THE BEHAVIOR OF LABOR-MANAGED FIRMS UNDER UNCERTAINTY
Product diversification, income insurance and layoff policy*

by

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I. An introduction to the institutions and the previous literature

In this paper we outline a program for the risk-averse labor-managed firm (LMF) to respond optimally to the presence of price uncertainty. Although it is a highly stylized program, it goes much farther to explain observed behavior of LMFs and their associations than previous models, while suggesting approaches for more systematic responses on their part.

The previous microeconomic analysis of the LMF under uncertainty has been comparatively limited in scope. The LMF is assumed to be risk-averse, to operate under conditions of price uncertainty, and to have as its objective the maximization of expected utility over per-worker income. 1 The "Illyrian" framework is almost always used (Ward, 1958), that is, the firm freely lays off workers to maximise ex post (expected utility of) income per worker.

The principal finding of this literature is that a new set of "perverse" behavior may characterize the LMF facing price uncertainty. In particular with labor variable, a change from certain price to uncertain price leads under standardized conditions to an expansion of the LMF where it leads to a contraction of the profit-maximizing firm (PMF). With labor fixed and capital variable the LMF contracts as would the PMF; with both factors variable, behavior parallels the PMF if the long-run supply schedule is upward slop-

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ing, and the "perverse" result that price increases cause employment and output declines, obtains when the long-run supply schedule is downward sloping. Similar "perverse" results are found for an increase in Sandmo risk and increased taxation. Note that while some factors under certainty tend to decrease the size of the LMF relative to the PMF, these uncertainty-related factors tend to increase the relative size.

Although this literature has yielded a number of interesting insights, it has had limited applicability to existing LMFs. First, membership in Spain's Mondragon coops, Italy's Lega industrial cooperatives and Yugoslavia's self-managed enterprises is inflexible on the downside; "solidarity" of the membership is an institutionally given feature. In Yugoslavia it is illegal to lay off any but a few temporary workers. In Mondragon, members may be laid off (about 2% are laid off at any one time) but one retains a membership right, which comes with compensation.

The Mondragon cooperative association in the Basque region of Spain is an especially interesting case of a labor-managed sub-economy with 70 cooperatively-run factories with a total workforce of more than 15,000 as well as a "Second Level Cooperative" including a credit union with nearly 100 branches and over 300,000 depositors (Bradley and Gelb, 1984; Thomas and Logan, 1982). The industrial part of the system is our central concern, but the bank's role as financial intermediary will also prove vitally important.

Here, we state some of the financial institutions found in the Mondragon associated cooperative enterprises, which are to be the framework around which we build much of the present analysis. (Further details are given when they are relevant to specific interpretations.) Worker-members receive income from three sources: from base-wages (called advances), from interest on mandatory individual internal capital accounts, and from a share (about 30%) of residual earnings. This share is divided in proportion to members' income from the first two sources.

Capital is classified as either individual or collective. Each member has a quota investments in what are called "individual internal capital account" (IICAs). These may be vested gradually through paycheck deductions after joining. The accounts are revalued in accordance with the CPI and IICAs are refunded upon leaving the firm. The Collective Capital Account (CCA) is financed out of retained earnings. Members have no property rights to CCA funds upon retirement. There are also legally mandated pension fund contributions; cooperators are considered self-employed in Spanish law and are not entitled to social security.

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2 The clearest statement of these results is found in Hey (1981b). For the results stated in the text decreasing absolute risk aversion and increasing relative risk aversion is assumed.
Both IICAs and CCAs are kept on deposit at the Association's financial institution, a credit union called the Caja Laboral Popular (CLP). Such institutions are sometimes referred to as "second level" cooperatives (SLCs), since their membership themselves are cooperatives. It has several functions relevant to the present study. 1. Each associated LMF makes a payment to the CLP, which is a fraction (currently 30%) of income net of materials costs, base wage payments and interest on IICAs. 2. Decisions of the CLP affecting the coops are made democratically by representatives of the member LMFs, as well as some outside representatives. It thus acts much like a producer cooperative. 3. The CLP and associated agencies provide product design, market research and other management services for member LMFs. 4. When a firm fails, the CLP provides financial and technical support in establishing a new coop for the unemployed workers in a new product line. Thus, members enjoy a very particular kind of job security, though they face some risk in their capital accounts.

