Measuring the Euro Area Output Gap using Multivariate Unobserved Components Models Containing Phase Shifts

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Abstract

This paper analyses the impact of using different macroeconomic variables and output decompositions to estimate the euro area output gap. We estimate twelve multivariate unobserved components models with phase shifts being allowed between individual cyclical components. As output decomposition plays a central role in all multivariate models, three different output decompositions are utilised; these are a first-order stochastic cycle combined with either a local linear trend or a damped slope trend, and a second-order cycle plus an appropriate trend specification (a trend following a random walk with a constant drift is generally preferred). We also extend the commonly used trivariate models of output, inflation and unemployment to incorporate a fourth variable, either investment or industrial production. We find that the four-variate model incorporating industrial production produces the most satisfactory output gap estimates, especially when the output gap is modelled as a first-order cycle. In addition, measuring phase shifts and calculating contemporaneous correlations between individual cyclical components provides a better understanding of the different gap estimates. We conclude that the output gap estimate in all models leads the cyclical components of inflation and unemployment, but lags those of industrial production and investment. Furthermore, the output gap estimates obtained from the four-variate model including investment present the longest leads-and-lags with respect to other cyclical components, implying that investment appears to be more of a leading indicator than a coincident variable for the euro area.

Keywords: output gap, higher-order cycle, industrial production, state-space, Kalman filter.
JEL classifications: C32, E32.

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1 Introduction

A fundamental objective of monetary and fiscal policy is to dampen economic fluctuations by keeping key macroeconomic variables, such as output and unemployment, close to their natural rates. To do this, economists need to be able to identify accurately the unobserved features of an economy, such as potential (trend) output, the output gap and the Non-Accelerating Inflation Rate of Unemployment (NAIRU), from observed macroeconomic data. Potential output is the maximum level of output that the economy can produce with stable inflation, and it should be accompanied by an unemployment rate that is consistent with the NAIRU. Deviations in output from this potential define the output gap, which is typically used as an indicator of inflationary pressures. A positive output gap implies that output is above its potential, in which case inflation will generally rise and a tighter monetary policy may then be needed to curb demand and inflationary pressures. Moreover, from a fiscal policy perspective, knowing the output gap will enable the cyclically adjusted budget deficit to be calculated, which is important since this provides a measure of the health of the underlying public finances.

In short, the output gap plays a central role in determining the stance of an economy's monetary and fiscal policies. A macroeconomic policy based on accurate output gap estimates can therefore attempt to mitigate the adverse effects associated with recessions and below trend growth and also help to provide sustainable economic growth. Conversely, basing economic policy on unreliable output gap estimates may damage the economy: for example, the surge in inflation during the 1970s was likely to have been, in part, due to monetary policy underestimating the size of the output gap.

It is well known that the output gap is notoriously difficult to measure and, as a consequence, output gap estimates differ widely depending on the methods used for their calculation (Canova, 1998). These methods can be divided broadly into three groups. The first relies on purely statistical approaches, such as the Beveridge-Nelson (1981) decomposition, the Hodrick-Prescott (1997) trend filter, the Baxter and King (1999) band-pass filter, and so on. These models simply 'let the data speak' and do not include potentially useful information about the supply side of the economy or business cycle information that might be contained in macroeconomic variables other than aggregate output. The second group employs the production function approach (PFA) that has been widely used by international institutions, such as the OECD (2001), the IMF (de Masi, 1997) and the European Commission (McMorrow and Roeger, 2001). Unlike the first group, PFA is a multivariate method that constructs potential output from the levels of its structural determinants, such as productivity and factor inputs. The third group is also multivariate but uses aggregate level data, incorporating a statistical output decomposition along with the inclusion of macroeconomic relations, such as the Phillips curve and Okun's law, and other variables that may contain business cycle fluctuations. Various multivariate unobserved components (UC) models are used by this group. A major advantage of a multivariate UC model over a purely statistical approach is that the former utilises a range of economic data. In addition, as a UC model can be cast in state-space form and estimated using the Kalman filter, estimates of unobserved components and their associated mean squared errors can be obtained. The latter provide an indication of how accurate potential output, and hence output gap, estimates are, something that cannot be provided by most PFA approaches.[1] Compared with other multivariate

[1] An exception is Proietti *et al*. (2007), who propose a multivariate UC model based on PFA.

decompositions, such as the multivariate Beveridge-Nelson decomposition (King *et al*. 1991; Proietti, 1997), multivariate UC models have more flexibility as they allow for dynamic interactions between observed and unobserved variables to be modelled in specific ways according to the objectives of the research.

An early example of this line of research was the bivariate UC model of US output and unemployment, based on Okun's law, that was proposed by Clark (1989) to estimate the output gap. Subsequently, Kuttner's (1994) bivariate UC specification combined Watson's (1986) output decomposition with a backward-looking version of the Phillips curve to relate changes in inflation to the output gap. A trivariate model of output, inflation and unemployment was used by Apel and Jansson (1999) to systematically estimate the NAIRU and the output gap for the UK, US and Canada. Rünstler (2002) extended Kuttner's (1994) bivariate model by including capacity utilisation and factor inputs to estimate the euro area output gap. Doménech and Gomez (2006) estimated the US output gap using a four-variate model of output, unemployment, inflation and investment in which volatility breaks were allowed. In addition, a hybrid version of the Phillips curve was proposed which contained several lagged inflation rates to account for backward-looking behaviour in forming inflation, while forward-looking behaviour was captured by the dynamics of core inflation. However, this hybrid version of the Phillips curve was criticised by Harvey (2008) on the grounds that using lagged dependent variables imposes restrictions on the estimates of core inflation: he therefore preferred to specify the Phillips curve in terms of additive components. Basistha and Nelson (2007) estimated the US output gap using a bivariate model that combines inflation and output through a forward-looking Phillips curve, with core inflation being a linear projection on survey expectations of inflation and a lagged inflation rate. Additionally, a non-zero correlation between the innovations of the trend and cycle components was allowed.

To the best of our knowledge, previous research, including that discussed above, has not specifically investigated the relationships between the output gap and any cyclical fluctuations contained in those additional variables, such as inflation and unemployment, used to identify the gap. However, knowing the phase shifts and correlations that exist between the various cyclical components could enable us to understand the properties of different output gap estimates and help to select the appropriate variables to use, along with output, in calculating the gap. To investigate this, we use stochastic cycle specifications, of the type developed by Harvey (1989), Harvey and Jaeger (1993), Trimbur (2001) and Harvey and Trimbur (2003), to model the dynamics of the output gap, and follow Rünstler (1997, 1998, 2004) in exploiting the phase shifts and contemporaneous cycle correlations implied by the various model parameter estimates. In addition, we extend the commonly used trivariate models of output, inflation and unemployment to incorporate a fourth variable, either investment, as suggested by Doménech and Gomez (2006), or industrial production. The importance of industrial production for indicating the cyclical state of the economy is emphasised by the business cycle dating committees of both the National Bureau of Economic Research (NBER), for the US, and the Centre for Economic Policy Research (CEPR), for Europe, but has generally been overlooked by academic research.

As output decompositions play a central role in all our multivariate models, we also investigate the impact of using different decompositions on output gap estimates. In particular, we compare three alternative combinations of the output trend and cycle. These are a first-order stochastic cycle with either a local linear trend or a damped slope trend, and a second-order stochastic cycle plus an appropriate trend specification (a random walk trend having a constant drift is always statistically preferred). Higher-order stochastic cycles have been little exploited in the literature. Although Harvey and Trimbur (2003) find that a second-order cycle produces good results in their univariate and bivariate models of US output and investment, they do not go on to assess the reliability of their resulting output gap estimates. Such reliability issues have been the concern of Orphanides and van Norden (2002), Camba-Méndez and Rodriguez-Palenzuela (2001), Rünstler (2002) and Jean-Philippe and van Norden (2005) and we also address these issues here.

To preview our results, we find that four-variate models with industrial production as the fourth variable and with the output gap modelled as a first-order stochastic cycle produce the most satisfactory gap estimates, both in terms of having only small revisions over time and in having only small errors left in the final output gap estimates. In addition, we observe that, when the output gap is specified as a second-order cycle, final estimation errors of the output gap increase considerably when compared to those from a first-order cycle specification. Finally, investment appears to be a leading indicator with respect to the other variables included in the model, since the leads-and-lags between the output gap and individual cyclical components are considerably larger in those models containing investment than in other models.

The rest of the paper is organised as follows. Section 2 presents twelve multivariate UC models for estimating the output gap. The parameter estimates and unobserved components obtained from these models are discussed in section 3. Section 4 focuses on analysing the contemporaneous correlations and phase shifts between these cyclical components. The reliability of the output gap estimates are assessed in section 5 and, finally, section 6 concludes.

2 The model

2.1 Output decomposition

Output decomposition obviously plays a central role in all the UC models that are investigated in this paper. A trend-cycle model of output can be set up as

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad t = 1,...,T, \qquad (1)$$

where output, y_t, is decomposed into a trend μ_t, a cycle (the output gap) ψ_t, and an irregular component, ε_t. The trend is modelled as

$$\mu_t = \mu_{t-1} + \beta_{t-1} + m + \eta_t, \qquad \eta_t \sim \text{NID}(0,\sigma_\eta^2), \qquad (2)$$

$$\beta_t = \phi\beta_{t-1} + \xi_t, \qquad \xi_t \sim \text{NID}(0,\sigma_\xi^2),$$

β_t is the slope of the trend, m is a constant drift and ϕ is the damping factor. The disturbances ε_t, η_t and ξ_t are assumed to be mutually and serially independent. As with Proietti *et al*. (2007), alternative specifications of the trend component are obtained by imposing restrictions on equation (2). First, by setting $m = 0$ and $\phi = 1$, the trend becomes a local linear trend (LLT). Second, a generalised local linear trend can be obtained by just imposing $|\phi| < 1$, which forces the trend component to have a damped slope (DST). The Hodrick-Prescott (1997) filter can be obtained by imposing, along with $m = 0$ and $\phi = 1$, the restrictions $\psi_t = 0$, $\sigma_\eta^2 = 0$ and $\sigma_\varepsilon^2 = 1600\sigma_\xi^2$ on equations (1) and (2), so that the UC model takes the form of a 'smooth trend plus white noise'. Proietti *et al*. (2007), however, have shown that this filter is inappropriate for the euro area data as the residuals from such a model exhibit strong autocorrelation.

The cycle, ψ_t, is a stationary process that has zero long-run persistence, strong autocorrelation and alteration of phases. We consider two specifications of a stochastic cyclical component, the first-order cycle introduced by Harvey (1989) and Harvey and Jaeger (1993) and the higher-order cycle proposed by Trimbur (2001) and Harvey and Trimbur (2003).

