes the price index of goods produced by industry  $i$ , and where  $Q_{i,t}$  denotes total output quantity of industry  $i$ . Looking at the evolution across years, we can decompose the change in input-output coefficient  $\Delta\mu_{ij,t} \equiv \mu_{ij,t} - \mu_{ij,t-1}$  in two components reflecting changes in prices and quantities respectively (see Appendix B):

$$\Delta\mu_{ij,t} = \underbrace{\left(\frac{P_{j,t}}{P_{i,t}} - \frac{P_{j,t-1}}{P_{i,t-1}}\right) \cdot \frac{1}{2} \left(\frac{Q_{ij,t}}{Q_{i,t}} + \frac{Q_{ij,t-1}}{Q_{i,t-1}}\right)}_{\Delta\mu_{ij,t}^P} + \underbrace{\frac{1}{2} \left(\frac{P_{j,t}}{P_{i,t}} + \frac{P_{j,t-1}}{P_{i,t-1}}\right) \cdot \left(\frac{Q_{ij,t}}{Q_{i,t}} - \frac{Q_{ij,t-1}}{Q_{i,t-1}}\right)}_{\Delta\mu_{ij,t}^Q}$$

Using this price-quantity decomposition of the change in direct coefficients, we thereby obtain a natural decomposition of the changes in the fragmentation index for each industry:

$$\Delta N_{i,t} = \underbrace{\sum_k a_{ik,t} \left[ \sum_j \Delta\mu_{kj,t}^P \left( \frac{N_{j,t} + N_{j,t-1}}{2} \right) \right]}_{\text{Price effect}} + \underbrace{\sum_k a_{ik,t} \left[ \sum_j \Delta\mu_{kj,t}^Q \left( \frac{N_{j,t} + N_{j,t-1}}{2} \right) \right]}_{\text{Quantity effect}}$$

where  $a_{ik,t}$  denotes the coefficients of the matrix  $(I - M_{t-1,t})^{-1}$  with  $I$  the identity matrix and  $M_{t-1,t}$  the average matrix for time  $t-1$  and  $t$  with coefficients  $\frac{\mu_{ij,t} + \mu_{ij,t-1}}{2}$  (see Appendix B).

To proxy for price ratios, I use data on producer price indices from the NBER CES database (for manufacturing industries) and from the BLS (for other commodities). Using data on relative price indices, quantity ratios are simply obtained by dividing the input-output coefficient by the relative price ratio  $\frac{Q_{ij,t}}{Q_{i,t}} = \frac{\mu_{ij,t}P_{i,t}}{P_{j,t}}$ .

In Table 8, I compute this decomposition to isolate the role of prices in explaining the within effect in the decomposition of the fragmentation index. Interestingly, price effects are very small except for transition period between 1972 and 1977, where the evolution of prices

Table 8: Price vs. quantity decomposition - Tradeable goods

Year	Within effect	Price effect	Quantity effect
67-72	-0.148	0.006	-0.154
72-77	0.035	0.076	-0.041
77-82	-0.049	0.008	-0.057
82-87	-0.070	-0.025	-0.045
87-92	0.001	-0.011	0.012

*Notes:* The within effect is the same as in Table 6, panel B, and equal the sum of the quantity and price effects.

(increase in the relative price of intermediates) can explain a large increase in the fragmentation index, even if we exclude petroleum. The quantity effect is however negative for 1972-1977, like other years, suggesting that the index of vertical fragmentation would have decreased during this period if relative prices had remained stable. This table shows that the negative trend in the index  $N$  cannot be simply explained by price changes.

While these results suggest that changes in relative price do not explain the overall decrease in vertical fragmentation, one must remain careful about potential price measurement errors. As measured by the BLS, price indices do not fully account for the introduction of new varieties.<sup>38</sup>

### Trade and vertical fragmentation

Trade can have two opposite effects. As trade barriers fall, production chains increasingly involve parties located in different countries (Yi, 2003). International trade provides new opportunities to reduce costs by shifting part or entire production abroad. It is thus natural to expect a positive effect of trade on the fragmentation of production. Note however that trade does not affect this measure of fragmentation if there is simply a substitution between domestic outsourcing and foreign outsourcing. As described in Section 2, the measure of fragmentation is based on the total use of inputs and does not differentiate shipments from another plant in the US and shipments from overseas. Hence, if trade is found to have a positive impact, it would suggest that it substitutes to tasks that were previously performed within the plant.

There may be also a negative effect of trade on this measure of fragmentation. If trade reduces the relative price of intermediate goods, there is a possibility that it also reduces the amount spent on these goods, and therefore reduces the share of value added associated with upstream stages.<sup>39</sup>

<sup>38</sup>This is known as the “outlet substitution bias” in the consumer price index literature. A similar issue arises with international trade and the availability of new imported varieties (Houseman *et al.*, 2011).

<sup>39</sup>The results from Table 8 suggest that price effects are small but these price indices do not perfectly account for new varieties of traded inputs. Note that this negative effect of trade can only occur if there is a low

Table 9: Import penetration and production stages

Dep. variable:	$N$	$D$	$\Delta N$	$\Delta N$	$\Delta N$	$\Delta N$	$\Delta N^{dom}$	$\Delta N^{dom}$
Import penetration	-0.032 [0.128]	-0.475 [0.307]						
$\Delta$ imports (same ind.)			0.218 [0.060]***	0.168 [0.076]**				
$\Delta$ imports (upstream)					0.260 [0.137]*	0.240 [0.204]	-0.393 [0.125]***	-0.343 [0.191]*
Controls	No	No	No	<b>Yes</b>	No	<b>Yes</b>	No	<b>Yes</b>
Nb industries	305	305	305	266	305	266	305	266
R-squared	0.01	0.01	0.03	0.16	0.01	0.13	0.04	0.16

*Notes:* OLS regressions. Dependent variables:  $N$  and  $D$  in 1992;  $\Delta N$ : increase in  $N$  between 1967 and 1992;  $\Delta N^{dom}$ : increase in average number of domestic stages  $N^{dom}$  between 1967 and 1992. Independent variables: import penetration (col. 1 and 2); increase in import penetration in the same industry (col. 3 and 4); increase in average import penetration in upstream industries (col. 5 to 8). Controls include all variables in column (1) of Table 7. Robust standard errors in brackets; \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%.

A first test is whether the number of production stages or the position on the value chain is correlated with import penetration across industries (in the cross section). Results presented in Table 1 already imply that is only a small correlation between either net imports and production stages  $N_i$  or net imports and the distance to final demand  $D_i$ . In Table 9, I confirm these results by regressing index  $N_i$  (column 1) and  $D_i$  (column 2) on import penetration across industries. Import penetration is defined as the ratio of imports to production plus imports minus exports in each industry. I find no significant correlation (OLS regression with robust standard errors). Interestingly, I even find a negative (but not significant) correlation between import penetration and  $D_i$ , suggesting that import competition is actually tougher in downstream industries than in upstream industries. Given this result, it appears also unlikely that import competition has induced a decrease in the relative price of upstream goods vs. compared to downstream goods. This is in line with the fact that price indexes have not decreased relatively faster in upstream industries.

From the non-significant correlation between trade and import penetration in a cross-section, we should however not conclude that trade does not affect vertical fragmentation. I now examine whether changes in import penetration are related to changes in the fragmentation of production. For this purpose, I regress the change in the measure of production stages ( $\Delta N_i$ ) by industry on the increase in import penetration between 1967 and 1992, by indus-

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elasticity of substitution (less than 1) between outsourced intermediate goods (domestically or internationally) and intermediate goods produced within the plant, otherwise a negative effect of trade on relative prices would also imply an increase in the share of outsourced intermediate goods.

try. In columns (3) and (4), I find a positive and significant effect which could suggest that trade indeed creates new opportunities to fragment production. Controlling for other industry characteristics does not greatly affect the main coefficient (column 4).

More importantly, we would like to test whether imports of *inputs* are associated with an increase in index  $N$ . For this purpose, I compute the change in average import penetration among upstream industries (import penetration weighted by direct input coefficients), and use it instead of the change in import penetration within the same industry. In columns (5), I find a larger coefficient but the associated beta coefficient is smaller (0.11 against 0.20) and less significant. As shown in column (6), the coefficient for imports is no longer significant when additional controls are added. Similarly, I find no significant effect after instrumenting the change in import penetration by transport costs and tariff decreases in upstream industries. Hence, while opening to trade seems to be positively associated with an increase in vertical fragmentation across plants, its effect is not large and robust.

Finally, the last two columns of Table 9 confirm that an increase in import penetration in upstream industries has a negative effect on the average number of *domestic* production stages, i.e. if we do not account for stages associated with imported intermediate goods.<sup>40</sup>

#### 4.4 A shift of value added towards downstream industries

As shown in Proposition 1, index  $N$  can be interpreted as the weighted average number of production stages, weighted by value being added at each stage. Hence, a decrease in  $N$  can be explained by a shift of value towards final stages. While plant-level data between buyers and suppliers are not available, we can still examine the shift of value added towards industries that are closer to final demand. This shift corresponds to the between effect associated with distance to final demand (decomposition 2) in Table 6.

A similar way to illustrate this shift is to examine the value-added-weighted average distance to final demand, using panel data on value added but using a reference value for the index of distance to final demand for each sector. To be more precise, I compute for each year:

$$\widetilde{D}_{v,t} = \sum_i v_{it} D_{i,1992}$$

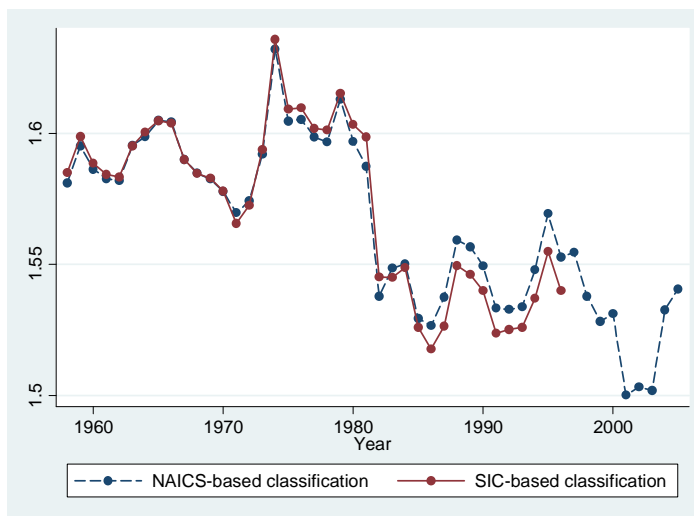
where  $D_{i,1992}$  is the distance index associated with industry  $i$  in year 1992 (or an alternative year) and  $v_{it}$  is the share of value added from sector  $i$  at time  $t$ . Hence, keeping the distance

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<sup>40</sup>In column (7) and (8), the dependent variable is the increase in  $N_i^{dom}$  defined as the solution of equation (1) after replacing the requirement coefficient  $\mu_{ij}$  by the domestic requirement coefficient  $\mu_{ij}^{dom}$ . The estimation of  $\mu_{ij}^{dom}$  is based on the standard assumption that the share of domestic purchases does not depend on the downstream industry:  $\mu_{ij}^{dom} = [1 - M_j / (Y_j + M_j - X_j)] \mu_{ij}$ .

index constant, the observed change in  $\widetilde{D}_{v,t}$  solely reflects a change in the industry composition. Moreover, we are no longer restricted to “benchmark” years since data on value added are available from other sources. Here, I use data on manufacturing value-added from the NBER-CES database available on a SIC-based classification between 1958 and 1996 (this dataset does not cover primary industries). To also examine what happened in subsequent years, I also compute the distance index using the 2002 input-output matrix (based on the NAICS classification) to be combined with NBER-CES data available on a NAICS basis until 2005.