In Italy, one of the second level associations of industrial cooperatives, the Lega Nazionale delle Cooperative e Mutue (Lega), plays a role analogous in some respects to that of the CLP in Mondragon. There are currently about 3,000 coops in the Lega. The national and especially regional Legas provide assistance with management, marketing and financing. They do not subsidize salaries for new or troubled firms; however, they do not have to, since the national government pays up to 80% of salaries in troubled firms, whether public, private or cooperative (this is called the "Cassa Integrazione"). Each member of the LMF pays dues to the regional and national Legas.

In Yugoslavia, a minimum reference wage is established by mutual agreement between firms and the government. Incomes are subsidized if they should fall below the reference levels.

In the Mondragon system, the LMF is not provided with a Yugoslav-type of a "soft budget constraint." A losing LMF must dip into its own reserve funds. If losses appear chronic, the "second level" coop has a special program to help workers start a new industry (Bradley and Gelb, 1984). A new firm is formed, rather than try to start to produce a new product within the old firm. Why this is done is not obvious, since the two possible approaches are formally equivalent. Possible reasons include the second level coop view that perhaps something is going wrong with the coop's internal "process", or that the second level coop feels it can play a more effective supporting "entrepreneurial" role if the firm is started from scratch.

3 We would like to thank Dr. Bruno Guilliani for pointing this out to us.
Feasibility study costs are split among three parties, the second level coop, the dying coop and potential founding workers in the new coop, at least half of which must come from dying coops. The initial capitalization shares are, 60% second level coop, 15% dying coop, 15% founding workers, and 10% other sources. The second level coop provides its share on a subsidized interest basis (8% in 1984). The new coop is required to gradually buy out the second level coop share, and thus it may be seen, as part of a "revolving loan fund," analogous to that of the Industrial Cooperative Association in the United States. Within five years, own-assets are expected to rise to 45-50% of total assets.

In this start-up period, pay is restricted to 85% of the overall Mondragon pay-scale. This reflects the technical assistance and capital subsidy they receive, and may be seen as an attempt to reduce "moral hazard." For the first three years, the replacement coops must use dying coop facilities on contract. Obviously the motive here is to save money and reduce risks of new outlays. Capital is thus at risk to members even if jobs are not. The LMF may wish to establish a new product line if it has "surplus" workers and membership may not be reduced. Or, the LMF may diversify its production to reduce uncertainty.

Taken together, these observations suggest behavior much different from the "Illyrian" predictions. It appears that LMF associations frequently act to reduce risk by promoting product diversification and cross-firm or cross-period income insurance. Layoffs are sometimes used as a last resort, but these come with compensation. In the following sections, we formalize these three responses into a general program of risk reduction. Further parallels are drawn to observed LMF behavior throughout the development. Finally, some general conclusions are drawn, and an agenda for further research, notably with respect to empirical testing and some moral hazard problems, is presented.

II. Product diversification

In Ireland and Law (1982, pp. 148-150) it is argued that labor-managed firms will wish to expand their product line to ensure "a more secure depository for savings" and "from a desire for a less speculative return on their labor."

