Public Debt and Risk Administration

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Introduction

The evolution of the international financial market has increased the volatility of financial assets and liabilities in general, elevating the risk associated with the instruments used by countries in financing their debts. The offices in charge of the public debt administration need to develop systems that identify, measure and, mainly, offer important information for an efficient administration of the risks associated with the debt.

Several countries have been debating this subject, discussing their experiences to improve the basic models of public debt administration and to allow better management of the risk and cost of their financing strategies. These discussions demonstrate the relevance of this subject in the context of the macroeconomic policies.

We intend, in this study, to contribute to the process of improving the methods and techniques to quantify the risk in public debt, considering the current experience of countries with a good reputation in the administration of their debts. The objective of this paper is to bring new considerations in relation to the current models, trying to adapt them to the particularities of Brazil.

The study is separated in five chapters. In the chapter one, we will present the evolution of the methods and instruments related to the administration of public liabilities, as well as their tendencies, in view of the new perspectives.
1. Historical Aspects and Tendencies

The concepts and models developed by the Modern Theory of Portfolios (TMP), thoroughly used by private investors, are becoming famous in the administration of public liabilities, especially in the last 10 years.

The process of modernizing the public debt administration began with the Government Borrowers Forum (GBF), in Helsinki, on April of 1989. There, about 80 managers of public liabilities, representing 30 countries and several organizations, had the opportunity to hear the detailed suggestions of private institutions for the development of advanced techniques related to public debt administration. (Nars, 1997)

In the middle of 90`, models of risk analysis and return, quite usual in the private sector, had been adapted by the public sector of several countries, which started to control the relation between risk and cost of their liabilities. As the identification of the investment efficient frontier assumes great relevance under the investor's optics, the determination of the financing efficient frontier became a fundamental tool for the public debt manager. This is the case, for instance, of countries like Australia and Finland that, among their instruments of debt administration, analyze averages and variances of the cost of that liability, based on the model developed by Harry Markowitz (pioneer of the Modern Theory of Portfolios) 

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\[ ^{1} \text{The model of Markowitz applied to the Brazilian public debt was analyzed by Silva (1997)} \]
Other sophisticated models of risk administration have been implemented in several countries. Besides the habitual use of the modified duration of the stock of the debt to analyze the sensitivity of that liability to changes in the interest rates, there is a clear tendency in the direction of the implementation of models similar to the Value-at-Risk (VaR).

According to Radstam and Claesson (1997), Sweden is considering the development of a system similar to VaR to control the market risk of debt. Portugal and Denmark use a methodology denominated Cost-at-Risk (CaR), quite similar to VaR. The distinction is that while CaR emphasizes the risk of changes in the financial cost of the debt, VaR consider the risk of changes in its market value.

The growing implementation of integrated analyses of assets and liabilities demonstrates the progress in the debt administration techniques. This kind of analyses tries to relate assets and liabilities to identify the government net exposure related with changes in the interest rates and exchange rates, among other risks.

In 1993, New Zealand through its Debt Administration Department - NZDMO, hired specialists to quantify the risks of the government's assets and liabilities. Since then, NZDMO considers that the amount and composition of its domestic debt should be based on the structure of the government's balance sheet.

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2 Vide Danmarks Nationalbank (1998) and Bushes (1999)
(Wheeler, 1997). South Africa and Belgium represent additional examples of countries that are using similar techniques in public debt administration.

In Brazil, the first step to modernize its techniques of public debt administration was taken at the end of the first semester of 1995. At the invitation of the Federal Government, International Monetary Fund specialists analyzed and proposed new guidelines for the debt administration, taking into account the economical stability provided by the “Real Plan”.

Several suggestions were presented for the improvement of debt administration and the development of the public bonds market in Brazil. Many of them, such as more frequent auctions and the reduction of the number of bonds used for financing of the debt, were accepted and implemented by the National Treasury and Central Bank.\(^3\)

Concern for developing debt administration capacity led the Brazilian Government to initiate studies at the National Treasury since 1995, with greater intensity after 1997. During this time the National Treasury developed: (i) the Map of National Treasury Net Liabilities – MELTN (described and analyzed in greater details in chapter 4); (ii) a multiannual planning to finance the public debt; and (iii) a

\(^3\) The auctions of National Treasury bonds, that were accomplished monthly, passed to biweekly periodicity in the second semester of 1995 and weekly after October of 1997. The objective is to avoid huge amount of bonds sold in each auction: “When reducing the volume of bonds to be refinanced in each auction, Treasury would be reducing its exposition to the risks of changing in the financial market…” (Silva, 1999)
study about the institutional experiences of several countries in the administration of their public debts.

In addition, it’s important to note the approaches between the National Treasury and the World Bank, starting in September of 1998. During that month, IMF specialists, aided by an external consultant, lectured about risk administration on the public debt for a wide group of analysts from the National Treasury and the Central Bank. Since then, contacts were deepened between the managers of the Brazilian debt and members of the World Bank, who, in August of 1999, presented an analytic report about the administration of the Brazilian public debt.

In general, a strong international tendency to improve techniques of public debt administration can be observed. Evidence of this tendency is the forum about public debt administration organized by the World Bank (with the presence of analysts of several countries), during the first week of November. Among the main topics that were discussed in that forum, the integrated administration of the government’s assets and liabilities and the development of models of risk administration in the molds of VaR, CaR and the traditional Markowitz method of analysis of averages and variances stand out.

In the next section we will discuss the limitations faced by underdeveloped countries like Brazil, as well as the objectives to be prioritized by the managers of the public debt, taking in account such limitations.
2. Risks and Objectives

The risks of the public debt are several. The financial risk, or market risk, is that related with the volatility of the interest or exchange rate (the most important) or other financial factor that can, directly or indirectly, affect the financial cost of the debt. Such a risk refers to the possibility of deviation of the public debt administration objectives taking in account an increase in: (i) the value of the stock of the debt; (ii) its flows of debt expenditures along a fiscal year (budgetary risk); and / or (iii) the debt expenditures in relation to the income tax or net available government income (liquidity risk). The refinancing risk refers to the probability of being the issuer unable to refinance the redeemed debt or able to just refinance at a high cost. Such a risk is directly proportional to the concentration of the debt in the short term or in few periods.

Some kinds of risks, as the financial one, are easily monetarily measurable. These risks could be compared with the costs incurred in the implementation of a specific strategy. Other risks, as the refinancing and budget one, despite that they can be measured, are strongly related to the economics or political expectations related with public debt, increasing the discretionary power for decisions about the relevance of these in relation to the others. In spite of the complexity of the measurement, those last ones cannot have their relevance minimized by the public debt manager, because they are, in many critical moments, more important than the first ones.
As previously mentioned, several countries are using and improving techniques that allow the monitoring of the risk and cost of their financing strategies. However, it is known that, at each exposure level of the debt to risks we can associate an expected cost. When issuing long term, fixed rate bonds, for instance, the government reduces the sensitivity (risk) of the debt in relation to short term movements in the interest rates. On the other hand, this strategy incurs an additional cost, considering that the transference of the interest rate risk to the market means an increase in the cost of the bonds with such characteristics.