Figure 3: VA-weighted distance to final demand



*Notes:* Distance index measured with the 1992 (SIC-based) and 2002 (NAICS-based) input-output tables. Value-added data are from the NBER-CES database.

Figure 3 illustrates the evolution of  $\widetilde{D}_{v,t}$ . We can indeed observe an overall shift of production towards downstream sectors during these five decades except between 1973 and 1981 when the price of oil and other basic commodities have dramatically increased.

Table 10 provides yet another way to examine the shift of value added. In columns (1) to (3), I test whether value added has grown significantly more in industries that are closer to final demand (OLS regressions with robust standard errors). The dependent variable is the growth in VA by industry between 1967 and 1992, while the independent variable is the distance to final demand by industry (1967-1992 average). The coefficient is negative and significant; the beta coefficient equals -0.221.

This result clearly confirms the shift of value added towards downstream industries, which is consistent with the negative “between” effect found in Table 6 (Panel B, Decomposition 2). In column (2), I control for the number  $N_i$  of production stages: the coefficient is not significant,

Table 10: Shift of value-added towards final stages

Dependent variable:	VA Growth	VA Growth	VA Growth	Increase in VA/GO	Increase in VA/GO	Price Growth
Stages to final demand	-2.927 [1.108]***	-2.951 [1.119]***	-2.365 [1.430]*	-0.403 [0.151]***	-0.270 [0.207]	-0.122 [0.597]
Number of stages		0.530 [2.931]	0.837 [3.245]			
Specificity			-5.985 [3.025]**		-0.400 [0.379]	
R&D intensity			0.323 [0.688]		-0.213 [0.084]**	
Capital intensity			-5.330 [2.088]**		-0.704 [0.253]***	
Skill intensity			8.126 [9.921]		3.110 [1.621]*	
Advertising intensity			0.712 [0.299]**		0.003 [0.077]	
Productivity growth			26.580 [11.381]**		0.456 [1.027]	
Financial Dependence			1.173 [0.522]**		0.210 [0.083]**	
Top 4 share			-0.065 [0.043]		0.003 [0.007]	
Import penetration			-36.838 [8.848]***		-2.022 [1.104]*	
Number of industries	305	305	266	305	266	305
R-squared	0.02	0.02	0.20	0.02	0.12	0.00

*Notes:* OLS regressions. Dependent variables: growth of value added by industry between 1967 and 1992 (columns 1 to 3); increase in the value-added-to-gross-output ratio (columns 4 and 5); growth of industry price index (column 6). Independent variables: averages between 1967 and 1992; data on industry characteristics are described in the appendix. Robust standard errors in brackets; \* significant at 10%; \*\* at 5%; \*\*\* at 1%.

reflecting the small “between” effect found in decomposition 1.<sup>41</sup> In column (3), I control for other industry characteristics: Product specificity, R&D intensity, capital and skill intensity, advertising intensity, productivity growth, financial dependence and industry concentration. The coefficient for distance to final demand remains significant but is now smaller. In particular, part of the negative correlation between value-added growth and distance to final demand can be attributed to a faster growth in advertising-intensive and less capital-intensive industries (which are closer to final demand). I also control for import penetration which has a negative effect on VA growth.

<sup>41</sup>Alternatively, we can use the growth of consumption as the dependent variable. The coefficient for the number of production stages is also not significant.



Interestingly, the ratio of value added to gross output (by industry) exhibits a similar pattern. In columns (4) and (5), the dependent variable is the increase (simple difference) in VA/GO between 1967 and 1992, regressed on the distance to final demand by industry. The coefficient is also significantly negative; the beta coefficient equals -0.242 in column (4). In this regression, the constant equals +1.30. We can test and verify that VA/GO has significantly increased for industries that are the closest to final demand, while it has significantly decreased for industries with a measure of distance to final demand equal to 3. These results remain fairly unaltered after controlling for other industry characteristics and import penetration.

Since the growth in value-added is mechanically affected by changes in prices, a natural question is whether the shift of value added does not simply reflect an erosion of the relative prices of intermediate goods. As in section 4.3, I use industry price data from the NBER-CES database and the Bureau of Labor and Statistics (BLS) over the same time period to examine this hypothesis. In column (6), I regress the change in the industry-level price index on the measure of distance to final demand. The coefficient is however very small and not statistically significant. Additional evidence on relative prices is provided in appendix section C, showing that the price of basic commodities and intermediate goods (compared to final goods) has not decreased over the past decades.

A straightforward explanation for the shift towards downstream industries is that value-added growth has been driven by other factors (e.g. shift towards high-tech industries) and that these factors are themselves related to the distance to final demand. In particular, value-added has grown faster in industries that are intensive in R&D, in skills, in advertising, in external finance, and less intensive in physical capital. In turn, these industries are generally closer to final demand (see Table 5) which can explain why value-added growth is negatively correlated with distance to final demand. To examine this explanation quantitatively, I perform the following exercise:

- i) First, I regress value-added growth on industry characteristics (all control variables from column 3 of Table 10: R&D intensity, skill intensity, etc.) excluding the two measures of fragmentation. The regression coefficients are almost identical to those in column 3 of Table 10 for the corresponding variables.
- ii) Then, I use the predicted value-added growth by industry from step 1 and regress the constructed variable on distance to final demand.

The resulting coefficient is -1.540 (s.e. = 0.597, significant at 1%). It is more than half of the magnitude of the main coefficient from Table 10, column 1. This result suggests that these industry characteristics can explain half of the negative correlation between value-added

growth and distance to final demand, which itself can explain the aggregate decrease in vertical fragmentation. These findings on the shift of value-added also demonstrate that the overall decline in fragmentation is not counter-intuitive if we see it from this angle: US activities that have grown the fastest are those at the last stages of production chains, which also implies that intermediate goods and early-stage production is becoming relatively less important.

## A global shift towards downstream industries

While the previous results document the fragmentation of production in the US, and particularly the shift of value-added towards downstream industries, one can ask whether similar results can be observed for other countries and for trade flows.

Production has become more fragmented *across borders* (Hummels, Ishii and Yi, 2001, Johnson and Noguera, 2012). This can be shown (see Johnson and Noguera, 2012) as an overall decrease of the ratio of value-added content of trade (VAX ratio) which can be interpreted (see Section 2.4) as the inverse of the average number of border crossings embodied in traded goods.

Surprisingly, I find however that trade flows have shifted towards *downstream* industries, in parallel to the shift of value-added in the US.<sup>42</sup> To document this fact, I construct the average of distance to final demand across industries weighted by the total value of world trade:

$$\widetilde{D}_{x,t} = \sum_i x_{it}^{world} D_{i,1992}$$

where  $D_{i,1992}$  is the distance index associated with industry  $i$  in year 1992 (or an alternative year) and  $x_{it}^w$  is the share of total trade of product  $i$  in world trade, at time  $t$ . To compute  $x_{it}^w$ , I use multilateral trade data from the UN-NBER database between 1962 and 1996.<sup>43</sup>

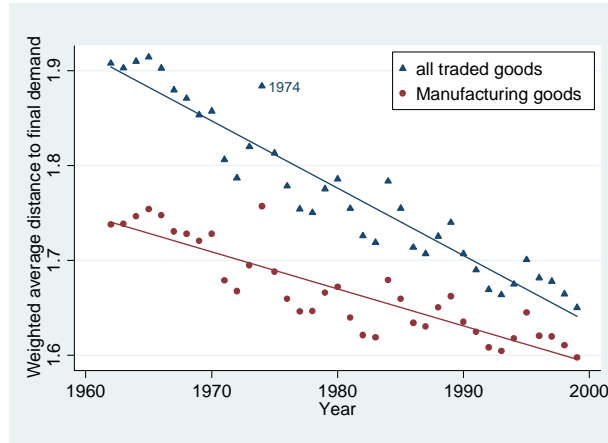
The evolution of  $\widetilde{D}_{x,t}$  is shown in Figure 4. The decline in average distance to final demand is even starker than for US value-added. The year 1974 is an outlier although petroleum-related trade flows have been dropped for the calculation of the weighted average. One may think that this shift simply reflects an increasing share of manufacturing goods relative to basic commodities, but a similar trend is obtained if we just look at trade flows in manufacturing industries.<sup>44</sup> It shows that the shift toward downstream activities is not unique to the US

<sup>42</sup>A similar finding has been pointed out by Hummels, Ishii and Yi (2001). Looking at trade across Broad Economic Classifications (distinguishing goods into capital, consumption and intermediate goods), the share of intermediate goods trade has been decreasing from 50% in 1970 to 40% in 1992. As discussed earlier, the BEC classification has some drawbacks while  $D_i$  better accounts for the position on the value chain.

<sup>43</sup>These trade data are available in the revision 2 of the SITC classification. I have used various concordance tables between SITC and SIC industries to combine the trade data with input-output measures. Alternatively, I obtain extremely similar results by using more precise concordance tables between SITC and HS product classifications, and then between HS and NAICS classification, to be finally combined with distance indexes constructed from the NAICS-based 2002 input-output table.

<sup>44</sup>Note that the fact that trade has shifted towards more downstream industries is not inconsistent with an

Figure 4: Trade-weighted distance to final demand (based on 1992 IO table)



economy and is also reflected in world trade flows. We obtain the same figure even US imports and exports are excluded in the computation above.