This common-sense idea, borrowed from portfolio theory, may in fact offer some explanation of broadly diversified product lines in LMF associations such as Mondragon. Yugoslav enterprises also have extremely broad product lines, crossing industrial classifications (see Prasnikar, 1983, pp. 46-47). However, unlike the portfolio problem, which leads optimally to diversifying into an infinite number of (non-dominated) securities, the LMF will be
restrained in its expansion by fixed costs. Suppose each product is statistically independent \(^4\) and the \(i^{th}\) product gives the LMF income \(y_i\), distributed with,

\[
\mu_i = E[y_i], \quad \sigma_i^2 = V(y_i), \quad i = 1,\ldots, N
\]

for a given number of workers and level of effort. Each product is characterized by the vector \((\mu_i, \sigma_i^2)\). The firm will be concerned only with products not "dominated" by another product. If \(\mu_i > \mu_j\) and \(\sigma_i^2 < \sigma_j^2\) then product \(i\) dominates product \(j\). In this way the LMF would narrow its interest to a set of relevant products. If the firm can produce only one good it will produce the one giving the highest (expected or Tobin-Markowitz) utility.

Now suppose the LMF can produce at most (its best) two products, \(i\) and \(j\), and must decide how much of each to produce. The firm may be modeled as maximizing Tobin-Markowitz utility given by,

\[
u_{ij} = \max_{\alpha} \left( \mu_{ij}, \sigma_{ij}^2 \right)
\]

subject to the constraints,

\[
\mu_{ij} = \alpha \mu_i + (1-\alpha) \mu_j
\]
\[
\sigma_{ij}^2 = \alpha^2 \sigma_i^2 + (1-\alpha)^2 \sigma_j^2
\]
\[
\alpha = \frac{i}{L}, \quad 1-\alpha = \frac{j}{L}, \quad 0 \leq \alpha \leq 1
\]
\[
i + j = L,
\]

where \(L\) is total membership and \(i\) and \(j\) are members assigned to the \(i^{th}\) and \(j^{th}\) production lines, respectively.

This set-up shows the formal equivalence between the LMF program and the standard approach in portfolio theory.

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\(^4\) If there is covariance between products \(i\) and \(j\), this may reflect either a positive or a negative correlation. Inclusion of this problem makes the "program" considerably more cumbersome to report, and unfortunately leads to few insights. The assumption of independence also enables us to reach sharper conclusions. We also implicitly assume either a truncated normal distribution or take the first two moments as an approximation of the distribution.
The program leads to the specification of the allocation of the membership
(α in the two-good case). For the special case of both y₁ and y₂ being
normally distributed, we have

\[ \alpha = \frac{1}{L} \cdot \frac{\left( \mu - \mu_j \right)}{\alpha + \sigma_j^2} / \left( \sigma_1^2 + \sigma_j^2 \right) \]

\[ 1 - \alpha = \frac{1}{L} \cdot \frac{\left( \mu_j - \mu \right)}{\alpha + \sigma_1^2} / \left( \sigma_1^2 + \sigma_j^2 \right) \]

If the firm can produce a third product, it will proceed in an analogous
maximization exercise to allocate labor over the three products providing
overall best utility. If utility can be increased further by expanding the labor
force size, it will expand, with workers allocated appropriately.

In general, the number of products will be added sequentially with labor
force expansion if appropriate, in this framework until net increments to utility
turn negative. (The full formal representation of this process is cumbersome
and hence is omitted; a mathematical appendix is available from the
authors.) Expansion will proceed until the marginal utility of diversification is
just offset by the disutility of the fixed cost incurred in establishing a new
product line. Obviously there are other factors leading to "classical diversifi-
cation," such as considerations of negatively correlated product prices,
production externalities and product complementarity.

Finally, we note three major differences between our analysis of product
diversification and Mario Nuti's (1984) study of mergers between LMIs. First
diversification within pure (egalitarian) LMIs will occur in our framework,
where mergers might not occur due to dilution of income for one of the firms.
High utility coops might not agree to a merger that would raise average
though not own-utility. In an "egalitarian" coop (see Meade, 1972), these
distinctions are less important. Incomes will not necessarily be equal after a
merger, if there are no institutional rules requiring this. In a cooperative
game, the necessary and sufficient condition for a merger is that the core is
non-empty. Second, if egalitarian results are allowed, there may be no limit
to the number of mergers, unless "rents" are being collected in a limited
number of utility-dominating enterprises. This is because the fixed cost, once
sunk, plays no role in restraining mergers the way it does in placing a cost on
diversification. Third, a firm may diversify into an independent product to
decrease uncertainty; mergers (generally) take place to increase mean
return through augmented technology, organization or market power.
Clearly, there will be socially optimal limits on this merger process, due to
potential anti-competitive effects. In order for Yugoslavia in particular, which
places great emphasis on market competition, to achieve an efficient
industrial organization, as well as to control workers' risks, it must instead
provide for a significant level of self-diversification. Unrestrained "merger mania", as it has been termed there, may be quite harmful to the Yugoslav economy in the longer term.