The statistical specification of a first-order cycle is

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ k_t^* \end{bmatrix}. \tag{3}$$

$$\begin{bmatrix} k_t \\ k_t^* \end{bmatrix} \sim \text{NID}\left(0, \begin{bmatrix} \sigma_k^2 & 0 \\ 0 & \sigma_k^2 \end{bmatrix}\right),$$

Both k_t and k_t^* are serially uncorrelated and mutually uncorrelated with ε_t, η_t and ξ_t. The parameters ρ and λ_c are the damping factor and cycle frequency, respectively, and satisfy $0 \le \rho < 1$ and , with values of ρ close to one yielding a more persistent cycle. The autocorrelation function (ACF), $\Gamma(s)$, for $\tilde{\psi}_t = \left[\psi_t, \psi_t^*\right]^{\mathsf{T}}$ is given by damped cosine and sine waves of length $2\pi/\lambda_c$,

$$\Gamma(s) = \sigma_\psi^2 \rho^{|s|} \begin{bmatrix} \cos(s\lambda_c) & \sin(s\lambda_c) \\ -\sin(s\lambda_c) & \cos(s\lambda_c) \end{bmatrix}, \tag{5}$$

where $\sigma_\psi^2 = \sigma_k^2 / \left(1-\rho^2\right)$ is the variance of the first-order cycle.

The first-order cycle can be generalised to a higher-order cycle as shown by Trimbur (2001) and Harvey and Trimbur (2003). Smoother cycle processes can be produced by specifying $\psi_t = \psi_t^{(n)}$, where

$$\begin{bmatrix} \psi_t^{(n)} \\ \psi_t^{*(n)} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(n)} \\ \psi_{t-1}^{*(n)} \end{bmatrix} + \begin{bmatrix} \psi_t^{(n-1)} \\ \psi_t^{*(n-1)} \end{bmatrix}, \tag{6}$$

for $n = 1, \ldots, k,$ and where $k_t = \psi_t^{(0)}$ and $k_t^* = \psi_t^{*(0)}$ are two mutually uncorrelated white noise disturbances with zero mean and common variance σ_k^2. Setting the order n to be greater than one leads to a greater concentration on a particular frequency band, and thus results in smoother cycle components than when $n=1$. The second-order cycle, $n=2$, is used here as it produces smoother cycles than the first-order cycle and may therefore give a better measure of the output gap.

The ACF matrix, $\Gamma(s)$, of the vector $\left[\psi_t^{(2)}, \psi_t^{*(2)}\right]^{\mathsf{T}}$ is

$$\Gamma(s) = \sigma_\psi^2 \left(1 + \frac{1-\rho^2}{1+\rho^2}|s|\right)\rho^{|s|} \begin{bmatrix} \cos(s\lambda_c) & \sin(s\lambda_c) \\ -\sin(s\lambda_c) & \cos(s\lambda_c) \end{bmatrix}, \tag{7}$$

where now

$$\sigma_\psi^2 = \frac{1+\rho^2}{\left(1-\rho^2\right)^3}\sigma_k^2 .$$

Expressions for larger n are derived in Trimbur (2001).

2.2 The Phillips curve

The Phillips curve used in this paper is specified as

$$\pi_t = \bar{\pi}_t + \gamma_t + \theta_\pi \psi_t^{(n)} + \theta_\pi^* \psi_t^{*(n)} + \upsilon_{\pi t}, \quad \upsilon_{\pi t} \sim \mathrm{NID}\left(0, \sigma_{\upsilon\pi}^2\right), \tag{8}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \omega_{\pi t}, \qquad \omega_{\pi t} \sim \mathrm{NID}\left(0, \sigma_{\omega\pi}^2\right),$$

where core inflation $\bar{\pi}_t$ follows a random walk and γ_t is a seasonal component that can be modelled as either a deterministic or a trigonometric seasonal component (Harvey, 1989). This formulation can be regarded as both a forward and backward-looking Phillips curve (Harvey, 2008). Moving average terms can be introduced into (8) if autocorrelation is present in the residuals. Harvey (2008) argues that (8) should be preferred to a specification using autoregressive terms as in the latter specification the dynamics of inflation are imposed on core inflation.[2,3]

[2] Harvey demonstrates this argument using the simple equation $\pi_t = \bar{\pi}_t + \alpha\pi_{t-1} + \varepsilon_t$, in which the dynamics of the inflation gap are picked up by a lagged dependent variable. The reduced form of this

Following Rünstler (1997, 1998, 2004), we link both $\psi_t^{(n)}$ and $\psi_t^{*(n)}$ to inflation via the loading parameters θ_π and θ_π^*. This allows us to analyse both phase shifts and the correlation between the output gap and the cyclical component of inflation (see Section 4).

2.3 Okun's Law

Apel and Jansson (1999), Rünstler (2002) and Camba-Méndez and Rodriguez-Palenzuela (2001) all propose trivariate models that incorporate both the Phillips curve and Okun's law along with an output decomposition to estimate the output gap. Here we specify the negative relationship between the output gap and cyclical unemployment by means of the following equation

$$ur_t = ur_t^{tr} + \theta_{ur}\psi_t^{(n)} + \theta_{ur}^*\psi_t^{*(n)} + \varepsilon_t^{ur}, \qquad \varepsilon_t^{ur} \sim \text{NID}\left(0, \sigma_{ur}^2\right) \tag{9}$$

where the unemployment rate, ur_t, is decomposed into a trend component ur_t^{tr}, a cyclical component, $\theta_{ur}\psi_t^{(n)} + \theta_{ur}^*\psi_t^{*(n)}$, that is a linear combination of both ψ_t and $\psi_t^{*(n)}$, plus an idiosyncratic term, ε_t^{ur}.

The NAIRU, the trend component in the unemployment rate, is specified as a LLT trend,

$$ur_t^{tr} = ur_{t-1}^{tr} + ur_{t-1}^{s} + \eta_t^{ur}, \qquad \eta_t^{ur} \sim \text{NID}\left(0, \sigma_\eta^{ur^2}\right) \tag{10}$$

$$ur_t^{s} = ur_{t-1}^{s} + \xi_t^{ur}, \qquad \xi_t^{ur} \sim \text{NID}\left(0, \sigma_\xi^{ur^2}\right)$$

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equation, $\pi_t = (1-\alpha L)^{-1}\bar{\pi}_t + (1-\alpha L)^{-1}\varepsilon_t$, shows that the dynamics of the inflation gap are imposed on core inflation, $\bar{\pi}$.

[3] We also tried the hybrid NKPC specification proposed by Doménech and Gomez (2006). However, we found that the autoregressive parameter estimates introduced a significant amount of uncertainty into the estimates of core inflation as both parameter uncertainty and final estimation errors were found to be larger than those produced by equation (8). Therefore, we choose to use equation (8) to model inflation dynamics in all the models analysed in this paper.

This specification is chosen in an attempt to model the upward trend in euro area unemployment during the sample period, which increased from the early 1970s to the mid-1980s and remained persistently high even when inflation stabilised at a low level. This specification for the euro area NAIRU is also supported by Fabiani and Mestre (2004) and Berger (2008).

2.4 Other variables

A variety of other variables may also contain valuable information for estimating the output gap. Rünstler (2002) concluded that more reliable output gap estimates for the euro area could be obtained by including capacity utilisation and factor inputs, while Doménech and Gómez (2006) emphasised the importance of using investment to estimate the output gap. However, both omit industrial production, whose importance in terms of indicating the cyclical state of the economy has been highlighted by both the NBER and the CEPR business cycle dating committees. We therefore extend our triviariate model outlined in Section 2.3 to include a decomposition of industrial production, given by

$$\mathrm{IP}_t = \mathrm{IP}_t^{tr} + \theta_{\mathrm{IP}}\psi_t^{(n)} + \theta_{\mathrm{IP}}^{*}\psi_t^{*(n)} + \varepsilon_t^{\mathrm{IP}}, \qquad \varepsilon_t^{\mathrm{IP}} \sim \mathrm{NID}\left(0,\sigma_{\mathrm{IP}}^2\right) \tag{11}$$

$$\mathrm{IP}_t^{tr} = \mathrm{IP}_{t-1}^{tr} + \mathrm{IP}_{t-1}^{s} + \eta_t^{\mathrm{IP}}, \qquad \eta_t^{\mathrm{IP}} \sim \mathrm{NID}\left(0,\sigma_\eta^{\mathrm{IP}^2}\right)$$

$$\mathrm{IP}_t^{s} = \mathrm{IP}_{t-1}^{s} + \xi_t^{\mathrm{IP}}. \qquad \xi_t^{\mathrm{IP}} \sim \mathrm{NID}\left(0,\sigma_\xi^{\mathrm{IP}^2}\right)$$

As with unemployment, the cyclical component of industrial production is also specified as a linear combination of both $\psi_t^{(n)}$ and $\psi_t^{*(n)}$, and an idiosyncratic shock $\varepsilon_t^{\mathrm{IP}}$ is also included.

In addition, we estimate a second four-variate model with industrial production replaced by investment. To preview our results, we find that the four-variate model with industrial production yields less parameter uncertainty and final estimation errors than the four-variate model incorporating investment.

To sum up, we investigate a number of UC specifications for identifying the output gap. Output decomposition plays a central role in all these models as linear combinations of the output gap variables $\psi_t^{(n)}$ and $\psi_t^{*(n)}$ are used to describe all the cyclical components appearing in the models. As a consequence, all cycles share the same autocorrelation function as specified in equations (5) and (7) for the first- and second-order stochastic cycles, respectively.