## 5 Vertical specialization and trade patterns

This section briefly describes alternative applications of the two measures of fragmentation examined in this paper. These two measures provide novel information on the position of each industry along production chains which is not captured by other indexes of fragmentation.

Table 9 shows that import penetration is not significantly correlated with either index  $N_i$  or  $D_i$  across industries. However, the patterns of trade and the source of imports may be related to the degree of fragmentation. A recent paper by Costinot, Vogel and Wang (2011) develops a simple model where stages along production chains are naturally sorted across countries depending on their productivities. They predict that poor countries specialize in early stages while more developed countries specialize in final stages. They also predict that poor countries should be involved in shorter production chains, while developed countries specialize in longer production chains.

To examine these predictions, I regress US imports in 1992 (by industry  $i$  and source country  $c$ ) on industry dummies, country dummies and two interaction terms: i) between GDP per capita of the source country  $c$  and the number of production stages  $N_i$  in industry  $i$

overall increase in *international* fragmentation (as shown for instance by the decrease in the VAX ratio, Johnson and Noguera, 2012). For instance, let us consider two industries: a downstream industry (e.g. assembly) and an upstream industry (e.g. components). If international trade is initially more concentrated in the upstream industry, with downstream activities taking place where final goods are consumed, the value-added content of trade would be large since they would be no “vertical specialization” as defined by Hummels *et al.* (2001). Now, if international trade occurs in both downstream and upstream industries, patterns of vertical specialization would appear and the value-added content of trade (VAX ratio) would decrease.

(measured for the US as above); ii) between GDP per capita and the distance to final demand (index  $D_i$ ):<sup>45</sup>

$$\log E[M_{ic}] = \beta_N \cdot N_i \cdot \log(\text{pcGDP}_c) + \beta_D \cdot D_i \cdot \log(\text{pcGDP}_c) + \alpha_i + \eta_c$$

Such approach using interaction terms has been put forward by Romalis (2004) and Nunn (2007) among others. If richer countries specialize in goods involving more stages, we could expect a positive coefficient  $\beta_N$ . If richer countries specialize in stages that are closer to final demand, we could expect a negative coefficient  $\beta_D$ . Since patterns of fragmentation are related to other industry characteristics such as capital and skill intensity (see Table 5), I further control for interactions between capital intensity and capital endowments, skill intensity and skill endowments (as in Romalis, 2004).

Table 11: Comparative advantage along supply chains

Dependent variable:	Imports	Imports	Imports	Imports
pcGDP <sub>c</sub> * production stages $N_i$	-0.420 [0.090]***		-0.421 [0.091]***	-0.209 [0.096]***
pcGDP <sub>c</sub> * stages to final demand $D_i$		-0.075 [0.033]**	-0.065 [0.033]*	-0.180 [0.042]***
Skill endowment <sub>c</sub> * Skill intensity <sub>i</sub>				6.118***
K endowment <sub>c</sub> * K intensity <sub>i</sub>				0.314***
Industry and country fixed effects	Yes	Yes	Yes	Yes
Observations	46412	46412	46412	31696
Log pseudolikelihood	-34632	-34647	-34629	-31032

Notes: Negative binomial PML regressions with robust standard errors in brackets; \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%.

Table 11 shows that, surprisingly, rich countries are more likely to export goods involving fewer production stages, with a negative and significant interaction terms in column (1). Also, richer countries seem to specialize in industries that are closer to final demand (column 2). The latter is consistent with Costinot et al (2011) while the former is not. In column (4), I further control for endowments in skilled labor and capital and interactions with skill and capital intensities (which are both positive and significant as in Romalis, 2004). With these controls, results are more in line with Costinot et al (2011) with a stronger coefficient for the interaction with the number of stages to final demand  $D_i$  and a smaller coefficient for the interaction with the number of production stages  $N_i$ .

Other applications of the second index  $D_i$  (“upstreamness”) are explored in Antràs, Chor,

<sup>45</sup>This equation with Negative-Binomial PML which allows for zeros and overdispersion. The same qualitative results are obtained with OLS (in log).

Fally and Hillberry (2012) where we examine the role of institutional quality in explaining patterns of specialization along value chains. In the working paper version (Fally 2012), I also examine the effect of distance depending on the position on production chains.

## 6 Conclusion

In this paper, I provide a novel measure of the fragmentation of production reflecting the average number of production stages by industry weighted by the contribution of each stage to value added. A variant of this measure reflects the number of stages between an industry's production and final demand. These indexes offer simple structural interpretations. These indexes only require input-output tables that are generally publicly available. They satisfy interesting aggregation properties: i) the weighted average equals the gross-output-to-value-added ratio in a closed economy; ii) at the industry level, these indexes are not likely to be biased by using more aggregated input-output matrices.

The key finding is that US industries have become less vertically fragmented over the past 50 years. The average number of production stages seems to have decreased according to the above fragmentation index computed using the BEA US input-output tables since 1947. This fact is not just limited to a composition effect between services and tradable goods. When I exclude services, I also find a decline in the number of production stages on aggregate. Among manufacturing industries, I find a relatively smaller declines in more specific, R&D- and capital-intensive industries, and larger declines in skill-intensive and financially dependent industries.

Trade and prices do not play an important role in explaining these results. While the commodity-price shock of the mid-70's can explain a temporary increase in measured fragmentation, long-term changes in fragmentation do not reflect systematic changes in relative prices of upstream vs. downstream goods. Also, import penetration in the US is not correlated with an industry's position on the value-added chain across industries, and the change in fragmentation is not strongly correlated with increases in import penetration in upstream industries.

In order to provide a more intuitive view on the decrease in vertical fragmentation, I examine the evolution of the relative contribution of stages to value added. In particular, I find a large and significant shift of value added towards production stages that are closer to final demand, which generates an overall decrease in weighted-average number of production stages. Half of this shift can be explained by observable industry characteristics such as intensities in the use of capital, skilled labor and advertising services.

While this paper mainly focuses on the vertical fragmentation of production in the US, the measures of fragmentation developed here may have various other applications. I illustrate one of those by investigating patterns of US imports depending on the position of industries along

value chains and the level of development of the exporting country. In particular, I find that rich countries have a comparative advantage in industries that are less vertically fragmented and closer to final demand.

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# Appendix sections for online publication

## A. Mathematical Appendix

**Proposition 1:** If  $N_i$  is defined recursively as in equation (1) and  $v_i^{(n)}$  is defined as above, then:

$$N_i = \sum_{n=1}^{\infty} n v_i^{(n)}$$

In other words,  $N_i$  is the average number of stages to produce good  $i$  weighted by the share  $v_i^{(n)}$  of value added at each stage  $n$ .

**Proof:** Suppose that  $N_i$  is defined by:  $N_i = \sum_{n=1}^{\infty} n v_i^{(n)}$  where the fraction  $v_i^{(n)}$  is defined recursively by  $v_i^{(n+1)} = \sum_j \mu_{ij} v_j^{(n)}$ , with  $v_i^{(1)} = \frac{V_i}{Y_i}$ . We need to show that  $N_i$  verifies the recursive definition of equation (1).

First, note that  $\sum_{n=1}^{\infty} v_i^{(n)} = 1$ . To see this point, note that

$$\begin{aligned} 1 - \sum_{n=1}^{\infty} v_i^{(n)} &= 1 - v_i^{(1)} - \sum_{n=1}^{\infty} v_i^{(n+1)} \\ &= 1 - \frac{V_i}{Y_i} - \sum_{n=1}^{\infty} \sum_j \mu_{ij} v_j^{(n)} \\ &= \sum_j \mu_{ij} - \sum_j \mu_{ij} \left( \sum_{n=1}^{\infty} v_j^{(n)} \right) \\ &= \sum_j \mu_{ij} \left( 1 - \sum_{n=1}^{\infty} v_j^{(n)} \right) \end{aligned}$$

Assuming that the identity matrix minus the input-output matrix is invertible (see footnote 11),  $1 - \sum_{n=1}^{\infty} v_j^{(n)} = 0$  is the only solution of the system of equation  $x_i = \sum_j \mu_{ij} x_j$ .

Using the above definition of  $N_i$ , we obtain successively:

$$\begin{aligned} N_i &= \sum_{n=1}^{\infty} n v_i^{(n)} \\ &= \sum_{n=0}^{\infty} (1+n) v_i^{(n+1)} \\ &= \sum_{n=0}^{\infty} v_i^{(n+1)} + \sum_{n=1}^{\infty} n v_i^{(n+1)} \\ &= \sum_{n=1}^{\infty} v_i^{(n)} + \sum_{n=1}^{\infty} n \sum_j \mu_{ij} v_j^{(n)} \end{aligned}$$

Then, using the recursive definition of  $v_i^{(n)}$  and using the fact that  $\sum_{n=1}^{\infty} v_i^{(n)} = 1$ , this becomes:

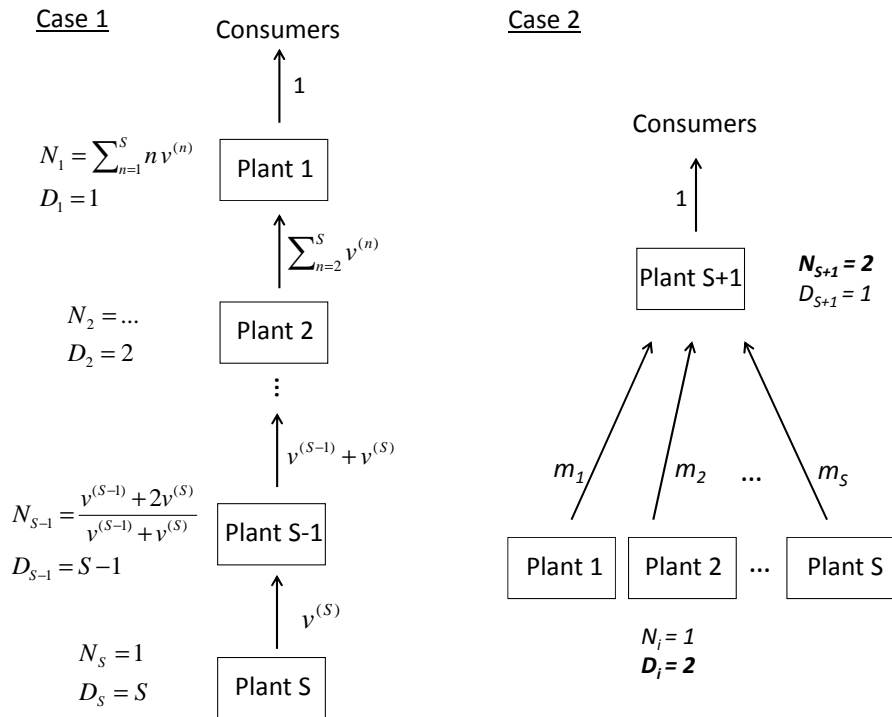
$$\begin{aligned}
 N_i &= 1 + \sum_{n=1}^{\infty} n \sum_j \mu_{ij} v_j^{(n)} \\
 &= 1 + \sum_j \mu_{ij} \sum_{n=1}^{\infty} n v_j^{(n)} \\
 &= 1 + \sum_j \mu_{ij} N_j
 \end{aligned}$$

which corresponds to equation (1).