III. Income Insurance

In this section, we consider broader strategies by LMFs or associations of LMFs for "income insurance". For the single-firm case, we consider the role of the collective reserve. These results are related to and generalize some of those in Miyazaki (1984), again without, however, the possibility of using hired labor. We then consider the role of the Second Level Coop (SLC). This corresponds to the Caja Laboral Popular (CLP) in Mondragon, the regional and national Lega associations in Italy, and regional government in Yugoslavia.

Rather than take out income in all states, coops in Spain and Italy set up an internal collective reserve (for the example of Mondragon, see Thomas and Logan, (1982)).

The essence of this may be illustrated with a simple static two-state model. Let A stand for the high-price ("good") state, with probability \( \Pi \); B is the low-price (bad) state (probability \( 1 - \Pi \)). Without self-insurance, expected utility of income may be written,

\[
E \{ U_Q \} = \Pi u \left[ \frac{P_A Q (L - F)}{L} \right] + (1 - \Pi) u \left[ \frac{P_B Q (L - F)}{L} \right]
\]

where \( Q \) is the production function and \( F \) stands for fixed costs. If the firm self-insures, it builds the collective reserve during good states and draws it down during bad states. Of course, the collective reserve may play other roles as well, but abstracting from these, the members pay a "premium", \( \rho \) in state A, draw a "subsidy", \( S \), in state B, which is expected to balance in the long-run, i.e.,

\[
\Pi \rho = (1 - \Pi) S,
\]

or,

\[
S = \frac{\Pi}{1 - \Pi} \cdot \rho.
\]

Expected utility in this model is then:

\[
E \{ U_I \} = \Pi u \left[ \frac{P_A Q - F - \rho}{L} \right] + (1 - \Pi) u \left[ \frac{P_B Q - F + S}{L} \right]
\]
The problem of the LMF is to decide how much income should be pooled to maximize $E [U_1]$ subject to the balance constraint which yields,

$$u' \left[ \frac{P_A Q - F - \rho}{L} \right] = u' \left[ \frac{P_B Q - F + S}{L} \right],$$

the familiar equating of marginal utilities across states. Monotonicity then implies that 5,

(1) \[ \rho^* = (1 - \Pi) (P_A - P_B) Q (L). \]

where $\rho^*$ denotes the optimal self-insurance premium. Self-insurance is undertaken providing $E [U_1] > E [U_0]$ which will hold for risk-aversion. In this case, the optimal premium is increasing in (a) the probability $(1 - \Pi)$ of the "bad state", (b) output and employment, and (c) a measure of the variance or dispersion of states (given here by $(P_A - P_B)$).

Note that the model may be interpreted equally well in terms of pooling across firms as well as in terms of cross-period self-insurance by one firm.

In Mondragon and Yugoslavia, workers lose their claim to the collective reserve upon retirement. Building up the reserve over the course of employment may be viewed as an "access fee" to the coop which members are obviously willing to pay (alternatively, leaving collective assets may be viewed as a "departure fee"). The reason workers do not choose to self-insure themselves rather than their firm may be explained by moral hazard, or time inconsistency. Workers would otherwise wish to join in good states but would quit in bad states; a collective reserve over which one does not have individual rights helps to circumvent this problem. Insurance across several firms is even more clearly superior.