2.4 State-Space form

All models can be recast into state-space form for estimation. For example, the state-space form for a four-variate model with a LLT and a second-order stochastic cycle is given by

$$\begin{aligned} Y_t &= Z\alpha_t + G\varepsilon_t \\ \alpha_{t+1} &= T\alpha_t + H\eta_t \end{aligned} \tag{12}$$

where Z, T, G and H are time-invariant matrices containing the hyper-parameters of the model:

$$Z = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_\pi & \theta_\pi^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{ur} & \theta_{ur}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \theta_{\mathrm{IP}} & \theta_{\mathrm{IP}}^* & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$T = \begin{bmatrix} T_\mu & 0_{2\times 4} & 0_{2\times 5} \\ 0_{4\times 2} & T_\psi & 0_{4\times 5} \\ 0_{5\times 2} & 0_{5\times 4} & \dddot{T}_\mu \end{bmatrix}, \qquad T_\mu = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

$$T_\psi = \begin{bmatrix} \rho\cos\lambda_c & \rho\sin\lambda_c & 1 & 0 \\ -\rho\sin\lambda_c & \rho\cos\lambda_c & 0 & 1 \\ 0 & 0 & \rho\cos\lambda_c & \rho\sin\lambda_c \\ 0 & 0 & -\rho\sin\lambda_c & \rho\cos\lambda_c \end{bmatrix},$$

$$\ddot{T}_\mu = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$G = \text{diag}\left\{\sigma_\varepsilon, \sigma_{\upsilon\pi}, \sigma_{ur}, \sigma_{\text{IP}}\right\} \text{ and}$$

$$H = \text{diag}\left\{\sigma_\eta, \sigma_\xi, 0, 0, \sigma_k I_2, \sigma_{\omega\pi}, \sigma_\eta^{ur}, \sigma_\xi^{ur}, \sigma_\eta^{\text{IP}}, \sigma_\xi^{\text{IP}}\right\}.$$[4]

The observation vector $Y_t = \left[y_t, \pi_t, ur_t, \text{IP}_t\right]^{\mathrm{T}}$ is known and the state vector

$$a_t = \left(\mu_t, \beta_t, \psi_t^{(2)}, \psi_t^{*(2)}, \psi_t^{(1)}, \psi_t^{*(1)}, \bar{\pi}_t, ur_t^{tr}, ur_t^{s}, \text{IP}_t^{tr}, \text{IP}_t^{s}\right)^{\mathrm{T}}$$

contains the unobserved components. This state-space representation can easily be expanded to accommodate seasonal components and MA terms in equation (8). Once the model is set up in state-space form, the hyper-parameters can be estimated by maximum likelihood using the prediction error decomposition produced by the Kalman filter. Since non-stationary variables μ_t, β_t, $ur_t^{tr}, ur_t^{s}, \text{IP}_t^{tr}, \text{IP}_t^{s}$ and $\bar{\pi}_t$ are appear in the state vector, the Kalman filter requires a diffuse initialisation and we use the initialisation method developed by Koopman and Durbin (2003). All the computations were performed using the library of state-space functions in SsfPack 3.0 developed by Koopman *et al.* (2008).

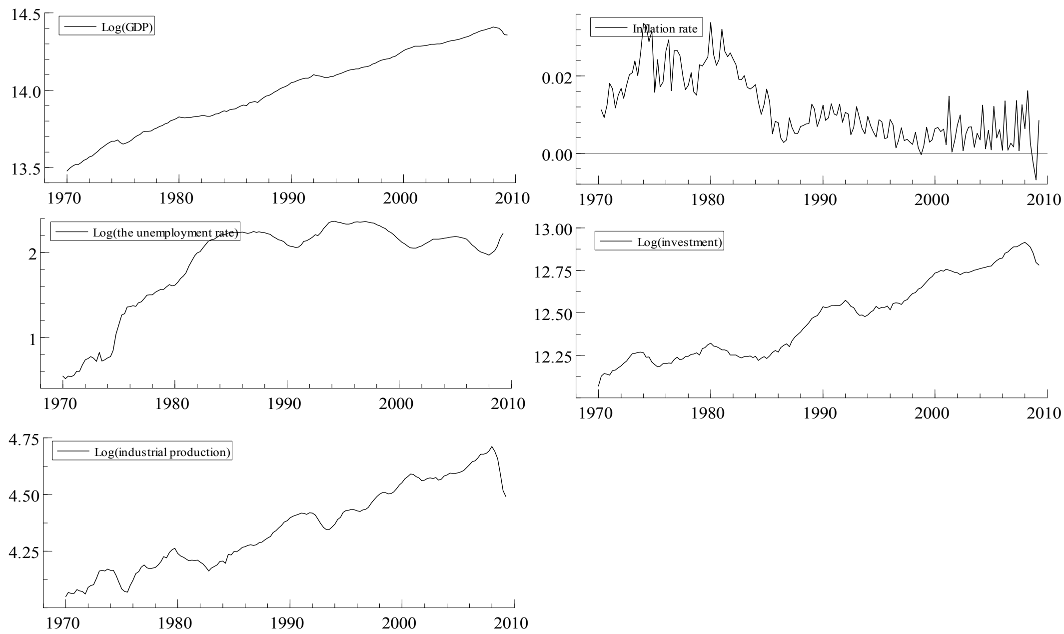
3 Data and estimation results

The data used in this paper are quarterly data for the aggregate euro area from 1970Q1 to 2009Q2. Historical data from 1970Q1 to 2007Q4 are taken from the area-wide model (AWM) database originally constructed by Fagan *et al.* (2001) and have

[4] Here we assume that all irregular terms are orthogonal and their variances are time-invariant. It would be interesting to see whether relaxing these assumptions alters output gap estimates, such as in Berger (2008) and Koopman *et al*. (2005). However, given that the focus of this paper is to assess the use of different variables and output decompositions on output gap estimates, we leave this exercise for future study.

been updated to 2009Q2 using the OECD database.[5] ADF test statistics indicate that (the log of) euro area GDP, (the log of) unemployment rate, the CPI inflation rate, and (the logs of) industrial production and investment (gross fixed capital formation) are all I(1) series.[6] The main data used in this paper are plotted in Figure 1.

Figure 1: the data



In total, we estimate twelve multivariate models containing different combinations of variables and output decompositions. These are set out in Table 1, where column headings list the variables used in each model and row headings give the type of output decomposition employed: for example, Model 1 uses the bivariate approach with output decomposed into a LLT and a first-order cycle.

[5] As the CPI itself is not seasonally adjusted, we adjust it using a deterministic seasonal component. Our deseasonalised series is consistent with that obtained by using the X-12-ARIMA seasonal adjustment procedure.

[6] Test statistics are available upon request.

Table 1: Model Specifications.

	output, inflation	output, inflation, unemployment rate	output, inflation, unemployment rate, Industrial production	output, inflation, unemployment rate, investment
A LLT + 1st-order cycle	Model 1	Model 4	Model 7	Model 10
A DST+ 1st-order cycle	Model 2	Model 5	Model 8	Model 11
A RW + 2nd-order cycle	Model 3	Model 6	Model 9	Model 12

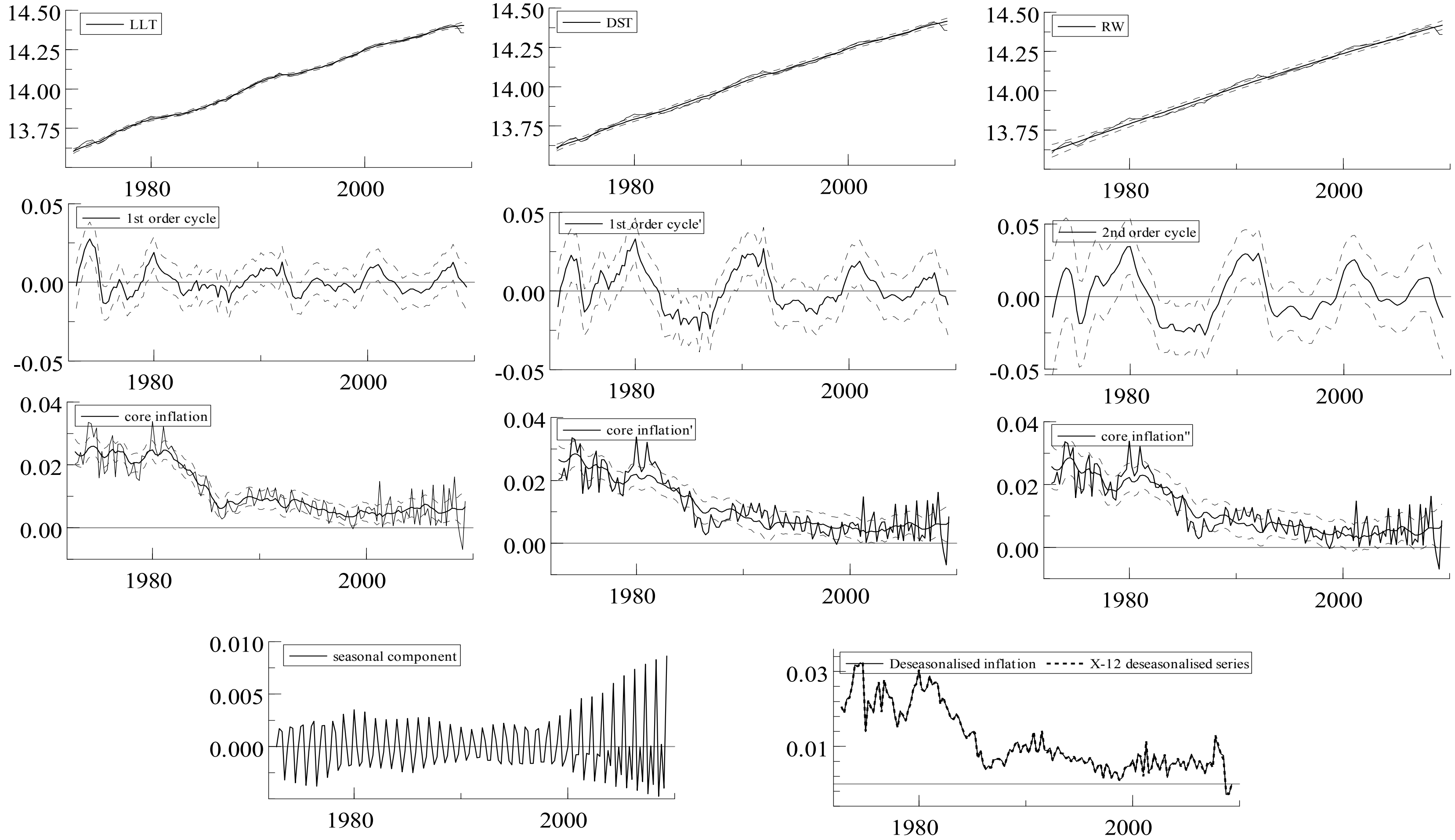
The parameter estimates obtained from each of the models are reported in Tables 2 and 3. Diagnostic checking of all models is performed using standardised prediction errors. Dummy variables are used to reduce the degree of non-normality in the model residuals, as detected by the Jarque-Bera (JB) test. These dummies are required for outliers occurring during the mid-1970s and during the current recession: specific dates are listed in Tables 2 and 3. All dummy variable coefficients are statistically significant.

Initial estimation indicated that there was significant third-order autocorrelation in the residuals of the inflation equation. A moving average term at lag three is therefore included to eliminate this autocorrelation: the estimates of φ_3, the parameter of the third-order MA term, are positive and statistically significant.

Parameter estimates for the three bivariate models (Models 1-3) are presented in Table 2. When the LLT (Model 1) is replaced by a DST (Model 2), the output cycle (output gap) becomes more persistent and volatile, as the values of the damping factor ρ, the period $2\pi/\lambda_c$, and the cycle variance $\sigma_k^2/\left(1-\rho^2\right)$ all increase. The use of a second-order cycle further increases the period to 56 quarters, which is much longer than the conventional view of the length of business cycle durations. The variance of the second-order cycle also exceeds that of the first-order cycle and, when the second-order cycle is used, the trend component becomes a random walk with constant drift as the variance of the slope, σ_{ξ}^2, becomes insignificant. In addition, the variance of the irregular component, σ_{ε}^2, is insignificant in Models 1 and 2, but is significant in Model 3. This reflects the fact that a higher-order cycle leads to more pronounced cut-offs of the band-pass gain function at both ends of the frequencies centered at λ_c and, therefore, more noise enters the irregular component.