This led us to the question—what would be the parameters used by the debt manager to delimit an ideal balance between the risk and the cost of the public debt. This question considers the identification of the most important risks, as well as for the measurement of the debt cost effect on other government policies, especially the fiscal and monetary policies.

Coessens and Montpellier (1997), for instance, affirm that, in Belgium, the macroeconomic situation and the monetary and fiscal policies determine the general lines for the public debt administration. According to the authors, considering those objectives of government, they try to minimize the financial cost of the debt in the medium term and, at the same time, to maintain the market risk of the debt at acceptable levels.

The objective of the fiscal policy can be to limit the budget deficit to a certain percentage in relation to the gross domestic product (GDP) and / or to maintain the relation between the stock of the net debt and GDP in a fixed interval.
In this case, the administrator of the debt should identify and quantify the risks that different administration strategies can bring in order to reach these fiscal objectives.

Limiting the budget deficit to a percentage of GDP imposes restrictions on debt lengthening, the composition between fixed and floating rate bonds and the debt amount to be refinanced during the year (Coessens and Montpellier, 1997). The lengthening and the composition influence the exposure of the debt to changes in the interest rates. The smaller the duration of the debt and/or bigger the participation of floating rate bonds, the larger the volatility of the debt cost. On the other hand, the volume of the debt to be refinanced during the year affects the exposure of the government to the refinancing risk. The larger the amount being refinanced at a given moment, the larger the exposure of the debt cost to movements in the financial market.

The fiscal objective of maintenance of the relation between the net debt and GDP in a fixed interval creates restrictions related to the volatility of the debt. In this aspect, besides other factors previously mentioned, the debt manager should try to monitor the exposure of the debt to variations in exchange rates. Considering that failures in the exchange policy adopted by the country can result in a strong depreciation of the domestic currency, the proportion of the debt in foreign currency should be controlled to avoid abrupt increases in the amount of the debt, certainly not followed by a correspondent increase in the GDP.

With relation to the limits imposed by the monetary policy, the distribution of the debt between fixed and floating rate bonds stands out again. It is
known that, the larger the proportion of floating rate bonds in the debt, the smaller the effectiveness of the monetary politics, because changes in the interest rates impact the debt cost in the short term, creating an inverse wealth-effect that was intended by the Central Bank. So, it’s important to impose limits on the proportion of floating rate bonds in the debt.

The strategy of public debt administration represents a problem of conditioned optimization. We identify below an optimal strategy that minimizes the debt cost function and, at the same time, satisfies restrictions: (i) to the maximum proportion of floating or exchange rate bonds; (ii) to the minimum duration of the debt; and (iii) to the distribution / concentration of debt along the periods. Notice that, while the first two restrictions are mainly related to the interest rate risk, the third refers to refinancing.

The optimal strategy takes into account that the government should possess an aversion degree to risk relatively larger than the other agents of the economy. This argument, as commented by Wheeler (1997), considers that: (i) individuals are averse to risks in their decisions and hope the government reflects this type of preference while managing their interests; (ii) losses incurred in the government’s portfolio provoke costs that most of the taxpayers are unable to avoid; and (iii) the government doesn’t possess comparative advantages in relation to other participants in the market in attempting to extract profits in its operations. This

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\[4\] Other restrictions could be considered such as the risk of credit of the debt or even operational limitations that affect the organ manager of the debt (vide Bushes, 1999).
dimension of the minimization of the risk under the social optics should be central for public debt administration and its main strategy.

Another aspect to be considered is that the degree of importance given to each restriction differs in each country. Emerging countries, characterized by periods of high volatility and trust crises, should prioritize strategies that reduce the debt exposure to adverse events. According to Giavazzi and Pagano (1990), “the probability of the authorities be capable of supporting a trust crisis is affected directly by the conditions that the government is going to the market to refinance its debt at each specific date.” Therefore, such countries should give high importance to lengthening and to the uniform distribution of the debt.

It’s important to note that the success of the macroeconomic stabilization policy of a country is the most important element for definition of the restrictions to the efficient public debt administration. As we will see in the following chapter, when analyzing the Brazil case, the stability increases the capacity to administrate the public debt efficiently and to improve the agents’ trust in the government’s capacity for financing its liabilities.

Finally, it is relevant to say that the objectives of the debt administration are not limited to cost minimization. Some underdeveloped countries, for instance, have domestic capital market that isn’t well structured. So, an additional objective of debt administration of those countries is the incentive to the development of market to long term government and corporate bonds.
3. Restrictions to the efficient public debt administration in Brazil

Many models of public debt administration, for simplification, assume the basic hypothesis of absence of restrictions that can hinder or disable the achievement of the great points. In practice, however, as previously mentioned, there are two types of restriction that limit the choice of the instruments of debt financing and the best debt distribution in terms of periods of time and volume in each period.

The first type of restriction is directly related to the model of economical policy chosen by the country. Considering that the administration of the public debt cannot conflict with the guidelines dictated by government economists, some limitations can appear from these guidelines. In that sense, it is fundamental that the government economists have the correct perception of the impact on the increase of the cost or risk that such limitations will bring for efficient debt administration. The second restriction type is directly related to the absence of macroeconomic conditions for the optimization, in the form proposed by the model. In the case of Brazil, several restrictions of this order exist, making public debt administration extremely complex.

Brazil, following the international tendency, has assumed the strategy of financing its fiscal deficits in the domestic market. Such a strategy has the advantage of reducing the dependence on the international capital market and the exchange risk, because of the increase of financing in local currency. On the other hand, this strategy creates the need for developing or improving the domestic
financial market and mobilizing huge amount of domestic saving, without which it won’t be sustainable in the long run.

Despite the great effort generated directly by all the agents involved for improving the finance market and of the observed progress in this area, some important restrictions still persist. Among these, ones are directly related to the financial market and the others are related with the macroeconomic situation, involving more complex questions and more difficult resolution of the problem in the short and medium term.

The first restrictions refer, mainly, to the absence of a well structured financial sector, able to improve the capacity of the financial institutions finance cash flows and administer risks efficiently, and to the deficiencies in the regulation and supervision of the bank system. The second restrictions are related to the sustainability of long term macroeconomic policies, as a *sine qua non* condition for the reduction of the market volatility and lengthening the maturity of the liabilities and assets of the financial system. Such changes would motivate, in addition, the development of a yield curve and the structuring of a market to the long term debt bonds, facilitating the lengthening of the public debt on consistent bases.