Some groups of labor-managed firms may also choose to pool some of their income as insurance against income shocks. Or, they may participate in mandatory schemes set out by the SLC. In Yugoslavia, this may be seen as taxation; when the firm does poorly enough that wages fall below target

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5 From monotonicity, $(P_A Q - F - \rho)/L = (P_B Q - F + S)/L$ when marginal utilities are equal.

Thus \[ P_A Q - P_B Q = \frac{\Pi}{1 - \Pi} \rho + \rho, \] or $P_A Q - P_B Q = \frac{\rho}{1 - \Pi}$. 
levels it is then subsidized in accordance with the famous "soft budget constraint" (see e.g., Tyson, 1977). In Mondragon, there is both mandatory and voluntary pooling. A certain percentage (currently 0.5%) of pay must be contributed to the SLC's "unemployment fund" (Bradley and Gelb, 1984). In return, as we have seen, the LMFs receive "job insurance".

In addition, some LMFs in Mondragon pool individual account surplus voluntarily. One subgroup pools all of their collective reserves, others pool some, and some pool none at all (Bradley and Gelb, 1984). Thus, the portion of income pooled is a choice variable within sub-groups. The problem is qualitatively analogous to the self-insurance problem, and pooling groups will be those with common attitudes toward risk, leading to $p^*$. A more general formulation has been relegated to the mathematical appendix and is available from the authors.

IV. Unanticipated shocks and compensated layoff policy

Suppose the firm or LMF association is faced with a major shock, after it has established its optimal product diversification and income insurance program. Diversification takes time, and in some cases, short-term layoffs will be optimal, especially if fixed costs are high. In this case, suppose there are L members, of whom L are to work in the coop. If members choose who shall work and who (if anyone) shall be temporarily laid off randomly, then probability of working in given period is $L/\ell$, while the probability of being laid off is $(L-l)/L$. (See e.g., Bonin (1981), Svejnar (1982), Miyazaki and Neary (1983), Smith (1984), and Miyazaki (1985)). The random layoff principle may be seen alternatively as a formalization of a layoff rotation, with $(L-l)/L$ the fraction in the layoff rotation at any time. If no compensation is paid, the laid off workers receive a (monetary equivalent) value "$v$", which may be an alternative wage, unemployment benefits, and/or the value of additional leisure time.

The firm, however, is likely to pay compensation to laid-off members if the value of $v$ is significantly lower than firm income. Currently in Mondragon, laid-off workers are receiving 80% of working members' pay. Let the fraction of income paid into the compensation fund, "c" and employment, l be the choice variables. For simplicity, let the objective be given as utility of member income. Initially let $v$ be known with certainty. The problem is:

$$\max_{l,c} \frac{1}{L} \left[ y (1-c) + \frac{L-l}{L} u (lc + v) \right]$$

$$PQ = F$$

where $y = \frac{Q}{L}$, P is output price $Q = Q(L)$ is the production function and,
F represents non-labor costs, fixed for expositonal ease. The solution to this
problem is to equate marginal utilities across working and laid-off states, so that the optimal compensation c and labor l satisfy (after some manip-
ulation)⁶:

\[(2) \quad y \left(1 - \frac{cL}{L-1}\right) = v\]

and,

\[(3) \quad PQ_l = y \left(1 - \frac{cL}{L-1}\right)\]

Thus,

\[(4) \quad PQ_l = v\]

Note that these allocational and distributional results are analogous to those
in Miyazaki (1984, p. 915), who uses a slightly different formulation; and to
the allocational rule in Svejnar (1982). Regardless of the contract form, the
optimal decision rules are invariant, giving allocative efficiency.

Comparative statics may be performed on these results. Where labor l is
variable, the findings confirm and extend related arguments in Domar
(1966), Svejnar (1982) and Smith (1984) that the "expected income-
maximizing" firm (here expected utility maximizing with compensation) will
not show the "pervasive" illyrian behavior, as \(\delta l/\delta P > 0\). See also Miyazaki
and Neary (1985).