It can be observed from Figure 2 that different output gap estimates lead to different estimates of core inflation. Greater deviations of the inflation rate from its core component are found for Models 2 and 3 than for Model 1, this being most clearly observed during the 1980s. The loading parameters of the output cycle (gap) and its adjacent auxiliary, θ_{π} and θ_{π}^*, are both significant in the inflation equation. The sign of θ_{π} implies that there is a positive correlation between the output gap and the cyclical component of inflation. Analysing θ_{π} and θ_{π}^* can give us valuable information about the relationships between cycles, such as phase shifts and correlations, and this is discussed further in Section 4. It appears that Model 2 provides the best fit to the data as it has the highest log-likelihood of the three models.

Figure 2: Unobserved components and their 95% confident intervals from Models 1-3.



Notes: the first six panels present estimates of the trend, the output gap and core inflation obtained from Models 1-3. When the unobserved components have the same name, ' marks the components estimated from Model 2, while '' denotes the components from Model 3. Since the estimated seasonal components, plotted in the last two panels, are consistent across different models, we only present them for Model 1 to save space.

Table 2: Parameters Estimates of Bivariate and Trivariate Models

	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
Output Decomposition												
σ_η	0.255	(0.213)	R(0)	-	0.271	(0.144)	0.372	(0.066)	0.321	(0.090)	0.340	(0.054)
σ_ξ	0.085	(0.048)	0.160	(0.109)	R(0)	-	0.056	(0.028)	0.108	(0.064)	R(0)	-
σ_k	0.342	(0.164)	0.431	(0.051)	0.276	(0.066)	0.212	(0.031)	0.218	(0.032)	0.189	(0.028)
σ_ε	R(0)	-	R(0)	-	0.149	(0.081)	0.095	(0.093)	0.135	(0.072)	0.125	(0.060)
σ_ψ	0.86	-	1.41	-	1.75	-	0.83	-	0.80	-	1.48	-
ρ	0.92		0.95		0.81		0.97		0.96		0.84	
$2\pi/\lambda_c$	24.36		41.73		56.41		22.03		21.72		31.30	
m	-	-	0.005	(0.000)	-	-	-	-	0.006	(0.001)	-	-
ϕ	-	-	0.581	(0.262)	-	-	-	-	0.840	(0.128)	-	-
1974q4	-0.010	(0.003)	-0.011	(0.003)	-0.010	(0.003)	-0.009	(0.003)	-0.009	(0.003)	-0.009	(0.003)
2008q4	-0.044	(0.008)	-0.050	(0.007)	-0.044	(0.007)	-0.040	(0.007)	-0.042	(0.007)	-0.039	(0.006)
2009q1	-0.019	(0.005)	-0.022	(0.005)	-0.018	(0.005)	-0.016	(0.004)	-0.017	(0.004)	-0.016	(0.004)
The Phillip Curve												
	Model 1		**Model 2**		**Model 3**		**Model 4**		**Model 5**		**Model 6**	
θ_π	0.175	(0.089)	0.141	(0.042)	0.156	(0.046)	0.133	(0.052)	0.126	(0.050)	0.133	(0.038)
θ_π^*	-0.139	(0.064)	-0.159	(0.060)	-0.104	(0.058)	-0.133	(0.055)	-0.132	(0.053)	-0.105	(0.043)
$\sigma_{\omega\pi}$	0.130	(0.031)	0.118	(0.030)	0.134	(0.032)	0.148	(0.025)	0.148	(0.025)	0.142	(0.026)
$\sigma_{\upsilon\pi}$	0.157	(0.034)	0.164	(0.031)	0.157	(0.033)	0.160	(0.032)	0.160	(0.032)	0.162	(0.032)
σ_γ	0.051	(0.011)	0.051	(0.011)	0.051	(0.011)	0.051	(0.011)	0.051	(0.011)	0.051	(0.011)
[a] φ_3	0.448	(0.197)	0.416	(0.179)	0.451	(0.193)	0.437	(0.185)	0.436	(0.185)	0.421	(0.190)
1975q1	-0.012	(0.002)	-0.012	(0.002)	-0.011	(0.002)	-0.012	(0.002)	-0.012	(0.002)	-0.012	(0.002)
2007q4	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)
2008q4	-0.008	(0.003)	-0.008	(0.003)	-0.007	(0.003)	-0.007	(0.003)	-0.007	(0.003)	-0.007	(0.003)
2009q1	-0.010	(0.003)	-0.010	(0.003)	-0.010	(0.003)	-0.010	(0.003)	-0.010	(0.003)	-0.010	(0.003)
Okun's law												
	Model 1		**Model 2**		**Model 3**		**Model 4**		**Model 5**		**Model 6**	
θ_{ur}	-	-	-	-	-	-	-5.496	(1.074)	-5.107	(1.079)	-5.512	(0.684)
θ_{ur}^*	-	-	-	-	-	-	3.474	(0.832)	3.553	(0.801)	2.482	(0.694)
σ_η^{ur}	-	-	-	-	-	-	R(0)	-	R(0)	-	0.844	(0.162)
σ_ξ^{ur}	-	-	-	-	-	-	0.540	(0.147)	0.580	(0.154)	0.321	(0.093)
σ_{ur}	-	-	-	-	-	-	R(0)	-	R(0)	-	R(0)	-
1973q2	-	-	-	-	-	-	0.106	(0.010)	0.107	(0.009)	0.110	(0.008)
1975q2	-	-	-	-	-	-	0.081	(0.011)	0.081	(0.010)	0.082	(0.010)
2009q1	-	-	-	-	-	-	0.051	(0.017)	0.052	(0.017)	0.045	(0.014)
LogL	1198.735		1208.944		1198.492		1588.627		1596.879		1603.420	
[b] LR0:	0.00		0.00		0.15		0.00		0.00		0.13	
[c] LR1: $\theta_\pi^*=0$	5.58*		5.38*		7.39**		5.63*		6.04*		5.49*	
[c] LR2: $\theta_{ur}^*=0$	-		-		-		10.85**		10.72**		11.36**	
Q(12) y_t	7.60		7.85		10.11		10.10		13.72		9.82	
Q(12) π_t	7.24		6.06		5.00		7.02		4.94		7.90	
Q(12) ur_t	-		-		-		14.73		15.69		23.57*	
JB y_t	4.80		4.03		7.02		4.77		2.69		5.23	
JB π_t	0.45		0.59		0.16		1.66		1.47		0.57	
JB ur_t	-		-		-		49.31**		20.71**		18.73**	

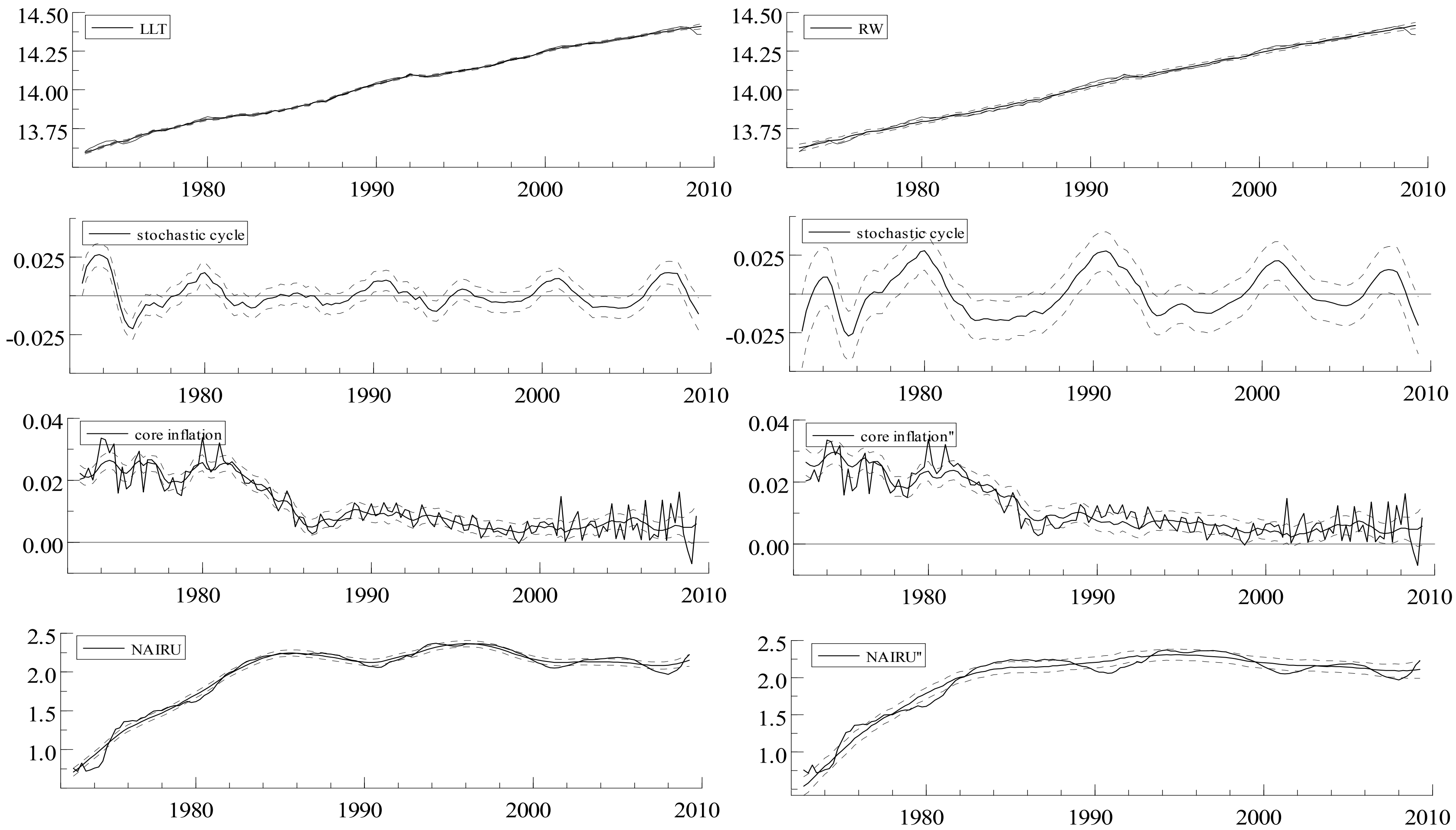
Notes: Standard errors in parenthesis are calculated using the delta method. The standard deviations of variance parameters are multiplied by 100. [a] The third-order moving average term is included in the inflation equation. [b] The Likelihood ratio (LR) test statistic LR0 suggests that the variance parameters restricted to zero are jointly insignificant. [c] LR1 and LR2 indicate the presence of phase shifts between the output gap and individual cyclical components. * and ** indicate significance at the 5% and 1% level, respectively. R(0) denotes that the parameter is restricted to zero a priori.