### Section 2.1: Illustration

Snakes or spiders? This point is illustrated in Figure 5, cases 1 and 2. Case 1 involves sequential production whereas case 2 involves simultaneous production.

Figure 5: Vertical vs. horizontal fragmentation: an illustration



In case 1, the measure of fragmentation increases with the number of suppliers because each of them enters sequentially in production. Each plant  $n$  contributes to a fraction  $v^{(n)}$  of the final value of the product ( $\sum_{n=1}^S v^{(n)} = 1$ ). According to Proposition 1,  $N$  equals  $\sum_{n=1}^S n v^{(n)}$  for the final product and increases with  $S$ .

In case 2, however, they all ship to the same plant, so the degree of verticality does not depend on how many of them ship to this plant. Index  $N$  does not depend on the number

of plants as long as they all contribute to a constant fraction  $\sum_j m_j$  of the value of the final product. In the example above, I set  $\sum_j m_j = 1$ , which implies that  $N = 2$  for the final good whatever the number of suppliers involved in parallel.

A production chain such as case 1 also provides a simple example to illustrate index  $D$ . In this example, plants are indexed from 1 to  $S$  depending on their position on the chain (with with one being the closest to consumers). We obtain that  $D_n = n$  for each plant  $n$ . Note that, in this example, the measure of production stages for the last stage  $N_n$  equals the average of the distance to final demand  $D_i$  across all plants  $i$  weighted by the contribution of each plant to value added. This result is a corollary of Proposition 1 and also holds for the aggregate economy (Proposition 2).

### Section 2.3: Structural interpretations of $N$ : details on examples ii) and iii)

**Example ii):** If the production function for product  $i$  has constant returns to scale in all inputs  $j$  plus labor, the unit cost is a homogenous function of degree one in prices of each input and labor, and is inversely proportional to productivity. Keeping wages constant, we obtain that the relative change in prices satisfies:

$$\hat{P}_i = -\hat{Z} + \sum_j \mu_{ij} \hat{P}_j$$

where hats denote relative changes and  $\mu_{ij}$  is the share of input  $j$  in total cost of production for  $i$ . We can see that  $\frac{-\hat{P}_i}{\hat{Z}}$  satisfies the same recursive definition as  $N_i$ . Hence:  $\hat{P}_i = -\hat{Z} N_i$ .

Concerning welfare, the result shown in equation (3) is obtained by considering the expenditure function and the envelop theorem. In equilibrium, quantities of goods for final consumption maximize utility given the set of prices. Hence the change in expenditures generated by a change in prices is given by:

$$\hat{e} = \sum_i \alpha_i \hat{P}_i$$

where  $\alpha_i = \frac{C_i}{\sum_j C_j}$  is the share of good  $i$  in final consumption. Using the previous results on price changes, we obtain the formula in the text.

**Example iii):** In a first case, let us consider an economy with  $J$  industries, in perfect competition, characterized by the following equations:

$$\begin{aligned} Q_i &= Q_i^F + \sum_j Q_{ji}^M \\ Q_i &= Z A_i \cdot \prod_{j=1}^J (Q_{ij}^M)^{\mu_{ij}} \cdot L_i^{1-\sum_j \mu_{ij}} \\ U &= \prod_{i=1}^J (Q_i^F)^{\alpha_i} \\ \bar{L} &= \sum_i L_i \end{aligned}$$

where  $U$  defines preferences in terms of consumption of goods  $i$ , with the sum  $\sum_i \alpha_i$  normalized to unity;  $Q_i^F$  refers to the quantity of final goods  $i$  whereas  $Q_{ij}^M$  refers to the quantity of goods  $j$  used as inputs for the production of good  $i$ . In addition, we normalize wages (and nominal income) to unity. Nominal GDP is therefore equal to population  $\bar{L}$ .

In this framework, final consumption (in value) is a constant fraction of total income:  $C_i \equiv P_i Q_i^F = \alpha_i \bar{L}$ . Intermediate demand (in value) is also a constant fraction of downstream

production  $Y_i$  (in value):  $Y_{ij} \equiv P_j Q_{ij}^M = \mu_{ij} Y_i$ . Hence, the value of production in sector  $j$  satisfies:  $Y_j = \alpha_j \bar{L} + \sum_i \mu_{ij} Y_i$ . Taking all sectors, this system of equation determines sectoral production as a function of total income and parameters  $\alpha_i$  and  $\mu_{ij}$ . In particular, the value of production does not depend on  $Z$ .

This framework is a special case of example ii). The result on prices applies:  $\frac{\partial P_i}{\partial Z} = -N_i$ . Since the value of production does not depend on  $Z$ , quantities should satisfy:

$$\frac{\partial Q_i}{\partial Z} = N_i$$

Now, suppose instead that we have the following Leontief production function:

$$Q_i = Z \cdot \min_j \left\{ \frac{Q_{ij}}{\alpha_{ij}}, \frac{L_i}{\alpha_{iL}} \right\}$$

with  $Q_i$  denoting the production (in quantity) of good  $i$ ,  $Q_{ij}$  is the quantity of input  $j$  used for the production of  $i$ ,  $Z$  reflects productivity,  $L_i$  is the amount of labor for the production of good  $i$ ,  $\alpha_{ij}$  and  $\alpha_{iL}$  are parameters.

In the spirit of the O-ring theory (Kremer, 1993) and Costinot Vogel and Wang (2012), we can interpret  $Z$  as being determined by the probability that no mistake arise, assuming that mistakes potentially arise at each stage of production (i.e. for the production of each good  $i$ , whether it is a final or intermediate good).

Suppose also that utility is a Leontief function of final consumption  $Q_i^F$ :

$$U = \min_i \left\{ \frac{Q_i^F}{\alpha_{iF}} \right\}$$

In this framework, we obtain that  $Q_i^F = \alpha_{iF} U$  where  $U$  is the level of utility attained at equilibrium. Total production quantities of good  $i$  satisfies:

$$Q_i = \alpha_{iF} U + \sum_j \alpha_{ji} Q_j / Z$$

Given a change in productivity  $\hat{Z}$  (generating a change in utility  $\hat{U}$ ), the effect on production is:

$$\hat{Q}_i = (1 - \sum_j \varphi_{ij}) \hat{U} + \sum_j \varphi_{ji} (\hat{Q}_j - \hat{Z})$$

where  $\varphi_{ij} = \frac{\alpha_{ji} Q_j / Z}{Q_i}$  denotes the share of production of good  $i$  absorbed as intermediate goods for industry  $j$ .

From the previous equation, we obtain that:

$$\hat{Q}_i - \hat{Z} - \hat{U} = -\hat{Z} + \sum_j \varphi_{ji} (\hat{Q}_j - \hat{Z} - \hat{U})$$

We can see that  $\frac{\hat{Q}_i - \hat{Z} - \hat{U}}{-\hat{Z}}$  satisfies the same recursive equation defining  $D_i$ , the index of “distance

to final demand”, and thus should be equal to  $D_i$ . Therefore:

$$\hat{Q}_i - \hat{Z} - \hat{U} = -\hat{Z}D_i$$

Taking the diifference between any two industries, we obtain:

$$\hat{Q}_i - \hat{Q}_j = -\hat{Z}(D_i - D_j)$$

which corresponds to the result shown in the main text.

**Proposition 2:** In a closed economy, the aggregate measure of fragmentation equals the gross output to value added ratio:  $\frac{\sum_i C_i N_i}{\sum_i C_i} = \frac{\sum_i Y_i}{\sum_i V_i}$  (part 1) and  $\frac{\sum_i V_i D_i}{\sum_i V_i} = \frac{\sum_i Y_i}{\sum_i V_i}$  (part 2).

**Proof:** We use two equalities: the definition of measure of fragmentation  $N_i = 1 + \sum_j \mu_{ij} N_j$ , and the link between final consumption, intermediate demand and production (in a closed economy):  $C_i = Y_i - \sum_j \mu_{ji} Y_j$ . We obtain:

$$\begin{aligned} \sum_i C_i N_i &= \sum_i \left( Y_i - \sum_j \mu_{ji} Y_j \right) N_i \\ &= \sum_i Y_i N_i - \sum_{i,j} \mu_{ji} Y_j N_i \\ &= \sum_i Y_i N_i - \sum_{i,j} \mu_{ij} Y_i N_j \\ &= \sum_i Y_i N_i - \sum_i Y_i \left( \sum_j \mu_{ij} N_j \right) \\ &= \sum_i Y_i N_i - \sum_i Y_i (N_i - 1) \\ &= \sum_i Y_i \end{aligned}$$

Similarly, for the other measure  $D_i$  (part 2), we obtain:  $\sum_i V_i D_i = \sum_i Y_i$  by using the definition  $D_i = 1 + \sum_j \varphi_{ij} D_j$  and the equality  $V_i = Y_i - \sum_j \mu_{ij} Y_j = Y_i - \sum_j \varphi_{ji} Y_j$ . The proof follows the same steps.

Finally, notice that the sum of final demand  $\sum_i C_i$  equals the sum of value added  $\sum_i V_i$ .

**Proposition 3:** In an open economy:

$$\begin{aligned} \frac{\sum_i C_i N_i}{\sum_i C_i} &= \bar{N} + \frac{\sum_i (M_i - X_i)(N_i - \bar{N})}{\sum_i C_i} \\ \frac{\sum_i V_i D_i}{\sum_i V_i} &= \bar{N} - \frac{\sum_i (M_i - X_i)(D_i - 1)}{\sum_i V_i} \end{aligned}$$

Where  $\bar{N}$  denotes the ratio of gross output to value added  $\frac{\sum_i Y_i}{\sum_i V_i}$ .