More originally, we may perform comparative statics to find the change in
compensation as the monetary equivalent value outside the firm increases.
With some algebra it may be shown that \(\delta c/\delta v = -1 - c/PQ\), which is
indeterminate.

One may also generalize the model to include a labor-leisure tradeoff. Let
\(u(y, h_0)\) be the utility of a working member, and let y in this set-up be the size

⁶ Monotonicity ensures that utility will be equated, or that \(y(1-c) = \ln y/k\cdot l + v\). Thus,
\(y(1-c/k\cdot l) = v\), or \(y(1-Lc/(L-I)) = v\), the result in the text.
of the compensation paid to laid-off members, with

\[ y = \frac{PQ - F - \gamma (L - l)}{l} \]  

and \( h_0 \) the leisure level. Let \( u(y, h_0 + h) \) be then the utility of the laid-off worker, with \( h_0 + h \) his or her leisure level and \( \gamma \) the compensation (there is no alternative wage as such). The model becomes:

\[ \max_{l, \gamma} \frac{1}{L} \sum u(y, h_0) + \frac{L - l}{L} u(y, h_0 + h) \]

If hours cannot be adjusted, we have, differentiating with respect to \( \gamma \),

\[ u'(y, h_0) = u'(y, h_0 + h) \]

The first order condition with respect to \( l \) gives us,

\[ u'(y, h_0) \cdot (PQ_1 + \gamma - y) + u(y, h_0) - u(y, h_0 + h) = 0 \]

If income and leisure are perfect substitutes\(^7\), equations (8) reduces to,

\[ PQ_1 = \frac{PQ - F - \gamma L}{l} \]

and thus,

\[ PQ_1 = y - \gamma \]

Here, \( y - \gamma \) is interpreted as the *monetary value* of being laid off.

However, given normal preferences, and appropriate restrictions on daily "set-up" costs, the optimal policy is not to lay off \( L-I \) workers. Marginal utility is equated across states at a maximum utility when all workers continue to work at reduced hours.

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\(^7\) This is a usual assumption in the implicit contracts literature. For a survey, see Rosen (1985).
We do not have evidence available to us on variation in hours worked in Mondragon. However, Berman (1982) argues that U.S. plywood coops do respond to changes in demand by variation in hours worked rather than by hires and layoffs, and this also appears to be the case in Italian industrial coops.

There will be a limit to this approach if individuals (or whole shifts in the case of round-the-clock operations) have significant "set-up costs". This may explain why there are some layoffs in Mondragon despite this finding. Note, however, that the incidence of layoffs in Mondragon during recessions is much lower than for the Basque or Spanish economies as a whole (Bradley and Gelb, 1984). Presumably, conventional firms could offer a similar contract; it is unclear why they do not do so.

Now allow that \( v \) may be a random variable; \( c \) is to be contractually set \textit{ex ante} but an unemployed worker realizes \( v \) \textit{ex post}. For simplicity of presentation suppose \( v \) takes on only two values, \( v_1 \) and \( v_2 \), with probabilities \( \pi \) and \((1 - \pi)\), respectively. The problem is to:

\[
(10) \quad \max_{l, c} \quad \frac{1}{L} \sum_{l} u \left( \frac{PQ - F - (L - l) c}{l} + \frac{L - IE (v + c)}{L} \right).
\]

which implies,

\[
(11) \quad u' (y) = E [u' (v + c)]
\]

\[
(12) \quad u' (y) (PQ + c - y) = E [u (v + c)] - u (y).
\]

Now what is the relationship between \( y, v \) and \( c \)? How does the outcome change with a change in \( \pi \)? With a change in the variance?

For \( v \), we have that

\[
E [v] = \pi v_1 + (1 - \pi) v_2
\]

\[
\text{Var} (v) = E [v - E (v)]^2.
\]

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\(^8\) Based on interviews with directors of two Lega cooperatives in Toscana and with regional officers of the Emilgiglia-Romano Lega in Bologna, in 1984. We would also like to thank the European University, Florence for their support, and Bruno Guilian, for his institutional guidance.
Now consider a mean-preserving spread random variable $v$. We may then demonstrate

**Proposition 1:**

$y < E[v] + c$ if and only if $u'' > 0$.