We now turn to the parameter estimates of the three trivariate models (Models 4-6) presented in Table 2. Although three dummy variables are used to pick up major outliers in the unemployment series, normality is still rejected for the unemployment residuals and autocorrelation is found in them when the second-order cycle is used (Model 6).[8] Compared with the output gap obtained from the bivariate models, we again observe that the second-order cycle produces longer cycle durations and greater volatility than the first-order cycle, but these values appear to be smaller than those obtained from the bivariate models. The loading parameters θ_{ur} and θ_{ur}^{*} are also found to be significant and the negative sign of θ_{ur} is consistent with Okun's law. The process generating the NAIRU is an integrated random walk in Models 4 and 5, while it becomes a LLT in Model 6. In all trivariate models, the idiosyncratic component of unemployment, $\sigma_{\varepsilon}^{ur}$, does not appear to be significant. This suggests that cyclical unemployment can be fully explained by the output gap with appropriate parameter loadings. Finally, Model 6 yields the highest log-likelihood of the three models.

Estimates of the unobserved components obtained from Models 4 and 6 are plotted in Figure 3, as these produce the most extreme differences: for example, larger deviations in the unemployment rate from the NAIRU are observed in Model 6 compared to those in Model 4.

[8] These diagnostic problems are also found in the unemployment residuals in the four-variate models to be presented in Table 3.

Figure 3: Unobserved components and their 95% confident intervals from Models 4 and 6.

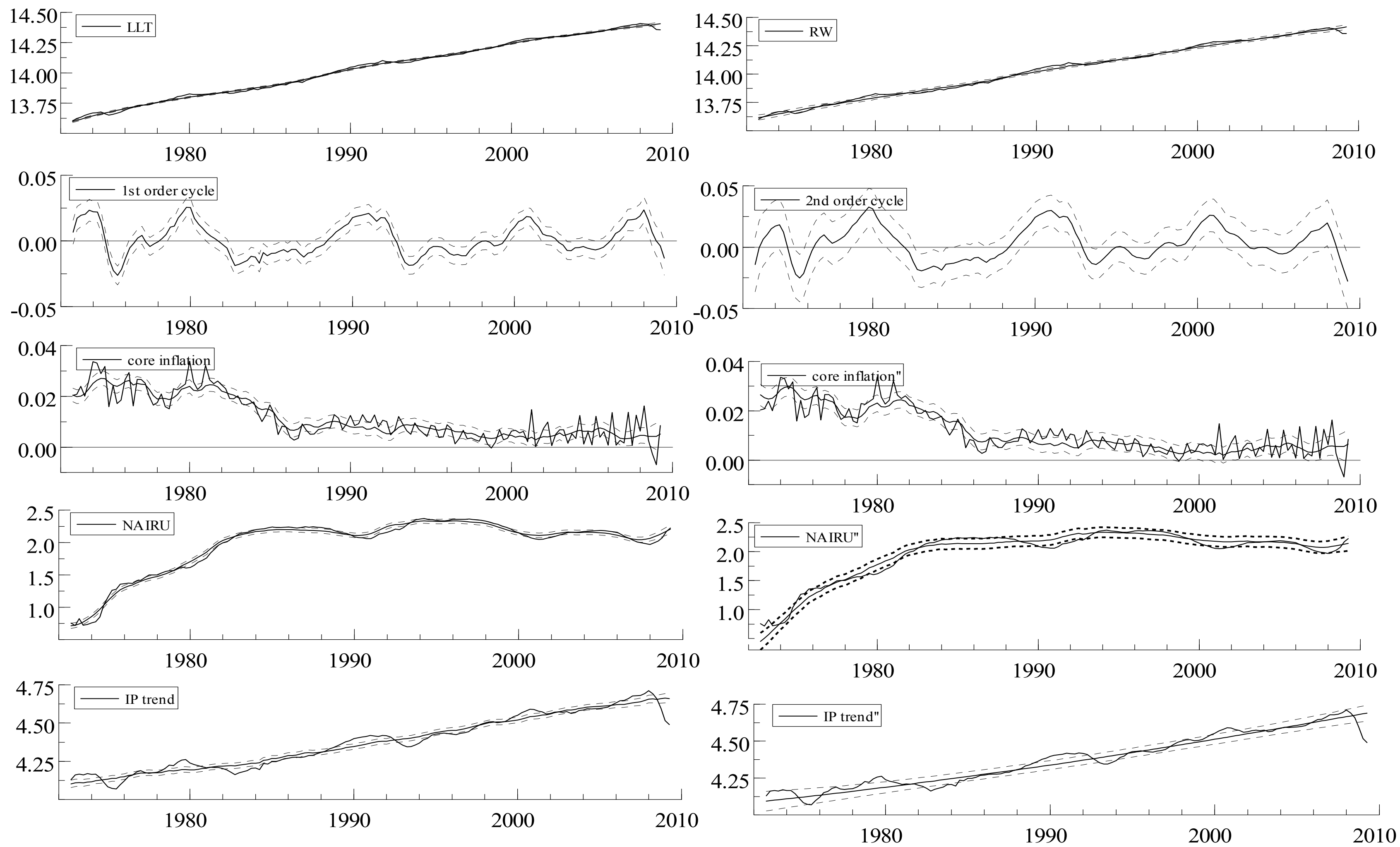


Notes: Graphs in the left column plot unobserved components from Model 4, while the right column displays those estimated from Model 6.

Table 3 reports the parameter estimates of the three four-variate models with industrial production included (Models 7-9). As expected, the output gap estimates obtained from these models have higher volatility than those from the corresponding bivariate and trivariate models, as industrial production exhibits greater cyclical fluctuations. Although Model 9, in which the output gap is specified as a second-order cycle, has a higher log-likelihood than either Models 7 and 8, which have first-order cycles, it is nevertheless unable to explain all the cyclical fluctuations in industrial production since the idiosyncratic component, $\sigma_{\varepsilon}^{\mathrm{IP}}$, becomes significant when the second-order cycle is used. This can also be seen in Section 4, where the degree of contemporaneous correlation between the output gap and the cyclical component of industrial production is reduced by 14% when the second-order cycle is used. Again, both loading parameters, θ_{IP} and θ_{IP}^{*} , in the industrial production equation are found to be significant with θ_{IP} being positive.

Figure 4 plots the unobserved components from Models 7 and 9. The trend component of industrial production estimated from Model 9 is very close to a linear time trend and this implies that more variations may be absorbed by the irregular component of industrial production.

Figure 4: Unobserved components and their 95% confident intervals from Models 7 and 9.



Notes: Graphs in the left column plot unobserved components from Model 7, while the right column displays those estimated from Model 9.

Table 3: Parameters Estimates of Fourvariate Models

Output Decomposition

	Model 7		**Model 8**		**Model 9**		**Model 10**		**Model 11**		**Model 12**	
σ_η	R(0)	-	0.123	(0.109)	0.224	(0.049)	0.252	(0.042)	0.180	(0.082)	0.258	(0.044)
σ_ξ	0.047	(0.012)	0.042	(0.018)	R(0)	-	0.017	(0.011)	0.160	(0.069)	R(0)	-
σ_k	0.330	(0.026)	0.331	(0.027)	0.267	(0.025)	0.387	(0.034)	0.393	(0.035)	0.387	(0.045)
σ_ε	0.224	(0.021)	0.207	(0.036)	0.173	(0.035)	R(0)	-	R(0)	-	R(0)	-
σ_ψ	1.33	-	1.34	-	1.84	-	1.59	-	1.57	-	1.76	-
ρ	0.97		0.97		0.82		0.97		0.97		0.76	
$2\pi/\lambda_c$	27.46		28.02		45.36		39.72		39.11		58.20	
m	-	-	0.005	(0.000)	-	-	-	-	0.006	(0.000)	-	-
ϕ	-	-	0.914	(0.036)	-	-	-	-	0.493	(0.210)	-	-
1974q4	-0.008	(0.002)	-0.008	(0.002)	-0.009	(0.002)	-0.008	(0.002)	-0.008	(0.002)	-0.009	(0.002)
2008q4	-0.038	(0.007)	-0.041	(0.006)	-0.033	(0.006)	-0.045	(0.007)	-0.048	(0.007)	-0.039	(0.007)
2009q1	-0.015	(0.004)	-0.016	(0.004)	-0.013	(0.004)	-0.018	(0.005)	-0.019	(0.005)	-0.016	(0.004)

The Phillip Curve

	Model 7		**Model 8**		**Model 9**		**Model 10**		**Model 11**		**Model 12**	
θ_π	0.142	(0.038)	0.146	(0.039)	0.131	(0.037)	0.116	(0.033)	0.110	(0.033)	0.079	(0.039)
θ^*_π	-0.098	(0.043)	-0.095	(0.044)	-0.063	(0.036)	-0.126	(0.036)	-0.121	(0.036)	-0.102	(0.040)
$\sigma_{\omega\pi}$	0.140	(0.024)	0.141	(0.024)	0.142	(0.024)	0.118	(0.027)	0.122	(0.028)	0.136	(0.028)
$\sigma_{\upsilon\pi}$	0.160	(0.031)	0.158	(0.031)	0.157	(0.031)	0.180	(0.028)	0.179	(0.029)	0.165	(0.031)
σ_γ	0.052	(0.011)	0.053	(0.011)	0.053	(0.011)	0.050	(0.011)	0.050	(0.011)	0.052	(0.011)
[a] φ_3	0.458	(0.176)	0.467	(0.175)	0.442	(0.182)	0.351	(0.160)	0.356	(0.165)	0.403	(0.179)
1975q1	-0.011	(0.002)	-0.011	(0.002)	-0.011	(0.002)	-0.012	(0.002)	-0.012	(0.002)	-0.011	(0.002)
2007q4	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)	0.005	(0.003)
2008q4	-0.007	(0.003)	-0.007	(0.003)	-0.006	(0.003)	-0.008	(0.003)	-0.008	(0.003)	-0.007	(0.003)
2009q1	-0.010	(0.003)	-0.010	(0.003)	-0.009	(0.003)	-0.010	(0.003)	-0.010	(0.003)	-0.010	(0.003)