In discussing the restrictions to the efficient public debt administration we must keep in mind the great interdependence among the three “anchors”: (i) macroeconomic stability; (ii) domestic bonds market; and (iii) public debt administration. The absence of long term macroeconomic stability elevates the uncertainty and, consequently, the preference for short term and indexed assets,
besides reducing the demand for financial assets in a general way. Macroeconomic instability also increases the volatility of interest rates and maintains them at higher levels, increasing the financial risk and cost of the debt.

In the case of the domestic bonds market, its underdevelopment reduces the public debt risk transferred to the society, maintaining it in the banking system and, consequently, increasing the systemic risk. On the other hand, such restriction, associated with higher internal interest rates, motivates the increase of the public and private loans abroad, elevating the exposure to the exchange risk.

Finally, in the case of the public debt administration, its inefficiency leads to the concentration of the debt in the short term, increasing the refinancing risk, and/or in indexed instruments, elevating the financial risk. Such factors reduce the Government's ability to develop a yield curve and to establish a market benchmark, delaying the development of the domestic market of corporate bonds.

In other words, macroeconomic instability limits the domestic bonds market and hinders the efficient administration of the public debt. On the other hand, without a public debt administration that manages efficiently the risks, a debt problem can easily transform itself in a macroeconomic crisis. It is important to point out that, of the three “anchors” cited before, the most important is the macroeconomic stability, an indispensable condition for the development of the other ones.
It is a fact that, despite the efforts to develop the financial market and of the evolution of the public debt administration, the moments of instability that occurred during the last years have been reducing the results considerably, destroying strategies already developed. Beginning in 1994, and more consistently after the second half of 1995, the National Treasury began a process of lengthening and, mainly, deindexation of the debt maturity, through the increase of the fixed rate debt in substitution of the other instruments used.

Until September of 1997, the process of lengthening and deindexation of the domestic debt issued by the National Treasury was led with great efficiency, as it can be observed in the following graph. During that month, the National Treasury auctioned, for the first time since the creation, in 1970, of the Letras do Tesouro Nacional – LTN (fixed rate), these bonds with a maturity of twenty-four months. This auction was considered a milestone in the evolution of the public debt administration. In the same auction, LTN of six and twelve months and exchange rate bonds of thirty six and sixty months were offered.
Such evolution corroborates the previous analysis on the interdependence among the three anchors mentioned. The short period of macroeconomic stability that lasted between March of 1995 (Mexico crisis) and October of 1997 (Asia crisis), created the necessary culture for the lengthening and deindexation of the public debt on consistent bases, giving conditions for efficient debt administration.

Parallel to this, the Treasury began the process of issuing long term fixed rate bonds motivated the appearance of a yield curve for longer periods, creating new possibilities of loans by the corporations in the internal bonds market.

In October of 1997 the Central Bank, considering the Asian crisis, was compelled to increase the interest rates from 21% p.a. to 40% p.a. If the macroeconomic stability observed in the two and a half previous years didn’t allow the Treasury to increase the maturity and the percentage of fixed rate debt, such increase in the interest rates would impact in the short run the cost of the debt. This fact would increase the perception of credit risk considerably, worsening the macroeconomic instability.

On the other hand, the fact that Brazil doesn’t have a financial market sufficiently developed eliminated the possibility of the financial institutions transferring the financial risk created by the increase of fixed rate debt (with a longer term than the term of the financial system liabilities), to the savers. Such a fact negatively affected the monetary policy as an instrument of control of the monetary
aggregates, because changes in the interest rates can increase the systemic risk, given the maintenance of great part of the interest rate risk in the financial system.

In that sense, despite the relevance of economic stability for the process, it urges the development of a domestic financial system on consistent bases. This way, the decisions of the public debt administrators that involve a trade-off between risk and cost, by transferring to the savers the earnings and risks associated with certain strategy, will not represent a concentration of these risks in the hands of the financial system.

After the Asian crisis, several other turbulent moments happened, resulting in a return to the previous strategy. The Treasury was “obliged” to change its strategy, considerably reducing the participation of the fixed rate debt in the total public debt, substituting it by floating rate bonds, updated daily by the open market operations rate (see previous graph). Such changes substantially elevated the interest rates risk of public liabilities.

This increase in the risk can be clearly observed in the following illustration. In two different moments, the Central Bank needed to elevate the short term interest rates. In October of 1997, considering the great concentration of the debt in fixed rate bonds, changes in the interest rates affected just a little the weighted cost of the debt. On the other hand, in September of 1998, when the debt was strongly concentrated in daily floating rate bonds, the abrupt movement in the short term interest rates considerably affected the cost of the debt, negatively altering the perception of the financial agents about government credit risk.
In general lines, based on the discussion presented in this section, the period under analysis proves the interdependence among the three “anchors” described previously and the relevance of the macroeconomic stability for the development of the finance market and the improvement of the administration of the public debt in efficient bases.
4. Integrated administration of assets and liabilities

Several countries have been implementing techniques of public debt administration, taking into account an integrated analysis of their assets and liabilities. They try to find a correct perception of the risks that the government would be exposed to, as a function of their sovereign position, as a function of the chosen instruments for financing their debts. Nevertheless, as we will describe in the next paragraphs, the implementation of this technique (used plenty in the private sector) demands certain considerations and adaptations when applied to the public sector.

4.1 Particularities of the Public Sector

For efficient public debt administration, it should not just be considered the liabilities under responsibility of the manager of the debt, but all the liabilities and assets that can affect an efficient administration. The identification of the items of the balance sheet that are important to the analysis of the risks associated to the debt is made considering two aspects: (i) the entities that should have their assets and liabilities considered; and (ii) the quality of those assets and liabilities.

Related to the first aspect, it is important to point out that the administration of the Brazilian debt, for legal determination, is the responsibility of Ministry of Finance and the National Treasury manages the external and domestic debts. In that sense, if there is a governmental entity (enterprise, autarchy or foundation) that has structural unbalances and periodically request that the National Treasury finance the presented unbalance through the emission of bonds or other
kind of debts, this entity should have its assets and liabilities considered by the manager of the debt, because the risks assumed by it mean risks for National Treasury.

The fact of considering assets and liabilities of other entities to measure the costs and potential risks for the debt doesn't mean that the manager of the debt will be responsible for the control of the balance sheet items and will participate in the decisions of composition of these items or, furthermore, will create any restriction for certain entity in the accomplishment of its functions. The function of the debt managers is only to have a strategic vision of the process as a whole, making movements with the objective of minimize the risks created by others in their institutional attributions.

For example, the states went frequently to the federal government along the last years to request the assumption and negotiation of their debts with other creditors. In that sense, if we assume the last events, the structure of assets and liabilities of the states, fundamental for perception of their structural unbalances, should be considered in the analysis of the federal government debt, because, in the end, it absorbs all the costs and risks.

On the other hand, if we assume that the states, considering the recent negotiation of their debts and the approval of laws that seek to discipline the public expenses and to impose limits to new borrowings (fiscal responsibility, administrative reform and social security laws), will be adjusted and will no longer
need the federal government's financial aid, then it isn't necessary to take into account their balance sheet items.