**Proof.** First take a Taylor expansion in (11).

This gives us

$u'(y) > u'(E[v] + c)$ if $u'' > 0$.

The proposition follows by the increasing monotonicity and the concavity of $u$.

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9 The following definitions are needed here. A random variable $v'$ is said to be mean-preserving if $E[v] = E[v']$ while $\text{var}(v') < \text{var}(v)$. Thus, if $v = E[v] + \Delta v_1$, with probability $\Pi$ while $v = E[v] + \Delta v_2$ with probability $1 - \Pi$, then we have

$$\Delta v_2 = \frac{\Pi}{1 - \Pi} \Delta v_1$$

10 Let $v_1 = E[v] + \varepsilon_1$ and $v_2 = E[v] + \varepsilon_2$, then

$$u'(v_1 + c) = u'(E[v] + \varepsilon_1 + c) = u'(E[v] + c) + u'' \varepsilon_1 + \frac{u'' \varepsilon_1^2}{2} + o(\varepsilon_1)$$

and

$$u'(v_2 + c) = u' + u'' \varepsilon_2 + \frac{u'' \varepsilon_2^2}{2} + o(\varepsilon_2)$$

Thus,

$$E[(u')] = \pi \frac{u'' \varepsilon_1^2}{2} + o(\varepsilon_1) + \frac{u'' \varepsilon_2^2}{2} + o(\varepsilon_2)$$

$$= \pi \varepsilon_1^2 + \frac{(1-\pi) u'' \varepsilon_2^2}{2} + o(\varepsilon_1, \varepsilon_2)$$
Proposition 1 states that if there is an uncertain \( v \), the laid-off workers should receive a higher expected compensation, provided \( u''' > 0 \). This implies decreasing Arrow-Pratt risk aversion, \( r \):

\[
r = -(u'/u) ; \quad r' = -(1/u') (u''' + r),
\]

and thus \( r' < 0 \) when \( u''' > 0 \). This also implies \( u' \) is convex. This is plausible as it is often considered that \( u'(x) \rightarrow 0 \) as \( x \rightarrow -\infty \) and \( u'(x) \rightarrow \infty \) as \( x \rightarrow 0 \).

To examine how an increase in the uncertainty of \( v \) affects compensation \( c \), we again consider a mean-preserving-spread random variable, and assuming constant Arrow-Pratt risk-aversion (CARA). We have the following proposition.

**Proposition 2:**

For a chosen \( l \), an increase in outside variance will lead the LMF to increase the compensation \( c \).

**Proof.** Totally differentiating equation (11) we have,

\[
-\left. \frac{d}{dc} \right|_{u''(y) \left[ \frac{-l}{-l} \right]} dc = \pi u''(v_1 + c) \left( dv_1 + dc \right) + (1-\pi) u''(v_2 + c) \left( dv_2 + dc \right).
\]

With a mean-preserving spread, we have that \( dv_1 = -((1-\pi)/\pi)dv_2 \), so that

\[
\left[ \frac{d}{dc} \right] \left[ \frac{-l}{-l} \right] u'(y) + (\pi u'(v_1 + c) + (1-\pi) u'(v_2 + c)) dc
\]

\[
= (1-\pi) [u''(v_2 + c) - u''(v_1 + c)] dv_2
\]

With CARA we have that,

\[
\left[ \frac{d}{dc} \right] \left[ \frac{-l}{-l} \right] u'(y) + (\pi u'(v_1 + c) + (1-\pi) u'(v_2 + c)) dc
\]

\[
= (1-\pi) r (u'(v_1 + c) - u'(v_2 + c)) dv_2
\]

But using (11), \( u'(y) = \pi u'(v_1 + c) + (1-\pi) u'(v_2 + c) \).