Okun's law

	Model 7		**Model 8**		**Model 9**		**Model 10**		**Model 11**		**Model 12**	
θ_{ur}	-2.535	(0.487)	-2.567	(0.502)	-3.732	(0.555)	-1.705	(0.462)	-1.564	0.465)	-1.669	(0.576)
θ^*_{ur}	3.173	(0.423)	3.168	(0.430)	3.095	(0.623)	2.837	(0.442)	2.819	(0.440)	3.486	(0.569)
σ^{ur}_η	R(0)	-	R(0)	-	0.780	(0.195)	R(0)	-	R(0)	-	R(0)	-
σ^{ur}_ξ	0.871	(0.157)	0.865	(0.159)	0.542	(0.176)	1.035	(0.180)	1.049	(0.182)	0.723	(0.281)
σ_{ur}	R(0)	-	R(0)	-	R(0)	-	R(0)	-	R(0)	-	R(0)	-
1973q2	0.108	(0.009)	0.108	(0.009)	0.111	(0.008)	0.108	(0.009)	0.108	(0.009)	0.111	(0.008)
1975q2	0.087	(0.010)	0.087	(0.010)	0.086	(0.010)	0.083	(0.010)	0.083	(0.010)	0.085	(0.010)
2009q1	0.052	(0.017)	0.054	(0.017)	0.047	(0.015)	0.051	(0.017)	0.052	(0.017)	0.045	(0.014)

	Industrial production							**Gross fixed capital formation**					
	Model 7		**Model 8**		**Model 9**			**Model 10**		**Model 11**		**Model 12**	
θ_{IP}	2.575	(0.187)	2.626	(0.203)	2.515	(0.170)	θ_{IN}	2.411	(0.242)	2.359	(0.251)	2.406	(0.228)
θ^*_{IP}	0.624	(0.183)	0.668	(0.194)	0.901	(0.178)	θ^*_{IN}	0.585	(0.210)	0.494	(0.217)	0.504	(0.217)
σ^{IP}_η	0.482	(0.095)	0.434	(0.114)	R(0)	-	σ^{IN}_η	0.409	(0.223)	0.295	(0.320)	0.412	(0.275)
σ^{IP}_ξ	R(0)	-	R(0)	-	0.016	(0.018)	σ^{IN}_ξ	0.039	(0.023)	0.052	(0.026)	0.067	(0.031)
σ_{IP}	R(0)	-	R(0)	-	0.379	(0.048)	σ_{IN}	0.280	(0.094)	0.356	(0.087)	0.347	(0.119)
1980q1	-0.016	(0.006)	-0.017	(0.006)	-0.014	(0.006)	2008q4	-0.089	(0.017)	-0.093	(0.017)	-0.076	(0.019)
2008q4	-0.123	(0.015)	-0.124	(0.015)	-0.106	(0.014)	2009q1	-0.033	(0.012)	-0.034	(0.011)	-0.026	(0.012)
2009q1	-0.052	(0.010)	-0.053	(0.010)	-0.044	(0.009)							

Notes: Diagnostics are presented in the following page. Standard errors in parenthesis are calculated using the delta method. The standard deviations of variance parameters are multiplied by 100. [a] The third-order moving average term is included in the inflation equation.

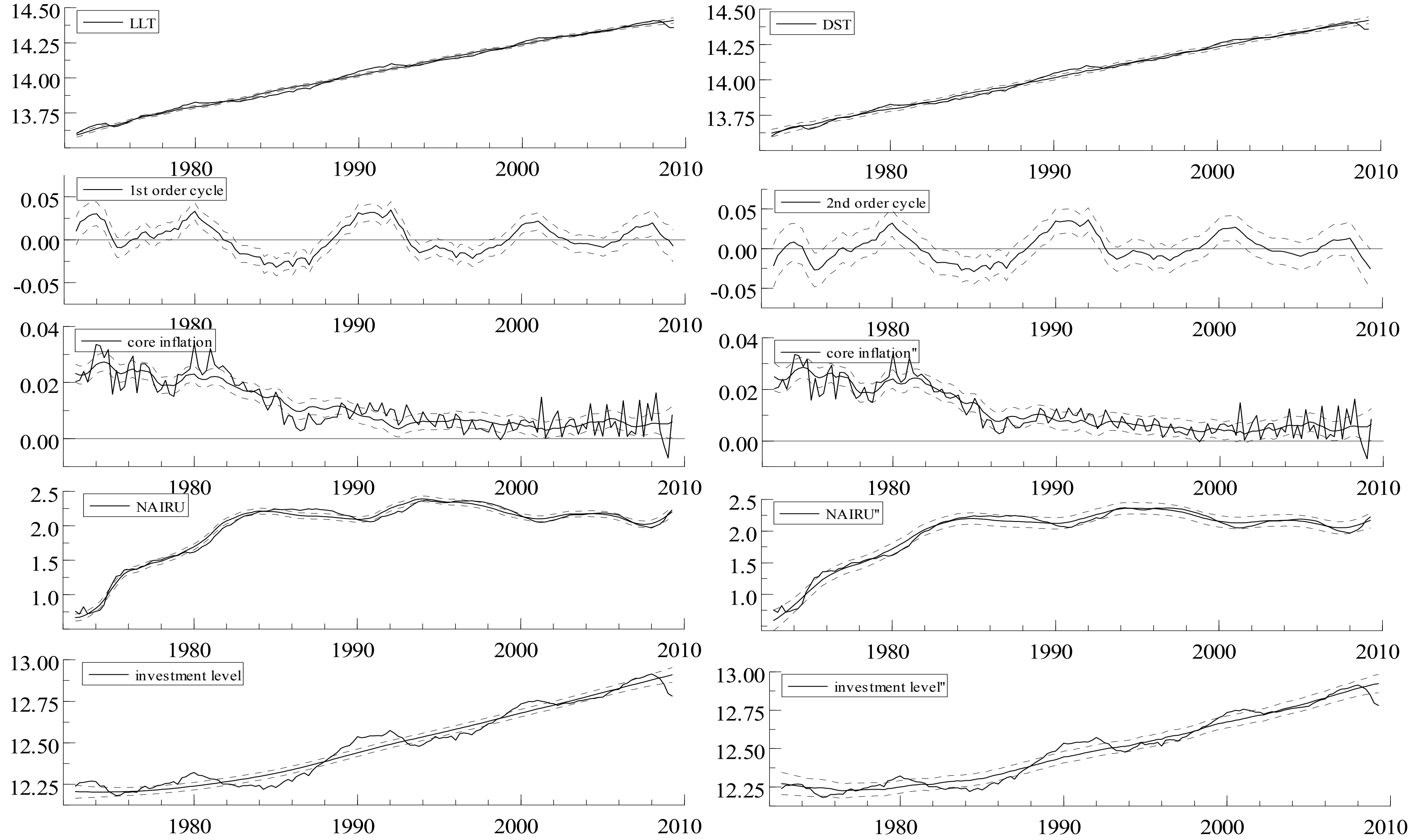
Table 3: Parameters Estimates of Fourvariate Models (Continued)

LogL	2088.921	2097.714	2100.397	LogL	2085.809	2092.1926	2072.815
[b] LR0:	0.00	0.04	2.39	[b] LR0:	0.00	0.24	2.63
[c] LR1: $\theta_{\pi}^{*}=0$	4.74*	4.32*	3.98*	[c] LR1: $\theta_{\pi}^{*}=0$	9.45**	9.00**	6.35**
[c] LR2: $\theta_{ur}^{*}=0$	28.83**	28.99**	23.09**	[c] LR2: $\theta_{ur}^{*}=0$	16.93**	16.47**	21.98**
[c] LR3: $\theta_{\text{IP}}^{*}=0$	10.95**	11.68**	15.87**	[c] LR3: $\theta_{\text{IN}}^{*}=0$	9.13**	7.52**	6.62**
Q(12) y_t	15.14	15.01	8.46	Q(12) y_t	9.69	8.66	18.10
Q(12) π_t	9.58	6.50	5.77	Q(12) π_t	7.94	6.14	5.60
Q(12) ur_t	22.21*	15.45	22.49*	Q(12) ur_t	18.16	13.37	25.95*
Q(12) IP_t	12.61	15.00	12.58	Q(12) IN_t	11.97	9.01	10.01
JB y_t	1.12	0.18	0.27	JB y_t	3.50	2.67	4.09
JB π_t	0.96	0.22	0.44	JB π_t	0.74	0.966	0.46
JB ur_t	47.18**	21.82**	45.26**	JB ur_t	38.89**	22.35**	18.83**
JB IP_t	1.56	5.41	15.24	JB IN_t	3.32	1.989	2.62

Notes: [b] The Likelihood ratio (LR) test statistic LR0 suggests that the variance parameters restricted to zero are jointly insignificant. [c] LR1, LR2 and LR3 indicate the presence of phase shifts between the output gap and individual cyclical components. * and ** indicate significance at the 5% and 1% level, respectively.

A further three four-variate models (Models 10-12) are also reported in Table 3, in which industrial production is replaced by investment. The output gap estimated from these four-variate models exhibits the highest volatility among all the models analysed. This is because investment has the largest cyclical swings of all the variables, and these swings are reflected in the output gap estimates. Consistent with the above findings, the second-order cycle again appears to be more persistent and volatile than the first-order cycle, as the period increases from 39 to 58 quarters, and the standard deviation of the output gap rises from 1.57 to 1.76. Both loading parameters θ_{IN} and θ_{IN}^{*} are statistically significant, suggesting that there is a phase shift between the output gap and the cyclical component of investment. The positive sign of θ_{IP} further supports a positive relationship between these two cycles. It is also worth noting that the trend of investment is well modelled by a LLT as both $\sigma_{\eta}^{\text{In}}$ and σ_{ξ}^{In} are significantly positive. The standard deviation of the idiosyncratic component of investment, $\sigma_{\varepsilon}^{\text{In}}$, is also found to be significant. Finally, the unobserved components obtained from Models 10 and 12 are presented in Figure 5.

Figure 5: Unobserved components and their 95% confident intervals from Models 10 and 12.



Notes: Graphs in the left column plot unobserved components from Model 10, while the right column displays those estimated from Model 12.