**Proof:** In an open economy, final consumption satisfies  $C_i = Y_i - \sum_j \mu_{ji} Y_j + M_i - X_i$ . Let us define  $F_i \equiv Y_i - \sum_j \mu_{ji} Y_j$ . We deduce that  $C_i = F_i + (M_i - X_i)$ . Following the same path as

in the proof of Proposition 2, we can show that  $\sum_i F_i N_i = \sum_i Y_i$ . Moreover, we can verify that  $\sum_i F_i$  equals total value added  $\sum_i V_i$  and thus:  $\bar{N} \sum_i F_i = \sum_i Y_i$ .

Using these three equalities above, we obtain:

$$\begin{aligned} \sum_i C_i(N_i - \bar{N}) &= \sum_i F_i(N_i - \bar{N}) + \sum_i (M_i - X_i)(N_i - \bar{N}) \\ &= \left( \sum_i Y_i - \bar{N} \sum_i F_i \right) + \sum_i (M_i - X_i)(N_i - \bar{N}) \\ &= 0 + \sum_i (M_i - X_i)(N_i - \bar{N}) \end{aligned}$$

After dividing by total consumption, this provides the first equality of Proposition 3.

Turning to the second equality, we use the following relationship between  $\varphi_{ij}$  and input-output coefficients in open economy:  $\varphi_{ij} = \frac{Y_j}{Y_i + M_i - X_i} \cdot \mu_{ji}$ . We obtain:

$$\begin{aligned} \sum_i V_i D_i &= \sum_i \left( Y_i - \sum_i \mu_{ij} Y_i \right) D_i \\ &= \sum_i Y_i D_i - \sum_{i,j} \mu_{ij} Y_i D_i \\ &= \sum_i Y_i D_i - \sum_{i,j} \mu_{ji} Y_j D_j \\ &= \sum_i Y_i D_i - \sum_{i,j} (Y_i + M_i - X_i) \varphi_{ij} D_j \\ &= \sum_i Y_i D_i - \sum_i (Y_i + M_i - X_i) (D_i - 1) \\ &= \sum_i Y_i - \sum_i (M_i - X_i) (D_i - 1) \end{aligned}$$

After dividing by total value added  $\sum_i V_i$  and using the definition of  $\bar{N} = \sum_i Y_i / \sum_i V_i$ , we get the second equality of Proposition 3.

## Section 2.5: From varieties to industries

In this appendix section I derive conditions under which the index measured at the industry level (equation 1) equals the average of an ideal index at the plant level weighted by the value of production by each plant that is sold to final consumers. If production techniques are homogenous across plants within each industry, this question would be irrelevant. However, Fort (2011) documents substantial heterogeneity within each industry in terms of fragmentation of production and sourcing strategies.

Some additional notation is needed for this appendix section only. Let us assume that each industry  $i$  is composed of a set of varieties  $\omega \in \Omega_i$ . These sets  $\Omega_i$  offer a partition of the set of all varieties produced in the economy. If we denote by  $y(\omega)$  the value of production of variety  $\omega$ , gross output  $Y_i$  of industry  $i$  can be defined as  $Y_i = \int_{\Omega_i} y(\omega) d\omega$ .

Without loss of generality, I assume that each variety is either sold to final consumers or

sold to a unique downstream industry  $j$ .<sup>46</sup> I denote by  $\Omega_{ij}$  the set of varieties in industry  $i$  that are sold as intermediate goods to industry  $j$ , and I denote by  $\Omega_{iF}$  the set of varieties in industry  $i$  that are sold as final goods. For a given industry  $i$ , the sets  $\Omega_{ij}$  and  $\Omega_{iF}$  offer a partition of  $\Omega_i$ . In particular,  $\Omega_{ii}$  refers to the set of varieties of industry  $i$  that are used as intermediate goods by industry  $i$  (e.g. chemicals used as inputs for other chemicals). Using this notation, industry-level input-output coefficients correspond to:

$$\mu_{ij} = \frac{\int_{\Omega_{ji}} y(\omega) d\omega}{\int_{\Omega_i} y(\omega) d\omega}$$

Now let us assume that  $N(\omega)$  is the “true” index of production stages at the variety level which could be measured if we had plant-level input-output matrices, i.e. data on the full supply chain for each variety  $\omega$ . Under the following conditions, the industry-level index equals a weighted average of the variety-level index in each industry:

**Proposition 4** *If  $(\int_{\Omega_{ij}} y(\omega)N(\omega)d\omega)/(\int_{\Omega_{ij}} y(\omega)d\omega)$  does not depend on the downstream industry  $j$ , for all  $j \neq i$  or  $j = F$ , then:*

$$N_i = \frac{\int_{\Omega_{iF}} y(\omega)N(\omega)d\omega}{\int_{\Omega_{iF}} y(\omega)d\omega}$$

*is the solution to equation (1) which characterizes index  $N_i$  at the industry level.*

In other words, the industry-level index defined by equation (1) provides an unbiased measure of the average of the “true” index at the variety level (weighted by final consumption) provided that the number of production stages does not depend on the buying industry  $j$ . Formally, it requires that:

$$\frac{\int_{\Omega_{ij}} y(\omega)N(\omega)d\omega}{\int_{\Omega_{ij}} y(\omega)d\omega} = N_i$$

whatever the downstream industry  $j \neq i$ . While plants may be heterogeneous in terms of production processes, such heterogeneity matters in terms of aggregation only if there is a systematic link between supply and demand across industries. For instance, if more productive firms are more likely to fragment their production, this would affect the measure of the industry-level index only if those firms are more likely to sell goods to a particular downstream industry rather than another.

Note also that these conditions do not impose any constraint on within-industry linkages and we may have:

$$\frac{\int_{\Omega_{ii}} y(\omega)N(\omega)d\omega}{\int_{\Omega_{ii}} y(\omega)d\omega} \neq N_i$$

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<sup>46</sup>While in practice the same type of product (e.g. tires) can be sold as an intermediate good to a downstream industry (e.g. the auto industry) and as a final good to consumers, for accounting purposes we can simply consider these products as different varieties that require the same production process (e.g. tires sold to final consumers vs. other tires).

In particular, if all varieties are aggregated into a unique industry (representing the whole economy), the measured index of production stages for an aggregate closed economy (GO/VA) is unbiased and equals the average of the index across all varieties that are sold to final consumers.

In order to mitigate the aggregation bias, more aggregation might be an answer instead of an issue. Indeed, if fragmentation depends on the buying industry, aggregating industries into larger industries might actually eliminate such patterns. For instance, if the production of auto parts is more or less fragmented depending on whether buyers are final consumers or plants in the auto industry, then aggregating auto parts with the rest of the auto industry would eliminate the bias that arises between the observed index of production stages and the true average across varieties of the number of production stages.

Similar properties can be derived for the distance to final demand  $D_i$ . Let  $v(\omega)$  denote the value added in the production of variety  $\omega$  and  $\mu_j(\omega)$  denote the use of inputs from industry  $j$  in the production of variety  $\omega$ . We obtain the following conditions for unbiased aggregation:

**Proposition 5** *If:  $(\int_{\Omega_i} y(\omega)\mu_j(\omega)D(\omega)d\omega)/(\int_{\Omega_i} y(\omega)\mu_j(\omega)d\omega) = (\int_{\Omega_i} v(\omega)D(\omega)d\omega)/(\int_{\Omega_i} v(\omega)d\omega)$  for all upstream industries  $j \neq i$ , then:*

$$D_i = \frac{\int_{\Omega_i} v(\omega)D(\omega)d\omega}{\int_{\Omega_i} v(\omega)d\omega}$$

*is the solution to equation (2) which defines index  $D_i$  at the industry level.*

In other words, the measure of the number of stages to final demand is unbiased at the industry level if there are no systematic differences in the distance to final demand depending on the use of inputs.

**Proof of Proposition 4:** If  $N(\omega)$  denotes the average number of stages required to produce variety  $\omega$  (same definition as for the industry-level index but at the variety- or plant-level), then  $N(\omega)$  equals 1 plus the weighted average of the index for inputs required to produce variety  $\omega$ . Aggregating over all varieties  $\omega \in \Omega_i$  in industry  $i$ , we obtain:

$$\int_{\Omega_i} y(\omega)N(\omega)d\omega = \int_{\Omega_i} y(\omega)d\omega + \sum_j \int_{\Omega_{ji}} y(\omega')N(\omega')d\omega'$$

where  $\omega'$  refers to varieties of inputs, and where  $\Omega_{ji}$  refers to the set of input varieties  $\omega'$  in industry  $j$  that enter the production of varieties in industry  $i$ . Note that the first term of the right-hand side corresponds to output in industry  $i$ :

$$\int_{\Omega_i} y(\omega)N(\omega)d\omega = Y_i + \sum_j \int_{\Omega_{ji}} y(\omega')N(\omega')d\omega'$$

If we exclude varieties in  $\Omega_i$  that are used as inputs for industry  $i$  (i.e. only consider varieties  $\omega \in \Omega_i \setminus \Omega_{ii}$ ), we have then:

$$\int_{\Omega_i \setminus \Omega_{ii}} y(\omega)N(\omega)d\omega = Y_i + \sum_{j \neq i} \int_{\Omega_{ji}} y(\omega')N(\omega')d\omega'$$



Let us denote by  $\tilde{N}_i = \frac{\int_{\Omega_{iF}} y(\omega)N(\omega)d\omega}{\int_{\Omega_{iF}} y(\omega)d\omega}$  the “true” average index across varieties in industry  $i$  weighted by final demand. If the conditions enounced in Proposition 4 are satisfied, the set  $\Omega_{iF}$  in the definition of  $\tilde{N}_i$  can be replaced by the set  $\Omega_i \setminus \Omega_{ii}$  that includes all varieties of industry  $i$  not sold as input for industry  $i$ . By using again the conditions enounced in Proposition 4 (between lines 3 and 4 in the following equalities), we obtain successively:

$$\begin{aligned}
\tilde{N}_i &= \frac{\int_{\Omega_i \setminus \Omega_{ii}} y(\omega)N(\omega)d\omega}{\int_{\Omega_i \setminus \Omega_{ii}} y(\omega)d\omega} \\
&= \frac{\int_{\Omega_i \setminus \Omega_{ii}} y(\omega)N(\omega)d\omega}{Y_i - \mu_{ii}Y_i} \\
&= \frac{Y_i + \sum_{j \neq i} \int_{\Omega_{ji}} y(\omega)N(\omega)d\omega}{(1 - \mu_{ii})Y_i} \\
&= \frac{Y_i + \sum_{j \neq i} \tilde{N}_j \int_{\Omega_{ji}} y(\omega)d\omega}{(1 - \mu_{ii})Y_i} \\
&= \frac{Y_i + \sum_{j \neq i} \tilde{N}_j \mu_{ij} Y_i}{(1 - \mu_{ii})Y_i} \\
&= \frac{1 + \sum_{j \neq i} \mu_{ij} \tilde{N}_j}{1 - \mu_{ii}}
\end{aligned}$$

After rearranging, we find:

$$\tilde{N}_i = 1 + \sum_j \mu_{ij} \tilde{N}_j$$

This shows that  $\tilde{N}_i = N_i$  if the condition in Proposition 4 is satisfied.