As we can observe, such analysis is totally related to the perception of the debt manager about the most important variables. The fact of a specific entity not to be considered in a first moment doesn't mean that it couldn't be included later and vice-versa. Among other reasons, this makes the debt administration, despite some universal basic rules, adaptable to the particularities of each country.

In relation to the second aspect, regarding the quality of the assets and liabilities that should be considered, it is taken as a general rule that the items that generate financial flows are the one that can be estimated. In that sense, assets that cannot contribute to the generation of future flows, like real assets, should not be considered. On the other hand, debts that are being discussed in the judicial or administrative branch, and guarantees, among others, should be monitored and have their risks quantified to avoid unpleasant surprises to the debt manager.

Having explained some of the particularities of the techniques of assets and liabilities administration as related to the public sector, we will make an example, practical and simplified, for the Brazilian case.

4.2. Practical example – the Brazilian Case

With the objective of simplifying risk analysis in relation to the relevant balance sheet for the Brazilian case, we assumed that the states, municipal districts,
social security system and other entities of the administration have their balance sheet well structured, not imposing on the Treasury future needs in financing their patrimonial unbalances. In that sense, it would not be necessary to consider their assets, liabilities and future cash flows for risk analysis.

So, the "single balance" will be represented by the items of the National Treasury and the Central Bank balance sheet. We will now study the structure and the relations among the assets and liabilities of these two entities, identifying the most important items and analyzing the risks of those liabilities in relation to the assets.

### 4.2.1 Important National Treasury and Central Bank Assets and Liabilities

Table 1. Consolidated National Treasury and Central Bank Balance Sheet

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic Assets</strong></td>
<td><strong>263.8 Domestic Liabilities</strong></td>
</tr>
<tr>
<td>Return of Operations (Treasury)</td>
<td>187.6 Monetary Base (Central Bank)</td>
</tr>
<tr>
<td>Law n° 7.976/89</td>
<td>11.1 Bonds held by the Market</td>
</tr>
<tr>
<td>Law n° 8.727/93</td>
<td>42.4 Competitive (Treasury)</td>
</tr>
<tr>
<td>Law n° 9.496/97</td>
<td>96.2 Special (Treasury)</td>
</tr>
<tr>
<td>Others</td>
<td>37.9 Competitive (Central Bank)</td>
</tr>
<tr>
<td>Other Assets (Treasury)</td>
<td>33.2 Contractual Debt (Treasury)</td>
</tr>
<tr>
<td>Category</td>
<td>Amount</td>
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<tr>
<td>----------------------------------------------</td>
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<tr>
<td>Counter-guarantees</td>
<td>28.3</td>
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<tr>
<td>Subrogation Rights</td>
<td>4.9</td>
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<td></td>
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<tr>
<td>Other Assets (Central Bank)</td>
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<tr>
<td>External Assets</td>
<td>96.6</td>
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<tr>
<td>Guarantees (Treasury)</td>
<td>5.7</td>
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<tr>
<td>Return Op. – PROEX (Treasury)</td>
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<tr>
<td>International Reserves (CB)</td>
<td>45.1</td>
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<tr>
<td>Other Assets (Central Bank)</td>
<td>39.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Net Worth</td>
<td>(357.6)</td>
</tr>
<tr>
<td>Total Assets</td>
<td>360.4</td>
</tr>
</tbody>
</table>

Sources: Mapa de Exibilidades Líquidas do Tesouro Nacional (MELTN) – STN
Central Bank Report (September 1999) – Banco Central do Brasil
4.2.1.1 National Treasury Assets

Returns of Operations - among the items of the assets, in financial terms, they are the most important (52% of the total assets). They include, mainly, credits against states and municipalities, acquired from the negotiation of their debts, determined by the Laws n° 7.976/89, 8.727/93 and 9.496/97. In correspondence to the credits related with the Law n° 8.727, there are liabilities to the original creditors in practically identical amount, being both canceled for risk analysis.

In that item, the assets related to the Law n° 9.496/97 are the most representative (27% of the total assets). This Law refers to the most recent program of refinancing the debts of the states and Federal District, which were renegotiated in 30 years, with monthly payments and updating factor IGP-DI (price index) + 6% p.a. For assumption of those debts, the National Treasury emitted several different bonds, the bigger part to Central Bank (R$ 80 billion) and the remaining to the market. Considering that the financial conditions of the assets and liabilities created by this Law aren’t equivalent, they cannot be, for analysis of the risk, mutually canceled.

Still classified under the denomination of "return from operations" there are other several operations that total R$ 37.9 billion (11% of the total assets), the main ones being: (a) Medium and Long Term and Paris Club, in the amounts of R$ 15 billion and R$ 3 billion, that can be canceled with part of the liability corresponding to the restructured foreign debt; and (b) public companies extinguished or privatized, in the value of R$ 7 billion, that can be compensated with liabilities classified as "Contractual Debt - New Debt".
Other assets – they correspond to counter-guarantees and subrogation rights. Both have a corresponding item in the liability side of the balance sheet and can be canceled mutually with such items.

Guarantees - they represent United States Treasury bonds, used as collateral of portion of the external bonds issued under the Brady Plan. These guarantees, therefore, can be compensated with part of the liabilities denominated "External Restructured Debt".

Return on PROEX operations - these assets are return from the government program that supports exports (PROEX).

The National Treasury still has, among its assets, a cash balance (called “conta unica”) in the Central Bank in the value of R$ 98 billion. When we consolidated the balances of both institutions, such an item, because it belongs to the assets of Treasury and the liabilities of the Central Bank, was canceled.

4.2.1.2 Central Bank Assets

The item “Other assets” of the Central Bank, belonging to the internal assets, includes the several credits to financial institutions: discount loans, reserve requirements and open market operations.
The item International Reserves (international liquidity concept) represents the available resources in foreign currency. The item “other assets”, in the case of external assets, refers, mainly, to stocks and quotas of international organizations, gold and SDR (IMF).

It is important to say that the item “National Treasury Bonds”, although it doesn’t consist of the consolidated balance, is the largest item of the Central Bank assets. Currently R$ 147 billion, it represents the bonds emitted by National Treasury for the Central Bank, in auctions and special operations. When we consolidated the balance sheet of both institutions, such an item, for belonging to the assets of the Central Bank and the liability of National Treasury, was canceled.

4.2.1.3 National Treasury Liabilities

Bonds issued in the domestic market – they represent the internal federal public debt (called DPMFi) in circulation in the market under the form of bonds, being divided in competitive (negotiated originally through auctions) and special (emitted for specific objectives, defined in law). It has been, in the last years, the main instrument used to finance the budget deficits, and is the most important item of the National Treasury balance sheet, for its volume (40% of the total liabilities) and for its strategic importance in refinancing the outstanding public debt.