4 Modelling phase shifts

When additional variables are included in the model to identify the output gap, it is important to investigate the relationship between the output gap estimate and the cyclical components of these variables. The individual cyclical components have therefore been specified as linear combinations of both $\psi_t^{(n)}$ and $\psi_t^{*(n)}$, as this allows us to assess phase shifts and cross cycle correlations using the parameter estimates.[9]

The ACF of the vector of cyclical components, $x_t^C = \left(\psi_t^{(n)}, \psi_t^{\pi}, \psi_t^{ur}, \psi_t^{\mathrm{IP}}\right)^{\mathsf{T}}$, is given by

$$\Gamma_x(s) = \tilde{\Theta}\Gamma(s)\tilde{\Theta}^{\mathsf{T}} \tag{14}$$

where $\Gamma(s)$ is the ACF of $\tilde{\psi}_t = \left[\psi_t^{(n)}, \psi_t^{*(n)}\right]^{\mathsf{T}}$ and

$$\tilde{\Theta} = \begin{bmatrix} 1 & \theta_{\pi} & \theta_{ur} & \theta_{\mathrm{IP}} \\ 0 & \theta_{\pi}^{*} & \theta_{ur}^{*} & \theta_{\mathrm{IP}}^{*} \end{bmatrix}^{\mathsf{T}}$$

contains the corresponding parameter loadings. To analyse phase shifts and cycle correlations among the cyclical components in x_t^C, we focus on the elements of

$$B(s) = \tilde{\Theta}\begin{bmatrix} \cos(s\lambda_c) & \sin(s\lambda_c) \\ -\sin(s\lambda_c) & \cos(s\lambda_c) \end{bmatrix}\tilde{\Theta}^{\mathsf{T}},$$

these being

$$B_{ii}(s) = \left(\theta_i^2 + \theta_i^{*2}\right)\cos(s\lambda_c), \tag{15}$$

$$B_{ij}(s) = \left(\theta_i\theta_j + \theta_i^*\theta_j^*\right)\cos(s\lambda_c) + \left(\theta_i\theta_j^* - \theta_i^*\theta_j\right)\sin(s\lambda_c) \tag{16}$$

$$= \theta_{ij}\cos(s\lambda_c) + \theta_{ij}^*\sin(s\lambda_c)$$

[9] Rünstler (1997, 1998, 2004) specifies individual cyclical components as linear combinations of both $\psi_t^{(1)}$ and $\psi_t^{*(1)}$ plus an idiosyncratic cycle, $\varphi_{i,t}$, that has the same damping factor and frequency as the output cycle (gap). We do not include such idiosyncratic cycles: instead, we use irregular terms to capture noise that cannot be explained by linear combinations of $\psi_t^{(n)}$ and $\psi_t^{*(n)}$, for $n = 1, 2$.

$$= \text{sign}\left(\theta_{ij}\right) r_{ij} \cos\left(\lambda_c \left(s - \xi_{ij}\right)\right),$$

where $r_{ij} = \sqrt{\theta_{ij}^2 + \theta_{ij}^{*2}}$ and $\xi_{ij} = \lambda_c^{-1} \tan^{-1}\left(\theta_{ij}^* / \theta_{ij}\right)$, $\left|\xi_{ij}\right| \leq \pi/2\lambda_c$, denotes the phase shifts among individual cyclical components. The property $\left(\theta_{ij}, \theta_{ij}^*\right) = \left(\theta_{ji}, -\theta_{ji}^*\right)$ implies that $\xi_{ij} = -\xi_{ji}$ and $r_{ij} = r_{ji}$. Equations (5), (7), (15) and (16) show that $\Gamma_x(s)$ consists of damped cosine functions, where the damping pattern depends on both ρ and n. The cross correlation function between individual cyclical components is given by

$$\text{corr}\left(\psi_{t-s}^i, \psi_t^j\right) = \rho^{|s|} \text{sign}\left(\theta_j\right) \text{sign}\left(\theta_i\right) \cos\left(\lambda_c \left(s - \xi_j + \xi_i\right)\right), \tag{17}$$

when the first-order cycle is used. With a second-order cycle specification, the cross correlation function becomes

$$\text{corr}\left(\psi_{t-s}^i, \psi_t^j\right) = \left(1 + \frac{1-\rho^2}{1+\rho^2}|s|\right) \rho^{|s|} \text{sign}\left(\theta_j\right) \text{sign}\left(\theta_i\right) \cos\left(\lambda_c \left(s - \xi_j + \xi_i\right)\right). \tag{18}$$

We have computed contemporaneous correlations and phase shifts among the cyclical components from all the multivariate models presented in Tables 2 and 3: however, statistics are only reported in Table 4 for eight of the models. Phase shifts are displayed in the cells of the upper triangle of the table and contemporaneous correlations in the cells of the lower triangle. The first number in each cell is taken from those models with output decomposed into a first-order cycle and a LLT, and the number in parentheses is taken from models using a second-order cycle specification.

All models imply that the output gap leads the cyclical fluctuations in inflation and unemployment, but lags those of industrial production and investment. Mixed results are obtained when comparing trivariate models with four-variate models, as the lead-and-lag relationship between the cyclical components of inflation and unemployment is reversed. Although the lead-and-lag relations between industrial production and investment are not directly measured, it can be seen that the four-variate models which use investment produce longer leads-and-lags between output gap estimates and individual cyclical components compared to those computed from the models using industrial production. This may suggest that investment can be thought of as more of a leading indicator than a coincident variable with respect to the other variables included in the model.

Table 4: Contemporaneous Correlation and Phase Shifts

	Bivariate Models			
	$\psi_t^{(n)}$	ψ_t^{π}		
$\psi_t^{(n)}$		-2.6 (-5.3)		
ψ_t^{π}	0.78 (0.83)			
	Trivariate Models			
	$\psi_t^{(n)}$	ψ_t^{π}	ψ_t^{ur}	
$\psi_t^{(n)}$		-2.8 (-3.3)	-2.0 (-2.1)	
ψ_t^{π}	0.70 (0.78)		0.8 (1.2)	
ψ_t^{ur}	-0.85 (-0.91)	-0.97 (-0.97)		
	Fourvariate Models with industrial production			
	$\psi_t^{(n)}$	ψ_t^{π}	ψ_t^{ur}	ψ_t^{IP}
$\psi_t^{(n)}$		-2.6 (-3.2)	-3.9 (-5.0)	1.0 (1.5)
ψ_t^{π}	0.82 (0.90)		-1.3 (-1.8)	3.7 (5.7)
ψ_t^{ur}	-0.62 (-0.77)	-0.96 (-0.97)		5.0 (7.5)
ψ_t^{IP}	0.97 (0.83)	0.67 (0.70)	-0.42 (-0.51)	
	Fourvariate Models with gross fixed capital formation			
	$\psi_t^{(n)}$	ψ_t^{π}	ψ_t^{ur}	ψ_t^{IN}
$\psi_t^{(n)}$		-5.2 (-8.5)	-6.5 (-10.4)	1.5 (1.9)
ψ_t^{π}	0.68 (0.61)		-1.3 (-2.0)	6.7 (10.4)
ψ_t^{ur}	-0.52 (-0.43)	-0.98 (-0.98)		8.0 (12.3)
ψ_t^{IN}	0.98 (0.98)	0.48 (0.44)	-0.30 (-0.24)	

Notes: phase shifts are presented in the upper triangle and contemporaneous cycle correlation are in the lower triangle. A positive phase shift indicates a lead of series column with respect to series row.

A mixed picture is presented when we try to analyse whether the use of a second-order cycle increases contemporaneous correlations. We find that, in bivariate and trivariate models, the use of a second-order cycle does increase the correlations between cyclical components. However, this is not so clear in the four-variate models which include industrial production, as the correlation between the output gap and the cyclical component of industrial production is reduced by 14% when the first-order cycle is replaced by a second-order cycle. Furthermore, it seems that a second-order cycle is not suitable for the four-variate model with investment, as most contemporaneous correlations are lowered. Finally, there is no doubt that industrial production and investment share more common cyclical fluctuations with output than with unemployment, as the correlation between the output gap and the cyclical component of unemployment declines dramatically when industrial production or investment is included in the model.

5 The accuracy of output gap estimates

Given the variety of models estimated in this paper, it is essential to have some criteria to judge which model provides the most reliable output gap estimates. Orphanides and van Norden (2002), Jean-Philippe and van Norden (2005) and Camba-Méndez and Rodriguez-Palenzuela (2001) all discuss the different sources of uncertainty arising from measuring the output gap. These include data revision, statistical revision and model and parameter uncertainty. Data revision refers to changes in output gap estimates produced by the revision of published data. Orphanides and van Norden (2002) used different vintages of US data to find that data revision did not appear to be the primary source of gap revisions for any of the models they examined. This is broadly consistent with Jean-Philippe and van Norden (2005), who found that, although the impact of data revision on Canadian output gap estimates varies across models, it was often dominated by other sources of gap revisions. Because of the lack of any real-time data for the aggregate euro area, we are unable to analyse the impact of data revision on the reliability of our output gap estimates, and so have to focus on assessing other sources of uncertainty.

The criteria commonly used to assess the reliability of output gap estimates are that there should only be small subsequent revisions caused by the arrival of new information, that only small errors should be left in the final estimates, and that estimates of the output gap should be able to forecast future inflation. We have performed all three exercises, but we only present results from the first two since none of the models that we have analysed can adequately predict inflation rates during the current recession.[10]

5.1 Size of revisions

When new observations become available, the output gap estimate is updated to incorporate this new information and therefore statistical revisions are produced. As shown by Orphanides and van Norden (2002), most revisions are due to uncertainty associated with the end-of-sample estimates of the output trend from univariate models. To analyse both the overall revision and the revisions caused by different factors, they produce four output gap estimates (i.e., Real Time, Quasi-Real, Quasi-Final and Final) at each point in time by using different information sets. The overall revision is the difference between the Real Time and Final estimates, which reflect both data and statistical revisions. However, given the unavailability of real-time data

[10] We do find that the four-variate models with investment perform slightly better than the other models in terms of inflation forecasting. This may be because output gap estimates obtained from these models exhibit longer leading periods with respect to the cyclical component of inflation, when compared to those obtained from other models.

for the euro area, we can only obtain Quasi-Real, Quasi-Final and Final output gap estimates.

Quasi-Real estimates, denoted $\hat{\psi}_{t|t}\left(\hat{\Xi}_t\right)$, are obtained by constructing an ensemble of 'rolling' estimates, where $\hat{\Xi}_t$ is the vector of hyper-parameters estimated using information up to and including time t. They are calculated by initially using observations up to 1995Q1 to compute the Quasi-Real estimate for 1995Q1. The sample is then moved forward quarter by quarter with the hyper-parameters being re-estimated at each step until the end of the sample is reached. The series of Quasi-Real estimates can be regarded as the first available estimate at each point in time. Although the choice of 1995Q1 is ad hoc, it does provide a sufficiently large sample for initial estimation and for analysing the subsequent revisions to the output gap estimates. The difference between the Quasi-Real and Final estimates reflect the overall revisions which have occurred in our models. Furthermore, the Kalman filter produces both filtered and smoothed output gap estimates, denoted as $\hat{\psi}_{t|t}\left(\hat{\Xi}\right)$ and $\hat{\psi}_{t|t+s}\left(\hat{\Xi}\right)$, for $s = 1, 2 \ldots T$. Both estimates are based on full-sample parameter estimates, $\hat{\Xi}$, the only difference being that they use different samples to form an optimal estimate at time *t*. The filtered and smoothed estimates using the final vintage of the data and the final data release are defined as Quasi-Final and Final estimates by Orphanides and van Norden (2002). The difference between $\hat{\psi}_{t|t}\left(\hat{\Xi}_t\right)$ and $\hat{\psi}_{t|t}\left(\hat{\Xi}\right)$ reveals those revisions produced by parameter instability, while the difference between $\hat{\psi}_{t|t}\left(\hat{\Xi}\right)$ and $\hat{\psi}_{t|T}\left(\hat{\Xi}\right)$ reflects those revisions arising from the filtering process. Since the differences between the Quasi-Real, Quasi-Final and Final estimates diminish over time as the information sets converge, we exclude the estimates from the last three years of the sample in our analysis.