**Proof of Proposition 5:** The proof follows the same logic and steps as for Proposition 4.

Propositions 4 and 5 can also be used to examine partial aggregation properties: what happens when two industries are merged together in the industry classification? Details are provided in the working paper version (Fally 2012).

## Section 2.5: Correspondence with the VAX ratio

Johnson and Noguera (2012) define fragmentation as cross-border production sharing. Their measure of fragmentation for the aggregate world economy is the ratio of total value-added content of exports to the total gross value of exports (“VAX\_world”). In keeping with Johnson and Noguera’s notation, this is:

$$VAX_{world} = \frac{\sum_{i \neq j} \sum_s va_{ij}(s)}{\sum_{i \neq j} \sum_s x_{ij}(s)}$$

where  $x_{ij}(s)$  denotes bilateral gross trade flows between countries  $i$  and  $j$  in sector  $s$  and where  $va_{ij}(s)$  the value-added content of trade between  $i$  and  $j$ .

There is a close link between the two measures of fragmentation, the VAX ratio in Johnson and Noguera (2012) and the gross-output-to-value-added ratio in this paper. In particular, Propositions 1 and 2 can shed light on the interpretation of the VAX ratio. To see the correspondence, one could treat each country as one plant where the counterpart of exports is gross output and the counterpart of value-added content of trade is value added.

Lemma 1 below formalizes the correspondence between gross output and gross exports derived as functions of the vector of final demand.

Lemma 2 below formalizes the correspondence between value-added at the plant level and the value-added content of trade at the country level.

Using Propositions 1 and 2, we can conclude that the inverse of the VAX ratio corresponds to the number of embedded border crossings in each dollar of the final good, weighted by the contribution of each country to total value-added content of trade (details are provided in the appendix section). Formally:

$$\frac{1}{VAX_{world}} = \frac{\sum_n \sum_{i \neq j} n \cdot va_{ij}^{(n)}}{\sum_n \sum_{i \neq j} va_{ij}^{(n)}}$$

where  $va_{ij}^{(n)}$  denotes the part of the value added by country  $i$  that is going to cross  $n$  borders before reaching final demand in country  $j$ . Hence the *inverse* of the VAX ratio is the analogous of the gross-output to value added ratio when focusing on cross-border transactions instead of transactions between plants.

The starting point in Johnson and Noguera (2010) is to construct a global input-output matrix  $A$  relating the use of input by destination and source country. They use this global IO matrix to derive output as a function of absorption in each country. Using their notation (with  $i$  and  $j$  being country subscripts):

$$y_j = (I - A)^{-1} f_j$$

where  $f_j$  is the vector of final goods to be purchased by final consumers in country  $j$ . Gross output  $y_j$  is the sum of both domestic sales and gross exports.

**Lemma 1:** *Gross trade  $x_j$  of goods absorbed in final destination  $j$  can be expressed as:*

$$x_j = (I - \tilde{A})^{-1} \tilde{f}_j$$

where  $\tilde{A}$  is an input-output matrix for trade flows, i.e. describing import requirement for each dollar of gross exports, and  $\tilde{f}$  is a vector of export to their final destination.

**Proof of Lemma 1:** Let us define  $A^D$  the domestic component of the global IO matrix (i.e. the block-diagonal matrix with blocks  $A_{ii}$  describing the use of inputs from country  $i$  by industries in  $i$ ) and let us define  $A^M$  the IO import matrix for the use of inputs from other countries:  $A^M = A - A^D$ .

Similarly, let us denote by  $f^D$  the vector of final goods consumption from domestic sources and by  $f^M$  the vector of imported final goods:  $f = f^D + f^M$ . Let us also denote  $x$  the vector of gross exports and  $h$  the vector of gross domestic shipments. We can obtain the following

accounting equality:

$$\begin{aligned}x &= A^M(x + h) + f^M \\h &= A^D(x + h) + f^D\end{aligned}$$

The first term in the first equation corresponds to imported intermediate goods and the second term reflects imported final goods, while the second equation reflects the purchase of intermediate and final goods from domestic sources. Solving for  $h$ , we obtain that:

$$h = (I - A^D)^{-1}A^Dx + (I - A^D)^{-1}f^D$$

Plugging in  $h$  back into the expression for  $x$ , we obtain successively:

$$\begin{aligned}x &= A^Mx + A^Mh + f^M \\&= A^Mx + A^M(I - A^D)^{-1}A^Dx + A^M(I - A^D)^{-1}f^D + f^M \\&= A^M \left( I + (I - A^D)^{-1}A^D \right) x + A^M(I - A^D)^{-1}f^D + f^M \\&= A^M(I - A^D)^{-1}x + A^M(I - A^D)^{-1}f^D + f^M \\&= \tilde{A}x + \tilde{f}\end{aligned}$$

where  $\tilde{A} \equiv A^M(I - A^D)^{-1}$  and  $\tilde{f} \equiv f^M + \tilde{A}f^D$ . In words,  $\tilde{A}$  is the matrix of imports directly required for each dollar of export  $x$  and indirectly for domestic output generated by this export through domestic requirements. We can then solve directly for trade:

$$x = (I - \tilde{A})^{-1}\tilde{f}$$

As in Johnson and Noguera (2010), we can also split trade and output depending on the final destination country  $j$  as:

$$x_j = (I - \tilde{A})^{-1}\tilde{f}_j$$

**Lemma 2:** *Total value-added content of trade from  $i$  to  $j$  (summed across all sectors  $s$ ) can be obtained as:*

$$\sum_s va_{ij}(s) = \sum_s x_{ij}(s) - \sum_s m_{ij}(s)$$

where  $x_{ij}(s)$  is the value of trade from  $i$  to final destination  $j$  in sector  $s$  minus the sum of import requirements  $m_{ij}(s)$  associated with these exports (summing across inputs), defined by  $m_j \equiv \tilde{A}x_j$ .

**Proof of Lemma 2:** Direct inputs required for output  $y_{ij}$  (output in country  $i$  for final absorption in country  $j$ ) are given by  $(I - A_i)y_{ij}$  where  $A_i$  is the global IO table component for country  $i$ . Output  $y_{ij}$  is the sum of exports  $x_{ij}$  and domestic output  $h_{ij}$  destined to final consumption in country  $j$ .

Note that if  $i \neq j$ , then  $h_{ij} = (I - A^D)^{-1}A^Dx_{ij}$  and does not depend on final goods purchased from domestic sources in  $i$ .

Combining these results, we can obtain the vector of output  $y_{ij}$  (production in country  $i$  destined to be absorbed in country  $j$ ) minus the vector of intermediate goods as a difference

between the vector of export from  $i$  (for final absorption in  $j$ ) and the vector of imported intermediate goods:

$$\begin{aligned}
(I - A_i)y_{ij} &= (I - A_i^D - A_i^M)(x_{ij} + h_{ij}) \\
&= (I - A_i^D - A_i^M) \left[ x_{ij} + (I - A_i^D)^{-1} A_i^D .x_{ij} \right] \\
&= (I - A_i^D - A_i^M) \left[ I + (I - A_i^D)^{-1} A_i^D \right] .x_{ij} \\
&= (I - A_i^D - A_i^M)(I - A_i^D)^{-1} x_{ij} \\
&= \left[ I - A_i^M (I - A_i^D)^{-1} \right] x_{ij} \\
&= \left[ I - \tilde{A}_i \right] x_{ij}
\end{aligned}$$

Then, by taking the column-sum of these vectors, the left-hand side gives the value-added content of trade from  $i$  to  $j$  as defined by Johnson and Noguera (2010): total output by country  $i$  to be absorbed in  $j$  minus total intermediate use by country  $i$  for the production of goods to be absorbed in  $j$ . Taking the column-sum of the right-hand side, we obtain total gross trade from country  $i$  to be absorbed in  $j$  minus the total use of imported intermediate goods related to these exports.

Hence, it is equivalent to measure the value-added content of trade by just looking at exports  $x_{ij}$  and the related use of imported goods using the IO matrix  $\tilde{A} = A_i^M (I - A_i^D)^{-1}$ .

**Interpretation of the VAX ratio:** Using these two lemmas we can deduce that:

- Exports can be derived from a purely international IO matrix  $\tilde{A} \equiv A^M (I - A^D)^{-1}$  and the vector of trade to be absorbed within the destination country  $\tilde{f} \equiv f^M + \tilde{A} f^D$
- The value-added content of trade (summed across sectors) can be simply obtained from trade flows and the international IO matrix  $\tilde{A}$ .

Hence to draw a parallel with Proposition 2, we can treat the world as one closed economy where only international shipments are observed, where both the value-added content of trade and the index of fragmentation can be constructed from the matrix  $\tilde{A}$  relating observed trade flows. The equivalent of an economy's gross output would be the total gross trade in this case, while total value added (GDP) would now correspond to the total value-added content of trade. Using Proposition 2, we thus obtain that the weighted number of border crossings embodied in trade flows for the world economy (weighted by value added at each "stage" i.e. each country) equals the VAX ratio.

## B. Data Appendix

### Treatment of “make” and “use” tables

“Make” and “use” industry-by-commodity tables are available from 1972 onward. I combine information from these two tables to construct a commodity-by-commodity table and estimate the amount of commodity  $j$  (input) used to produce commodity  $i$  (output).<sup>47</sup>

“Use” tables describe the value of purchases  $u_{kj}$  of input  $j$  by industry  $k$ , while “make” tables describe the value of production  $m_{ki}$  of output  $i$  for each industry  $k$ . I construct commodity-by-commodity input-output coefficients  $\mu_{ij}$  by taking the average share of input  $j$  in production of industry  $k$  weighted by the contribution of industry  $k$  to the production of output  $i$ :

$$\mu_{ij} = \sum_k \left[ \frac{m_{ki}}{\sum_{k'} m_{k'i}} \frac{u_{kj}}{\sum_{j'} m_{kj'}} \right]$$

where  $\sum_{k'} m_{k'i} = Y_i$  corresponds to total production of output  $i$  and  $\sum_{j'} m_{kj'}$  corresponds to total production of industry  $k$  – this method is based on the “industry-technology assumption” (see Guo *et al.*, 2002).