Contractual Internal Debt – it refers mainly to the debts renegotiated in the National Program of Privatization.
Other obligations - they include mainly debts to official institutions, renegotiated under the Law 8.727/93, and others that represent, mainly, guarantees of the federal government to internal and external financing operations of the states, municipalities and government enterprises.

Bonds issued in the external market - in the amount of R$ 86.8 billion (12% of the total liabilities), this item takes into account the restructured debt, also known as Brady bonds, and the new competitive global issues, occurred since 1995.

Contractual External Debt - it includes the new debts with multilateral organizations and the debt renegotiated with private banks, governmental agencies and Paris Club.

4.2.1.4 Central Bank Liabilities

Monetary Base - in the value of R$ 33.2 billion (5% of the total liabilities), it includes domestic currency issued and banking reserves.

Bonds issued in the domestic market - in the value of R$ 84 billion (12% of the liabilities), it considers the bonds emitted in the market, by the Central Bank, with objective of monetary and exchange policies.

“Other obligations”, in the case of the internal liabilities, correspond to the required and voluntary deposits and funds and programs resources. In the case of the
After examining the several items of both institutions in detail, the next step will be to structure a consolidated balance sheet, for analysis of the financial differences between both sides. Extensive connections exist between the assets and liabilities of the balance. Some items (Treasury bonds in the Central Bank portfolio and cash balance) were already canceled on both sides. For simplification, with the objective of better analyzing the possible existent unbalances among the two sides of the equation, we will eliminate the other items correlated and some ones that we considered without strategic relevance.

After such adjustments, the Central Government consolidated balance sheet could be visualized as following:

Table 2. National Treasury and Central Bank Simplified Balance Sheet

<table>
<thead>
<tr>
<th>Date: June, 30th, 1999</th>
<th>R$ billions</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic Assets</td>
<td>Domestic Liabilities</td>
</tr>
<tr>
<td>Assets Law n° 9.496/97</td>
<td>96.2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2.2 Risks Associated to the Simplified Structure of the Balance Sheet

Having defined the most important assets and liabilities, the next step would be to identify the risk associated to the balance structure and its flows. Among the assets, international reserves, indexed to the exchange rate, could be considered a hedge (protection) for the external liabilities. Occurs that the National Treasury and the Central Bank issue bonds indexed to the exchange rate in the domestic market. So, international reserves (R$ 45,1 billion) are in a quite inferior level to the sum of the bonds in external market (R$ 86,8 billion) and in domestic market indexed to the exchange rate (R$ 82 billion, being R$ 28,9 billion in National Treasury bonds and R$ 53,1 billion in Central Bank bonds).
Some strategies could be developed to reduce the big current exchange rate risk shown in the analysis, such as the emission of bonds in foreign currency that have a very low or negative correlation. Such a strategy reduces the risk of changes in a specific exchange rate in relation to the internal currency, by creating compensations inside of the basket of currencies chosen. However, it doesn’t eliminate the systemic risk (i.e., the risk of changes in the relation among the domestic currency and all the other ones). Therefore, it would be preferable to reduce the net liabilities in foreign currency, considering that the high historical volatility of the exchange rates means high currency risk.

Such reduction in the amount of bonds related to exchange rate would mean not only to eliminate all the domestic bonds with these characteristics, but also to reduce the volume of external bonds considerably. As previously mentioned, the great solution of the strategic point of view rarely has conditions to be implemented because of the several restrictions to the public debt risk minimization.

In this case, some limitations should be considered, such as the capacity of the internal market in absorbing an important part of the foreign debt and the need of maintaining constancy in the external emissions to consolidate the international market for the public bonds and for the corporative issues.

Another solution for the unbalance between exchange assets and liabilities could appear. Considering the new exchange policy (floating rate) in practice since January of this year, a high correlation between the domestic inflation rate and the exchange rate is expected. If we assume that hypothesis, we could
consider that the assets created by the Law 9.496/97 (R$ 96,2 billion), linked to an inflation index (IGP-DI), will be a hedge alternative for the majority of the liabilities excess in foreign currency (R$ 123,7 billion).

In addition to the problem previously analyzed, the fact that the totality of the assets of our simplified balance (R$ 141,3 billion) is considerably inferior to the totality of the liabilities (R$ 459,6 billion) makes the issue more complex. As the basic premise of immunization of a balance is the equilibrium among the characteristics of the assets and liabilities, when there aren’t enough assets to balance the equation, we should consider the present value of the future revenues as the solution for the problem.

So, the present value of the central government future surpluses could be considered as additional item of the balance sheet, for debt administration policy. In this regard, the financial characteristics of the portion of the debt represented by the difference among the two sides of the balance (R$ 318,3 billion) should not add volatility to the flows, that is, they should maintain narrow correlation with the financial characteristics of the future surpluses.

Regarding that federal fiscal revenues (the basis for the surpluses) should be considered: well distributed along the time, increasing as the nominal growth of GDP and in domestic currency, the risk would be minimized when the volume of bonds corresponding to the mentioned difference would maintain similar characteristics. It means, the risk could be minimized by the sale of long term fixed rate bonds in domestic currency. This argument can be reinforced by the fact that
short term interest rates are more volatile than long term interest rates. So, debt in long term fixed rate bonds would bring less volatility, adjusting the flow of outstanding debt to balance to the reduced flotation of the fiscal revenues.

The above mentioned considerations demonstrate that, for efficient immunization of the debt, we should be aware not only of the financial equilibrium of both sides the balance sheet, but also, and mainly, of the flows generated by the assets and liabilities items considered in the previous analyses.

The immunization of the flows has great relevance in the Brazilian case, because of the existence of high net debt, the big difference between the remuneration of the assets and liabilities and the concentration of the debt in the short term. So, for correct measure of the financial, refinancing and budget risks, among others, we should structure a system of forecast of the assets and liabilities flows, capable of calculate the sensibility of these flows to variations in the factors of remuneration of the debt.

In the next section we will describe some of the existent tools for the measurement of these risks, using a model based on Value-at-Risk concept and a methodology to evaluate the quality of distribution of the outstanding debt.
5. Analysis of Debt Administration Instruments

In this section we described three theoretical tools that can be used by the public debt manager. The first measure to be analyzed is Duration. After, we will analyze a dispersion (with entropy) index - ID$_e$ and finally we will deal with a particular case of Cost-at-Risk (CaR).

We will demonstrate that each one of the three techniques approaches different dimensions to the risks faced by the debt managers. It is important to point out that the use of these techniques follows the international tendency of improving debt administration, emphasizing the methods used by private agents.

Although the emphasis of this section is of theoretical nature, we will illustrate, in the subsection 5.4, the practice of those debt administration techniques based in an example for the Brazilian case.