Thus,

\[
dc \frac{1}{l} \left[ \frac{u'(v_1 + c) - u'(v_2 + c)}{u'(y)} \right] dv_2.
\]

If \( v_2 > v_1, u'(v_1 + c) > u'(v_2 + c) \) or \( dc/dv_2 > 0 \). If \( v_2 < v_1, dc/dv_2 < 0 \).

Thus, increasing variance increases \( c \).
Proposition 3:

The greater is the degree of risk aversion, \( r \), the smaller is the effect of changes in \( \pi \) on \( c \), or,

\[
\frac{dc}{d\pi} = \frac{1}{L} \left[ \frac{u'(v_1+c) - u'(v_2+c)}{ru'(y)} \right]
\]

Proof: Omitted. (The procedure is analogous to the proof of Proposition 2.)

We may now consider the policy where \( l \) and \( c \) may be simultaneously adjusted for the case of constant absolute risk aversion. Define \( \omega = (-1/r) \ln M_v (-r) > 0 \) where \( M_v (-r) \) is the moment generating function of \( v \) evaluated at \( r \), where \( v \) is now defined as a continuous random variable. Then we have,

Proposition 4:

The optimal choices in \( l \) and \( c \) are given by, in the CARA case,

\[
PQ' = \omega \\
y = \omega + c.
\]

Proof. Letting \( r \) stand for the measure of absolute risk aversion, \( u(x) = -e^{-rx} \) and \( u' = re^{-rx} = ru \). Thus (11) implies that

\[
u(y) = E[u(v+c)].
\]

From (12), we have \( PQ' = y - c \). But \( u(y) = E[u(v+c)] \), which implies that

\[
y = c - \left( \frac{1}{r} \right) \ln M_v (-v)
\]

Corollary: \( y < c + E(v) \).

Proof: by Jensen's inequality,

\[
E[e^{-rv}] > e^{E[v]}
\]

Here we require CARA; \( u = -e^{-rx} \) \( u'' > 0 \), consistent with Proposition 1.
Note that if layoffs are random, $P$ is stochastic and $v$ is given deterministically, we have

$$E[u'(y)] = u'(v+c),$$

and increases in the variance of $P$ imply a decrease in $c$:

$$\frac{dc}{dP_2} = \frac{(1 - \pi) Q (y'(y_1) - u'(y_2))}{L u'(v+c)} < 0.$$ 

Finally, suppose workers are randomly laid off while $v$ and $P$ are both stochastic. The rule then is:

$$E[u'(y)] = E[u'(v+c)]$$

A detailed decomposition of this latter case is found in Ye (1986).

V. Conclusion

In the previous sections, the behavior of the risk-averse LMF and associations of these firms facing uncertain output price has been examined. It was assumed that these firms could not hire labor and that membership provided certain basic rights, including an *ex ante* vote about just compensation for layoffs, equal probability of layoff or rotation, and maintenance of membership during layoffs; and it was assumed firms could voluntarily enter into mutually beneficial "insurance" agreements with each other. Finally, it was allowed that a "second level coop" might exist, with certain "welfare" goals; its possible policies were examined. All of these analyses were intended to reflect actual features of LMFs and LMF associations, especially as seen in Spain, Italy and Yugoslavia. The models demonstrate that observed behavior is consistent with "rational" behavior under risk-aversion, and also suggest at least rules of thumb for improving the structure and performance of these programs.

One key problem for further analysis is the "moral hazard" dilemma. Though members' collective reserves are to some extent at stake, income insurance across firms could lead to "free riding." In China, a closely related concern is known as the "iron rice bowl" problem. Optimal policies in the face of this problem need to be worked out. In general, however, the moral hazard problem is solved through "partial coverage" schemes. In Mondragon, for example, one receives an income subsidy only in extreme circumstances when the need for extensive reorganization has been reached.
The analysis has also led to a number of specific predictions in the degree of diversification, self-insurance and layoff compensation. Another major direction for future work will be to put these predictions to empirical testing.

BIBLIOGRAPHY