Note that this way of constructing input-output coefficients  $\mu_{ij}$  is consistent with the construction of coefficients  $\varphi_{ij}$  measuring the fraction of output  $i$  used for production of output  $j$  if they are defined as:

$$\varphi_{ij} = \sum_k \left[ \frac{u_{ki}}{(\sum_{k'} u_{k'i} + u_{Fi})} \frac{m_{kj}}{\sum_{j'} m_{kj'}} \right]$$

where  $\sum_{k'} u_{k'i} + u_{Fi}$  includes the use of product  $i$  by all industries plus final demand. In an open economy, this corresponds to total absorption  $Y_i + M_i - X_i$  i.e. domestic production plus net imports, as discussed in section 2.2. We can verify that:

$$\varphi_{ij} = \frac{Y_j \mu_{ji}}{Y_i + M_i - X_i}$$

Note also that this method to construct input-output coefficients is consistent with aggregation properties discussed in the text. In particular, we find that total value added  $\sum_i V_i$ , where value-added is defined by  $V_i = (1 - \sum_j \varphi_{ij})Y_i$  as in the text, equals total production  $\sum_{k,i} m_{ki}$  minus total use of inputs  $\sum_{k,j} u_{k,i}$ .

### Treatment of “non-comparable” and “transferred” imports

In the 1972 table and after, the sum of each column of the use table provides production for each industry (sum of value-added and intermediate purchases). Intermediate goods imports are reported as part of input usage  $u_{kj}$  as described above. Total imports and exports by product are also reported in two of the last columns.

A small share of imports, however, are reported as “non-comparable” and correspond to a distinct row in the list of inputs. These non-comparable imports correspond to products that are different from any product produced in the US such as coffee and cocoa beans. Since I need

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<sup>47</sup>The 1967 input-output table is treated as a commodity-by-commodity table. I obtain very similar results by extrapolating a “make” table from other years to adjust input-output coefficients.

to have an estimate of the number of production stages necessary to produce all inputs (even if those goods are imported), I make changes in the data for two industries: I assume that all non-comparable imports by the coffee-roasting industry (industry 142800) and the chocolate industry (industry 142002) correspond to imports of coffee and cocoa beans respectively and are comparable to “tree nuts” (commodity 020401). These two changes reduce the amount of non-comparable imports of intermediate goods by more than half and the remaining non-comparable account for less than half a percent of total production value (and are thus dropped).<sup>48</sup>

The 1967 input-output table has a different treatment for imports and a few other corrections are needed. Imports are classified in two categories: “non-comparable” imports as described above and “transferred” imports. “Transferred” imports are recorded in two places and would be double-counted if not carefully taken into account. In particular, the column-sum of the 1967 I-O table gives the sum of domestic production plus “transferred” imports classified in the same product category. Hence we need to subtract “transferred” imports to obtain domestic output. Note however that “transferred” imports of intermediate goods also appear in input-output coefficient for each input category.<sup>49</sup> In terms of final consumption, some imports destined for final consumption are classified as “non-comparable” imports (while being actually quite comparable) and may account for a large share of absorption in these industries: for instance, most imports of cars are missing in the 1967 consumption data. I thus use import data from the NBER trade database (Feenstra, 1996) to impute the amount of imports for consumption.

## Other data sources

Industry characteristics are obtained from various sources. I use the NBER-CES database (Bartelsman, Becker and Gray, 2000) to construct an index of capital intensity (value of capital stock over wages), skill intensity (share of non-production-worker wages in total wages) and productivity. The NBER-CES database is available for manufacturing industries in the SIC 1987 classification and includes all benchmark years between 1967 and 1992. Data on R&D intensity are obtained from the National Science Foundation and is available from 1982. An index of product specificity has been developed by Rauch (1999). Rauch (1999) classifies goods into three categories: goods traded on integrated markets, goods with reference prices and other goods classified as specific. I simply use a dummy being equal to one when goods are specific.<sup>50</sup> I also use an index of dependence in external finance following Rajan and Zingales (1998) methodology. Concentration indexes are obtained from the Census, which provides the Herfindahl index and the share of production by the 4 largest companies for each 1987 SIC manufacturing industry. An index of advertising intensity for manufacturing industries is constructed using the input-output coefficient for advertising-related services in 1992. Note

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<sup>48</sup>Note that the 1992 table significantly reduced the “non-comparable imports” category by associating these imports with other classified commodities. In particular, the coffee-roasting and chocolate industries in 1992 exhibit large uses of inputs classified as “tree nuts” instead of non-comparable imports, which is consistent with the changes made on earlier tables.

<sup>49</sup>For instance, imports of crude petroleum to be used by the petroleum refinement industry appear twice: in the row for transferred imports in the column of crude petroleum, and also in the row for crude petroleum in the column for petroleum refinement.

<sup>50</sup>Rauch classification follows SITC revision 2. My final index is then the fraction of goods within each 1987 being categorized as specific in the SITC classification.

Table 12: Mean and standard deviation of industry variables

Variable	Mean	Std. Dev.
Number of stages	1.684	0.251
Stages to final demand	1.574	0.672
Specificity	0.744	0.386
R&D intensity	1.944	1.942
Capital intensity	1.124	0.615
Skill intensity	0.357	0.112
Advertising intensity	1.479	2.119
Productivity	0.978	0.113
Productivity growth	0.024	0.081
Financial Dependence	0.166	1.490
Top 4 share	40.36	19.84
Import penetration	0.096	0.110

*Notes:* Mean and standard deviation of the main variables across industries.

finally that the main results presented throughout the paper are robust to dropping extreme observations for each variable (extreme percentiles).

US trade data are available in the 1972 SIC classification (after 1958) and 1987 SIC classification (after 1972) for manufacturing industries from Feenstra (1996). For section 4.5 (on imports across source countries) I complement the trade data by source country with Penn World Table data on GDP per capita (average between 1990 and 1994), physical distance (CEPII) and data on endowments in capital and skilled labor from Hall and Jones (1999).

### Section 4.3: Price decomposition

To see how the change in the input-output coefficient  $\Delta\mu_{ij,t}$  impacts the within-industry change in the fragmentation index, we can write:

$$\Delta N_{i,t} = \sum_j \Delta\mu_{ij,t} \left( \frac{N_{j,t} + N_{j,t-1}}{2} \right) + \sum_j \left( \frac{\mu_{ij,t} + \mu_{ij,t-1}}{2} \right) \Delta N_{j,t}$$

We can see this equality as a linear equation in  $\Delta N_{i,t}$ , the change in the fragmentation index for each industry. Inverting this equation, we can write the change in the index as a function of the change in input-output coefficients:

$$\Delta N_{i,t} = \sum_k a_{ik,t} \left[ \sum_j \Delta\mu_{kj,t} \left( \frac{N_{j,t} + N_{j,t-1}}{2} \right) \right]$$

where  $a_{ik,t}$  denotes the coefficients of the matrix  $(I - M_{t,t-1})^{-1}$  where  $I$  is the identity matrix and  $M_{t,t-1}$  is the matrix with coefficients  $\frac{\mu_{ij,t} + \mu_{ij,t-1}}{2}$ . Using the price-quantity decomposition of the change in direct coefficients, we thereby obtain the decomposition described in the text for the changes in  $N_i$  for each industry  $i$ .

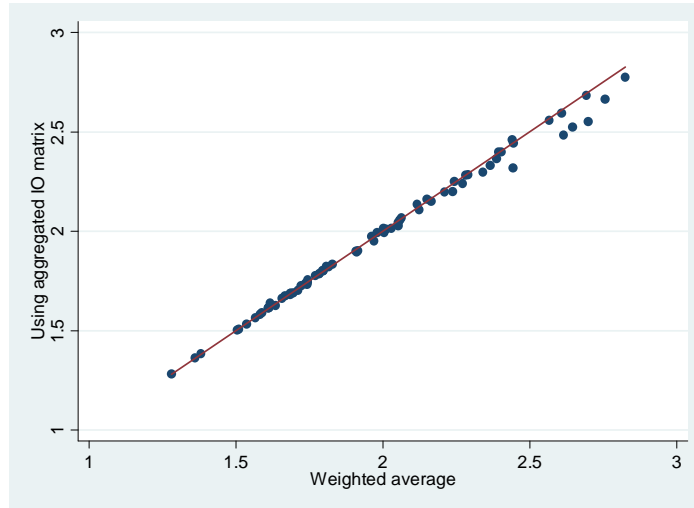
## C. Other robustness checks

A first robustness check examines aggregation properties of index  $N$  by comparing the index calculated with an aggregated input-output matrix with a weighted-average of the index calculated with a finely disaggregated matrix. A second and third robustness check examine the evolution of the relative price of intermediate goods and the evolution of transport and distribution margins. A fourth robustness check examines alternative indexes to confirm the patterns in vertical fragmentation across industries and over time.

### Aggregation

As shown by Proposition 4 and 5 in appendix section A, results at the industry-level might be sensitive to the level of disaggregation when characteristics of production across varieties within an industry are systematically related to characteristics of the buying industry. In order to check whether the level of aggregation matters, I artificially construct an aggregated input-output matrix at the 3-digit level (similar results are obtained at the 2-digit level), I reconstruct the index of fragmentation using this aggregate matrix, and I compare with the appropriately-weighted average of the disaggregated measure.

Figure 6: Aggregation at the 3-digit level



I find that the new index is always very close (less than a 1% difference on average) to the average of the disaggregated ones. This is depicted in Figure 8 where I plot the measured index using the aggregated input-output table as a function of the average of the index calculated across sub-industries using the disaggregated input-output table. We can see that the two measures differ only for extreme industries (generally belonging to the food industry).