5.1 Duration and Modified Duration

In the classic case of "discount bonds" the maturity date measures the time that the investor has his capital invested. But in the case of "coupon bonds" the maturity date is not a good measure of this period of time because part of the flows of the "coupon bond" occurs before its expiration. Macaulay (1938) created the concept of Duration to best evaluate the involved period. In this case Duration

\footnote{For an excellent explanation about Duration see Bodie, Kane and Marcus (1999).}
not only measures the maturity but the flows that will happen during this period, too. More specifically, Duration can be defined as the weight average of the flows of maturity where the weights correspond to the present value of the flows. In this case, we can write Duration as:

\[ D_{cn,t} = \frac{C \sum_{i=1}^{n} i}{P_{cn,t}} \frac{1}{(1+Y_{cn,t})^i} + \frac{n}{(1+Y_{cn,t})^n} \]

\( C \) is the coupon rate for period, \( P_{cn,t} \) is the price of the "discount bond" and \( Y_{cn,t} \) is the yield to maturity. Notice that the maturity of the first "discount coupon" is one period and it receives a weight equal to \( C/(1+Y_{cn,t}) \). When the bond is a "discount bond" we have \( C = 0 \) and the Duration is equal to maturity. In practice, it is used most time a concept similar to Duration, called Modified Duration:

\[ \frac{D_{cn,t}}{1+Y_{cn,t}} = -\frac{dP_{cn,t}}{dY_{cn,t}} \frac{1}{P_{cn,t}} \]

Modified Duration measures the sensitivity of the price of the bond (in proportional terms) to a small change in its return. For instance, if Modified Duration is 10, then an increase of one base point in the interest rate (of 3% for 3,01%) will cause a decrease of 0,10% in the price of the bond. Others measures related to Duration were created to analyze other types of more sophisticated bonds, but in the case of government bonds those two measures are quite useful, given knowledge of the volatility of interest rates.
5.2 Dispersion (ID) and Dispersion with Entropy (ID_e) Indexes

Another important measure cited in the literature about debt administration is the Dispersion Index. The idea is relatively simple: we try to compare the structures of maturity. So, a classification of the different maturity structures is obtained on way that the more well distributed the debt, the better the government can avoid confidence crises. ID is defined as 1 minus the relation between the standard deviation of the percent of debt maturing in each period and the standard deviation of the worst case of concentration, which is, 100% of the debt expiring in a single period. We can write ID as:

\[ ID = 1 - \left[ \frac{\text{sd} (\% \text{maturing in } t=1,T)}{\text{sd (worst case)}} \right] \]

Sd is the standard deviation, T is the maximum period of maturity and “period” is the unit of time for which we will divide T. Both variables are expressed in working days. In this case ID \( \in [0,1] \) and the better the debt is distributed the more ID goes to 1 (the same occurs to ID_e). The extreme cases are the following ones: if the debt is equally distributed \( \text{sd} = 0 \), so that ID = 1. On the other hand, if the debt is all concentrated in a single period, the quotient becomes 1, so that ID = 0.

Barcinski (1998) applies the concept above for the period after Real Plan in Brazil. The author shows the importance of this indicator, especially in crisis situations, pointing out that ID doesn't show in a satisfactory way the decrease of Duration. In our simulation, we used the entropy function of ID (ID_e). Again, the
index continues restricted to the interval [0,1] and the closer to 1, better distributed
the debt, with a percent 1/t maturing in each period. In this case, we can write
adjusted ID for entropy as:

\[ ID_e = \frac{\sum_{i=1}^{T} p_i \ln(1/p_i)}{\ln(T)} \]

\( P_i \) is the percent of the debt maturing in the period i, for positive values
of \( p_i \). As argued by Barcinski (1998), the indexes ID and ID_e present similar
behavior. So, when using ID_e to structure the examples in the subsection 5.4, the
conclusion of our analyses will not lose quality.

5.3 Cost-at-Risk (CaR)

The idea of CaR is derived of the Value-at-Risk concept (VaR) used by
financial institutions to manage risks. Some pioneering countries, like Denmark and
Portugal, adapted the notion of VaR, but with a slight change in the focus. In the
case of CaR, it’s intended to measure the risk by calculating a distribution of
probability estimated on different strategies, based in several yield curves. In a
simple case, we will assume that, for a specific stock of the debt (R$100 billion), the
yield curve is constant and equal to 5%. Besides, suppose that the interest rate is
normally distributed, so that we can generate the distribution of probabilities of the
cost of the debt. With the distribution of probabilities determined, we can compute
the different probabilities associated with the different results. That it is CaR.
In the case of this example, we have the following distribution:

Table 3. Cost in R$ billion

<table>
<thead>
<tr>
<th>Cost</th>
<th>1-2</th>
<th>2-3</th>
<th>3-4</th>
<th>4-5</th>
<th>5-6</th>
<th>6-7</th>
<th>7-8</th>
<th>8-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0,1%</td>
<td>2,1%</td>
<td>13,6%</td>
<td>34,1%</td>
<td>34,1%</td>
<td>13,6%</td>
<td>2,1%</td>
<td>0,1%</td>
</tr>
</tbody>
</table>

Given the simplicity of the example, notice that the distribution is symmetrical and the values are relatively concentrated around the average. In reality, the distribution will depend on the yield curve shape as well as of the government financing strategy. The idea is to simulate the behavior of the yield curve using historical data and, considering the assumptions about the initial stock of the debt and the financing strategies, to determine the CaR.

Despite the use of historical data to simulate the behavior of the interest rates, it is important to comment that, in countries traditionally sensitive to international adverse economical moments, the use of projections with different probabilities and, mainly, stress hypotheses scenarios are very important for a better measurement of the risks associated with a specific governmental debt strategy.

Returning to the previous subject, in the next subsection we will apply the model using the methods described before. To facilitate the explanation, we will demonstrate, step by step, the simulation used in this study. Initially, we will need to structure a yield curve model. The idea is to estimate one factorial model to obtain the coefficients and use them in a structural model (in the case of Denmark the
model is Cox, Ingersoll and Ross (1985)). When simulating the structural model we will obtain the prices of the bonds and their respective yields to maturity. So, we can obtain a "yield curve" in each simulation. In this paper, we will use a discreet time approximation of the Vasicek model (1977), proposed by Singleton (1990)\(^6\).

A common element of these yield curve theoretical models is the supposition of the existence of a factor that determine the interest rate movements. Generally speaking, the factors are originated of terms that involve the marginal utility of consumption, parameters of risk average aversion of the economy, among others\(^7\). In the Vasicek model, the factor is not specified explicitly. In this case we used the Brazilian discount rate, known as “taxa SELIC”, as our empirical factor.

It is convenient to assume that the factor (state variable) follows a simple stochastic process. In our case, we assumed a process of the type AR(1) with average \( \mu \) e persistence \( \phi \):

\[
(5) \quad x_{t+1} = (1 - \phi) \mu + \phi x_t + \xi_{t+1}
\]

It's important to notice that the innovation \( \xi_{t+1} \) isn't necessarily a “white noise”. Besides, it could be correlated with certain variables in the period \( t \). Singleton

\(^6\) For larger details on the models to consult Vasicek (1977) and especially Singleton (1990) and Sun (1992).