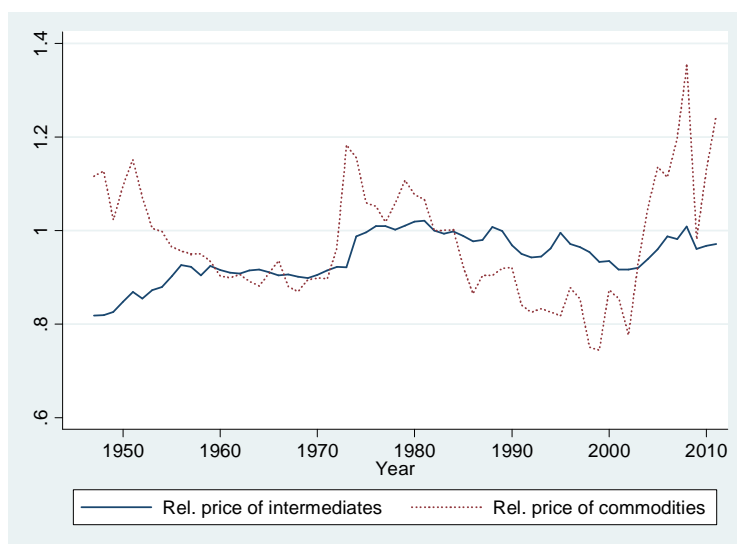
This robustness to aggregation is comforting and promising for future studies as most countries beside the US do not have precise input-output tables. For the US, where more precise but still imperfect input-output tables are available, this suggests that the results of this paper would probably not be very different if even more detailed tables were available.



## Intermediate vs. final goods prices

A first concern is that commodity prices and intermediate goods prices might have decreased compared to final goods prices. Keeping quantities constant, this could explain a downward trend in the fragmentation index. To investigate this issue, I compare producer price index series from the Federal Reserve Economic Database (FRED) for different types of goods. In particular, I consider the following series: i) “Finished Consumer Goods”; ii) “Intermediate Materials: Supplies & Components”; iii) “Crude Materials for Further Processing”. Figure 6 plots the ratio of the price index of the second and third category over to the first one (yearly average).

Figure 7: Relative price of commodities and intermediate goods compared to final goods



There is no evidence that intermediate goods prices have declined compared to final goods over the 1947-2002 period. As shown in Figure 6, there has been instead an overall increase in the relative price of intermediate goods. Concerning the relative price of basic commodities, there is no decline over the period 1967-1992 (period corresponding to the results presented in Table 1 to 10) and only a small decline if we compare 1947 to 2002. Given the relatively small share of commodities in total production (10% of value added and gross output), this change is not large enough to explain the decrease of the measure of fragmentation.

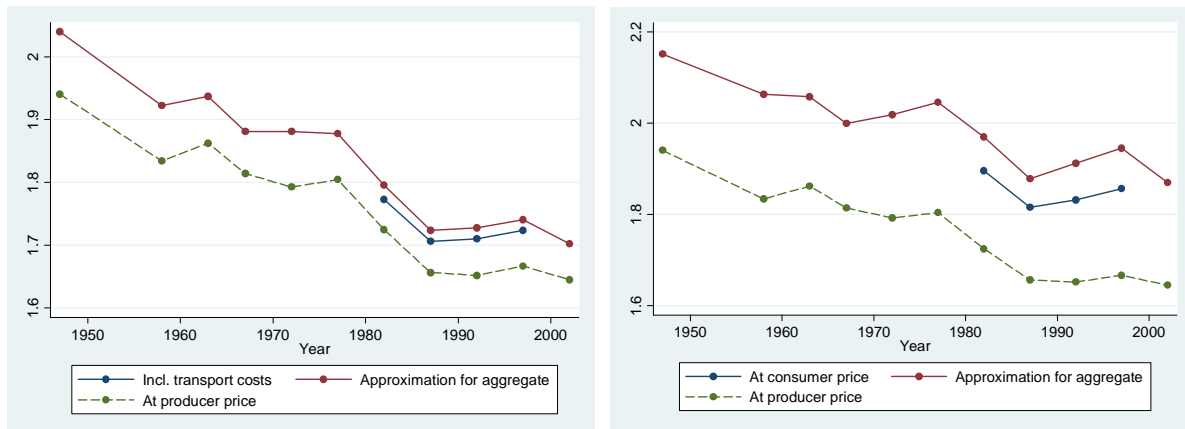
## Consumer vs. producer prices

A second issue is that the BEA input-output tables are mainly based on producer prices. This might be a concern if the main focus is the decision to outsource by the downstream firm: purchasing prices could be more appropriate. From 1982 onward, the BEA input-output tables include coefficients based on consumer price, with details on transport margins, retail and wholesale margins. Such data are not available for previous tables (1947-1977) at the industry level. For the aggregate economy, we can however approximate the index of fragmentation.

If  $\mu$  is the ratio of intermediate goods use to gross output, and  $\tau$  the total amount of spent on trade costs divided by gross output, the corrected measure of fragmentation equals  $\frac{1}{1-\mu-\tau}$  instead of  $\frac{1}{1-\mu}$ . In order to approximate  $\tau$ , I use input-output coefficients associated with the use of retail, wholesale and transportation industries as inputs.

Figure 7 (a) plots the measure of fragmentation after incorporating transportation margins only. The corrected index of fragmentation is larger as it puts more weight on intermediate goods. The approximated curve is even above the curve using actual consumer prices, but not by far. As Figure 7 (a) shows, transportation margins have remained fairly constant over the past decades and thus the negative trend in vertical fragmentation is confirmed. Similarly, the negative trend still appears after incorporating retail, wholesale as well as transportation margins (Figure 7b), even if retail and wholesale margins have slightly increased.

Figure 8: Incorporating (a) transportation and (b) retail margins



## Alternative indexes of vertical fragmentation

While the main index examined in this paper captures a weighted-average number of production stages, one may be also interested in the actual unweighted number of production stages. This would however require plant-level data with match information between buyers and suppliers, which are not available.

One could however get an idea of the number of stages by looking at input-output tables. A natural way to define the maximum number of stages involved in the production of product  $i$  could be:

$$N_i^{\max} = \max \left\{ N; v_i^{(N)} > 0 \right\}$$

where  $v_i^{(N)}$  is defined as in Section 2.1. I obtain, however, that  $N_i^{\max} = \infty$  for almost all products  $i$  because of the presence of loops within the input-output table (e.g. industries buying inputs classified within the same industry). An alternative way to define such an index would be define the minimum number of stages required to for 95% of the value of production:

$$N_i^{95} = \min \left\{ N; \sum_{n=1}^N v_i^{(n)} > 0.95 \right\}$$

More generally, we could define  $N_i^\alpha$  as the minimum number of stages required to produce a fraction  $\alpha$  of the value of production:

$$N_i^\alpha = \min \left\{ N; \sum_{n=1}^N v_i^{(n)} > \alpha \right\}$$

While these previous measures of fragmentation aim at reflecting the number of plants that production is sequentially going through, they might not well reflect whether production is actually dispersed along the value chain. For instance, if plant A ships one dollar of an intermediate good to plant B, and plant B only add one cent of value added to the product, index  $N$  associated with the final product will be equal to 2 whereas production is mostly concentrated within just one plant.

For this purpose, I construct an alternative measure of fragmentation inspired from the Herfindahl-Hirschman Index (HHI). For each product  $i$ , I define  $H_i$  by:

$$H_i = \frac{1}{\sum_n^\infty (v_i^{(n)})^2}$$

where  $v_i^{(n)}$  is defined as in Proposition 1 and corresponds to the share of the value added that has gone through  $n$  stages. Note that the sum of these shares equal one for each industry:  $\sum_n^\infty v_i^{(n)} = 1$ , hence  $H_i \geq 1$  by construction. This index can be interpreted as a HHI-index of the concentration of value added across production stages. If value added originates from only one stage (i.e. if  $v_i^{(n)} = 1$  for a particular stage  $n$ ), this index equals 1. If the source of value added is rather dispersed across production stages, this index will take larger values.

Table 13: Industries with the largest values

Index	$N_i$	$N_i^{90}$	$N_i^{95}$	$N_i^{99}$	$H_i$
Meat packing plants	2.67	4	5	7	4.27
Sausages and other prepared meat products	2.65	5	5	7	4.69
Leather tanning and finishing	2.43	5	5	7	4.12
Poultry slaughtering and processing	2.36	4	5	7	4.05
Creamery butter	2.35	4	5	6	4.09
Max across all industries	2.67	5	5	7	4.69

I calculate indexes  $N_i^{90}$ ,  $N_i^{95}$ ,  $N_i^{99}$  and  $H_i$  for all tradable goods (excluding services and petroleum-related industries as in previous tables).<sup>51</sup> Each of these indexes exhibit a large correlation with weighted-average number of stages  $N_i$ . For each year, the correlation between  $H_i$  and  $N_i$  is above 90% and the correlation between  $N_i^\alpha$  and  $N_i$  is above 80% for each  $\alpha = 0.90, 0.95$  and  $0.99$ . This confirms that all these indexes capture very similar aspects of fragmentation.

<sup>51</sup>In practice, I compute the H-index by summing up to  $n = 20$ , but this captures more than 99.99% of the value-added.

Moreover, these alternative indexes confirm that the most vertically fragmented industries are those identified with index  $N_i$ . The industries associated with the highest values in  $N_i$  are also associated with the highest values for indexes  $N_i^{90}$ ,  $N_i^{95}$ ,  $N_i^{99}$  and  $H_i$  as shown in Table 13.

The evolution of these indexes across years also confirm the downward trend observed with  $N_i$ , i.e. that production has become less vertically fragmented. Table 14 shows the average of each of these indexes across industries for each year. The unweighted average of  $H_i$  across industries has steadily decreased from 2.68 in 1967 to 2.18 in 1992. The average of  $H_i$  weighted by final consumption has decreased from 3.01 to 2.45. The (unweighted) average of  $N_i^{90}$ ,  $N_i^{95}$ ,  $N_i^{99}$  also decreased between 1967 and 1992.

Table 14: Alternative indexes of vertical fragmentation

Year	1967	1972	1977	1982	1987	1992
Index $H$	2.68	2.36	2.38	2.36	2.15	2.18
Weighted	3.01	2.76	2.77	2.56	2.43	2.45
Index $N$	1.85	1.68	1.70	1.68	1.58	1.59
$N^{90}$	3.37	2.99	3.03	2.99	2.77	2.81
$N^{95}$	4.18	3.76	3.80	3.70	3.41	3.40
$N^{99}$	6.23	5.47	5.54	5.37	5.02	4.98

*Notes:* First row: average across industries of index  $H_i$  for each year; second row: average of index  $H_i$  weighted by final consumption; third row: weighted-average number of stages  $N$ ; fourth to sixth row: indexes  $N^{90}$ ,  $N^{95}$  and  $N^{99}$ .