\(^7\) More precisely, some models are based on the representative agent, so that the aversion to this imaginary agent's risk corresponds the average of the economy.
(1990) shows that considering this specification of the model we can write the price of a “zero-coupon” bond as:

(6) \[ p_{t,n} = -x_t + \beta^2 \sigma^2 / 2 \]

where the parameters \( \beta \) and \( \sigma^2 \) can be estimated from the factorial model given in (5). So, we can write the yield as:

(7) \[ y_{t,n} = x_t - \beta^2 \sigma^2 / 2 \]

In this model, the short term interest rate assumed the dynamic of the factorial model. In fact, the short term interest rate reflects the economy current situation, given the stochastic movement of the factor. In general, we intend to determine the price of a “n maturity” bond. Then, we propose a conjecture involving the factor and we derive a recursive equation, where we obtain a price and the corresponding return for each maturity. We use the equation (7) as initial condition. In this case, we can write the “pricing function” as:

(8) \[ -p_{n,t} = A_n + B_n x_t \]

(9) \[ y_{n,t} = -p_{n,t} / n \]
where the coefficients $A_n$ and $B_n$ are functions of the parameters of the model.

Notice that the coefficient $B_n$ measures the sensitivity of the bond price to changes in the yield to maturity and that this converges for $B = 1/(1-\phi)$ when $n$ tends to infinite. This way, the bond prices fall in response to an increase in the short term interest rates and the sensibility of the yield increases when the maturity of the bond increases.

After the simulations described above, we obtain a variety of scenarios for the yield curve. The natural step suggested by the Denmark and Portugal models would be to compute CaR using this group of yield curves. However, we want to incorporate explicitly in our model the effect of the refinancing risk in the cost of rollover the public debt.

Based on that idea, we used the interest rate binomial model proposed by Black, Derman and Toy (1990), to determine the different paths (scenarios) for the interest rates and, consequently, for the cost of the debt during the period. Such scenarios represent a “tree” of possibilities that will be used in the next step.

---

8 The coefficients are given for $B_n = (1-\phi^n)/(1-\phi)$ and $A_n = A_{n-1} + (1-\phi)\mu B_n - (\beta + B_{n-1})^2 \sigma^2 / 2$

9 It's important to observe that in the binomial model, the number of possible yield curves is equal to $2T$, where $T$ is the number of periods.
5.4 A practical example based on the Brazilian data

Based on the model cited in the previous subsection, we will illustrate the application of CaR, as well other mentioned methods. In the case of CaR, the idea is to simulate a certain number of scenarios for the yield curve, to calibrate the interest rates tree and then to refinance the debt based in the different financing strategies. In the case of the simulations, we simulated the stochastic process of the factor given by the equation (5) and then we calculated the returns based on the equation (9). As the intention of this subsection is just to illustrate the methodology, we simulate 3 samples, with hundred yield curves each, and, after, we calculate the average of the 3 samples. We used these yield curve to calibrate the interest rates tree and, then, to compute CaR.

In this example, we used the stock of LFT (floating rate bond remunerated by a daily interest rate, similar to the discount rate) and LTN (fixed rate bond) held by the market in June of 1999. With amounts of R$ 130.5 billion and R$ 25.8 billion, respectively, these bonds represented approximately 66% of the total bonds issued by the National Treasury to the domestic market. To simplify our demonstration of the model, we considered the following hypotheses:

- the date is December, 31st, 1999;
• the amount previously mentioned, R$ 130.5 billhões, represents the total debt of National Treasury held by the market on December 31st, 1999. So, we will compute CaR regarding the first six months of the year 2000.

• the LTN and LFT maturity dates happen in the first day of every month, and LTN matures in three equal and consecutive portions beginning in the January 1st, 2000. On the other hand, LFT matures in the next 18 months, in portions that vary taking in account the month of reference. More specifically, it is assumed that: 7,5% of the stock of LFT will expire in each one of the months of January, April, July and October; 3,5% will expire in each one of the months of February, May, August and November; and 5,5% will expire in each one of the months of March, June, September and December. In order to assure the consistence of this maturity structure with the total of 18 portions, we made an adjustment in the last maturity, June/2001, in which will matures 6.5% of the LFT.

Based on those hypotheses, we calculated the average maturity and the dispersion index of the current debt (hypothetical). These measures serve as point of reference to define the strategies to be adopted by the government and for the comparative evaluation of their results.

The current debt average maturity simply corresponds to the weight average of the flows assumed previously, in this case 7,23 months. Using the cited methodology to calculate the dispersion index (adjusted by the entropy) we found a value of 0,969. Remember that this index is defined for the interval [0,1], and close results of 1, as this one, indicate that the debt maturity is well distributed.
With these initial data, we simulated the effects of two strategies that could be used by National Treasury in these auctions. In practice, obviously the debt manager can analyze several different strategies. However, following our strictly illustrative propose, we defined two extreme strategies to understand better all the steps of the process.

The first strategy considers that the Treasury will refinance all the future debt maturity using three-month fixed rate bonds (LTN). This period is consistent with the most recent issuing of this bond. Like this, we can observe what would be the effects on the cost and risk of the debt if the Treasury started a process of strong deindexation of the debt. In an opposite way, the second strategy supposes that Treasury will issue only eighteen-month floating rate bonds (LFT). In this extreme case, the Treasury would be giving preference to the lengthening of the debt, despite an increase in the exposure to the interest rate.

To determinate the need for financing during the first semester of 2000, we considered that the stock of the debt will remain constant. This assumes that every month the government primary surplus will be enough to pay the interest rate expenses (accrual basis). In spite of this hypothesis not be necessary to calculate the CaR, its use is quite usual, because besides simplifying the model, it allows us directly estimate the need of primary surplus to maintain the debt in the current real amount.
The effects of the strategies described above in the medium term and distribution of the debt maturity at the end of June, 2000 (represented by term and ID_\text{e}, respectively) are described in the following table.

Table 4. Dispersion index and Debt Medium Term

<table>
<thead>
<tr>
<th></th>
<th>Strategy 1</th>
<th>Strategy 2</th>
<th>Current Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID_\text{e}</td>
<td>0.889</td>
<td>0.968</td>
<td>0.969</td>
</tr>
<tr>
<td>Debt Term (months)</td>
<td>3.46</td>
<td>9.16</td>
<td>7.23</td>
</tr>
</tbody>
</table>

Related to the dispersion index, it can be observed that there is little change on it, mainly in the case of the second strategy. The more accentuated effect (fall) verified in the first strategy occurs because of the refinancing of the debt with short term instruments (three-month fixed rate bonds). In relation to the medium term of the debt, it is noticed that the strategies change it significantly, when compared to the current situation.

In a certain way, this result was expected, given the discrepancy among the maturity of the bonds considered in the two strategies (LTN of 3 months versus LFT of 18 months). However, its important to remember that, in the case of the second strategy, this increase in the term brings negative consequences because it exposes the public debt to a larger interest rate risk.

Based in the three yield curves considered in this example, we built three interest rates trees (following the methodology previously described) that resulted in 96 possible scenarios at the end of the six months analyzed. These
scenarios were used to generate the Probability Density Function (PDF) of the cost of the public debt under the two strategies. The following table presents the descriptive statistics and the illustrations that contain the PDFs.

Table 5. Descriptive statistics of the strategies

<table>
<thead>
<tr>
<th></th>
<th>Strategy 1</th>
<th>Strategy 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>14,605,722</td>
<td>14,482,274</td>
</tr>
<tr>
<td>Median</td>
<td>14,546,576</td>
<td>14,401,294</td>
</tr>
<tr>
<td>Maximum</td>
<td>16,719,282</td>
<td>16,876,618</td>
</tr>
<tr>
<td>Minimum</td>
<td>12,713,067</td>
<td>12,374,552</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>952,007.3</td>
<td>1,043,267.0</td>
</tr>
<tr>
<td>Asymmetry Index</td>
<td>0.177616</td>
<td>0.190707</td>
</tr>
<tr>
<td>Kurtosis Index</td>
<td>2.385592</td>
<td>2.438165</td>
</tr>
</tbody>
</table>

The numbers above create conditions to calculate the CaR of each strategy. Our tests don’t reject the assumption of normality. In this case we can calculate CaR based on the average and standard deviation of each strategy. Then, the CaR (level of confidence: 95%) is R$ 16,177 million and R$ 16,204 to strategies 1 and 2, respectively. CaR can also be measured in relative terms, corresponding to the difference between the upperlimit of the confidence interval and the average of

10 The distributions are calculated considering a gaussian “kernel” taking in account the 96 observations. It is important to point out that the obtained results are robust to different choices of the “kernel.” For better details, consult Pagan and Ullah (1999).

11 Based on the Jarque-Bera test, with 5% of significance.
the distribution. So, the relative CaR for the strategy 1 is R$ 1,571 million, while for
the strategy 2 is R$ 1,720 million.

Considering the data generated, we can analyze the trade-off between
risk and cost of each strategy. The risk would be measured by the difference
between the relative CaRs and the cost would be calculated by the difference
between the basic financing cost of the debt (discount rate) and the cost of each
strategy (average of the distribution). Considering as basic cost, for simplification,
the financing strategy with LFT, we could use it as reference to calculate the
additional cost of the other strategies.

So, the additional cost of the strategy 1, compared to the basic cost
(strategy 2), is R$ 123 million and its financial risk is smaller in R$ 149 million. In
other words, strategy 1, in a strictly financial point of view, is preferable, because it
reduces the financial risk more than it increases the cost. Then, it would be
preferable to accelerate the process of debt deindexation, lengthening the maturity
of the fixed rate bonds issued, to approximate more the costs and risks.

In the example above, the difference between the cost and the risk isn’t
high enough to determine changes in the public debt administration objectives. Also,
other discretionary variables (not measurable) should be considered, such as the
importance of the process of deindexation of the public liabilities for the
implementation of the monetary policy and the increase of the refinancing risk.
Illustration 3. Probability Density Function of the Debt Costs

The PDFs obtained reinforce the previous comments, when demonstrating that, from the point of view of the cost and risk, the two strategies don’t show us economically significant differences. This is due mainly to the fact that a three-month fixed bond doesn’t represent risks fundamentally different from a floating rate bond. Another point to be pointed out is that in our analysis we considered CaR for a period of six months only. This assumption underestimates the impacts of these strategies in the cost of the debt, because relevant part of their effects will be felt only in the medium and long terms.

For comparative effect, we examined a third financing strategy: issues of six-month fixed rate bonds to refinance all the debt. In this case, we obtained a average cost and CaR (95%) of R$ 14,700 million and R$ 16,138 million, respectively. This means a larger average cost in R$ 218 million, considering the basic strategy, and smaller risk in R$ 718 million. In other words, the deindexation process would be, considering the assumed hypotheses, reducing the risk much more quickly than the increase of the financing cost of the debt, what would reinforce the argument in favor of the maintenance of such strategy.
In a certain way, the methodology CaR should be preferably employed longer planning horizons. Considering that, generally speaking, the debt administration policies are formulated for a period of at least one year, in conformity with the budget and fiscal policies, that should be the minimum period for a program of risk administration based on CaR.

It's important to notice that, when incorporating the binomial model of Black and other (1990), our methodology, applied to a one year period, would result in the existence of 2,048 scenarios, what would make possible larger precision in the estimation of the PDF and, consequently, of the CaR. Moreover, this would also conduct to a larger volatility of the costs that would increase the power of our methods in predict the consequences of the two strategies.
6. Conclusions and Recommendations

Nowadays, the sophistication of the techniques of public debt risk administration is undoubtedly one of the topics most discussed at the international level. Based on theoretical concepts, the government financial analysts of several countries are adapting private sector models to use in the public sector. This is the case, for instance, of the implementation of integrated analyses of the government assets and liabilities and of models developed under the Value-at-Risk foundations.

In this work, we demonstrated that the decision process related to the appropriate debt administration strategy can be understood as a problem of conditioned optimization, in which we try to minimize the cost of the debt considering restrictions imposed by fiscal and monetary policies, mainly.

The first years of “Real Plan” and the period after October of 1997 prove that the efficient administration of the debt depends of the development of the capital market and, mainly, of a sustainable macroeconomic policy.

The description and analysis of the National Treasury and Central Bank assets and liabilities in the chapter 4 demonstrates the current exposure of the balance of these institutions to variations in the exchange and interest rates. Such risks suggest that National Treasury should try to reduce its exchange rate liabilities and, as far as possible, to prioritize the use of long term fixed rate bonds, considering the restriction related to the domestic savings.
It is known, however, that strategies of risk reduction commented in the previous paragraph impact in a negative way the cost of the public debt. To aid the monitoring of this balance between cost and risk it is fundamental that the Treasury continues in its process of modernizing debt administration instruments.

Finally, it is important to point out the absence of an universal model for the administration of risk in the public debt. Although based on performance measurement, it is important to note that risk management is not an exact science. It draws as much on experience and judgement as it does on the numbers that reflect performance. So, the process for managing risk should not be overly prescriptive, but flexible enough to reflect the practical realities of dealing with risk within a structured framework.

For this reason, we incorporated in the CaR model analyzed in the chapter 5 a risk factor (refinancing risk), that, as we could see, it wasn't considered integrally in previous models. Underdeveloped countries, like Brazil, should prioritize the control of this risk to reduce the probability of trust crises. We can conclude, therefore, that, despite of the process of sophistication of the public debt administration, the decision related to the instruments and appropriate strategies continues to be an art for those involved in the formulation of policies related to the debt.
References