We compute the revisions between the Quasi-Real, Quasi-Final and Final estimates of the output gap in relative terms using the formula $SR = \sigma\left(z_{t|t} - z_{t|T}\right) / \sigma\left(z_{t|T}\right)$, where $\sigma(x)$ is the standard deviation of the variable x. This ratio gives us a proxy for the noise-to-signal ratio. The column labelled 'total revisions' in Table 5 reports the relative revision between $\hat{\psi}_{t|T}\left(\hat{\Xi}\right)$ and $\hat{\psi}_{t|t}\left(\hat{\Xi}_t\right)$, which reflects revisions produced by both filtering uncertainty and recursive estimation of the hyper-parameters. In general, four-variate models yield smaller revisions than bivariate and trivariate models, with the relative revisions in most four-variate models being smaller than 0.5 (i.e., the standard deviation of total revisions is half the size of those of the Final estimates). Furthermore, in the four-variate models, the size of the total revision does not alter significantly when different output decompositions are used, which is the opposite to what is observed for the bivariate and trivariate models. The bivariate model using a DST and a first-order cycle (Model 2) gives the largest revision among the bivariate Models 1-3, while the trivariate model using a second-order cycle (Model 6) significantly reduces the size of revisions. The difference between the largest and smallest revisions is found to be 0.4 in the bivariate models and 0.3 in the trivariate models, but only 0.1 in the four-variate models.

The columns labelled 'parameter instability' and 'filtering uncertainty' present the relative revisions of $\hat{\psi}_{t|t}\left(\hat{\Xi}\right) - \hat{\psi}_{t|t}\left(\hat{\Xi}_t\right)$ and $\hat{\psi}_{t|T}\left(\hat{\Xi}\right) - \hat{\psi}_{t|t}\left(\hat{\Xi}\right)$ respectively. The first impression is, again, that for the four-variate models the size of revisions produced by either parameter instability or filtering uncertainty does not change significantly when different output decompositions are used. However, it is large parameter instability that increases the total revisions produced by Model 2, while the reduction in total revisions found in Model 6 is due to small revisions occurring in the filtering process.

We also present correlations between Quasi-Real, Quasi-Final and Final estimates in parentheses and these are consistent with the relative size of revisions.

Table 5: Revisions 1995q1-2006q2

	Overall revisions		Parameter stability		Filtering uncertainty	
Model 1	0.442	(0.90)	0.445	(0.92)	0.287	(0.98)
Model 2	0.838	(0.63)	0.767	(0.72)	0.317	(0.96)
Model 3	0.530	(0.86)	0.440	(0.92)	0.401	(0.97)
Model 4	0.895	(0.52)	0.225	(0.98)	0.940	(0.52)
Model 5	0.804	(0.62)	0.235	(0.97)	0.843	(0.60)
Model 6	0.609	(0.86)	0.403	(0.95)	0.408	(0.93)
Model 7	0.525	(0.85)	0.280	(0.96)	0.547	(0.85)
Model 8	0.444	(0.90)	0.235	(0.97)	0.498	(0.88)
Model 9	0.438	(0.92)	0.334	(0.94)	0.309	(0.96)
Model 10	0.385	(0.93)	0.276	(0.96)	0.261	(0.98)
Model 11	0.447	(0.92)	0.301	(0.96)	0.281	(0.97)
Model 12	0.429	(0.91)	0.414	(0.96)	0.371	(0.97)

Notes: First number in each cell is the size of revisions, while the number in the parentheses is correlation coefficient.

5.2 Parameter uncertainty and final estimation errors

The above exercise investigates the size of the relative revisions produced by incorporating new observations to update output gap estimates. However, as emphasised by Orphanides and van Norden (2002) and Jean-Philippe and van Norden (2005), analysing such revisions can only be a minimum requirement and other criteria should also be considered to provide a joint assessment of the reliability of output gap estimates. This is important as smaller revisions do not necessarily guarantee that there will be less uncertainty in the Final estimates. In this section, we therefore present final estimation error variances and associated measures of parameter uncertainty.[11]

As defined above, the Kalman filter produces Final estimates, $\hat{\psi}_{t|T}\left(\hat{\Xi}\right)$, based on full-sample parameter estimates and data from 1 to T. It also constructs mean squared errors for these Final estimates, denoted as $\hat{P}_{t|T}\left(\hat{\Xi}\right)$, which indicate the final estimation error in the model without considering parameter uncertainty. In order to assess the final estimation errors after taking account of parameter uncertainty, we follow the approach of Hamilton (1986) and evaluate the impact of parameter uncertainty using Monte Carlo simulation. The overall error variance $\hat{P}_{t|T}\left(\Xi\right)$, where Ξ is the vector of true parameters, can be approximated by

$$\hat{P}_{t|T}\left(\Xi\right)=\frac{1}{K}\sum_{k=1}^{K}P_{t|T}\left(\Xi^{(k)}\right)+\frac{1}{K}\sum_{k=1}^{K}\left[\mathrm{E}\left(\psi_{t|T}\left|\Xi^{(k)}\right.\right)-\hat{\mathrm{E}}\left(\psi_{t|T}\right)\right]^{2} \qquad (19)$$

where $\hat{\mathrm{E}}\left(\psi_{t|T}\right)=\frac{1}{K}\sum_{k=1}^{K}\mathrm{E}\left(\psi_{t|T}\left|\Xi^{(k)}\right.\right)$ and $\Xi^{(k)}$ are independent draws from the multivariate normal density of hyper-parameters, $\hat{\Xi}\sim \mathrm{N}\left(\hat{\Xi},\mathrm{V}_{\hat{\Xi}}\right)$. The first term on the right hand side of equation (19) represents the final estimation error allowing for parameter uncertainty, while the second term measures the extent of parameter uncertainty. Both terms are computed using $\Xi^{(k)}$, which is generated by 1000 random draws from the multivariate normal density of hyper-parameters.

In Figure 6 we plot the standard errors of the final estimates from our twelve models for the period 1975Q1 to 2006Q2, so that the first and last three years of the

[11] Proietti *et al*. (2007) suggest examining model uncertainty by applying model averaging, such as $\bar{\psi}_t=\sum_i c_i\hat{\psi}_t^i$, for $i=1,2\ldots K$, where c_i is the weight assigned to the ith output gap estimate. However, this assessment involves a number of subjective elements and we do not pursue this approach here.

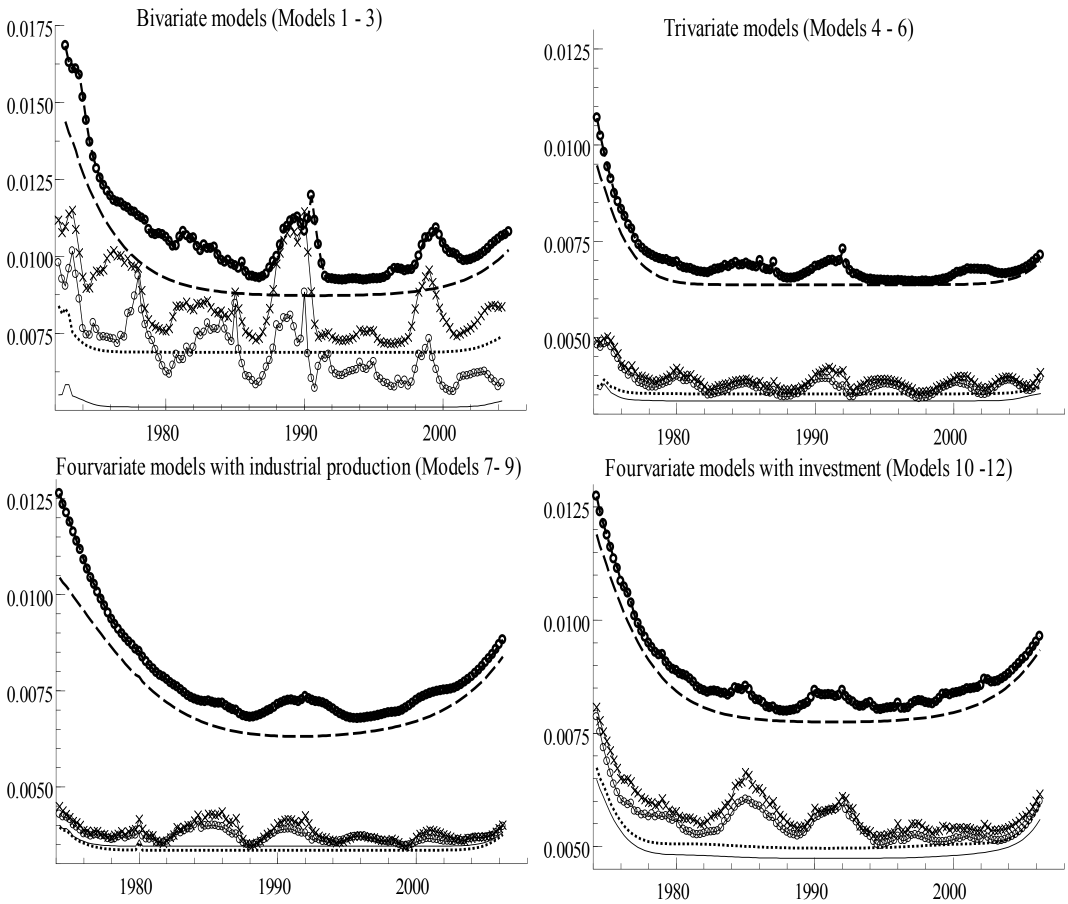
complete sample are excluded. In all panels of Figure 6, the $\left[\widehat{P}_{t|T}\left(\widehat{\Xi}\right)\right]^{1/2}$ are plotted as lines only, while the $\left[\widehat{P}_{t|T}\left(\Xi\right)\right]^{1/2}$ are plotted using lines with symbols.

Comparisons are first made across models containing the same variables but different output decompositions. The use of a second-order cycle significantly increases the level of uncertainty observed in the final estimates of the output gap, especially at the beginning and end of the sample. With the exception of the bivariate models, other multivariate models with a first-order cycle and either a LLT or a DST yield similar degrees of uncertainty.

It can also be seen that the output gap estimates obtained from the trivariate and four-variate models exhibit smaller final estimation errors than those estimated from the bivariate models. This is especially true for the two trivariate (Models 4 and 5) and two four-variate models (Models 7 and 8) using a first-order cycle. It also appears that Models 7 and 8 are effective in reducing the level of parameter uncertainty observed in Models 4 and 5. However, this is not achieved by Models 10 and 11, which include investment, as the level of overall uncertainty in them is considerably higher.

To sum up, we have performed two exercises in this section to assess the reliability of the output estimates obtained from our models. First, we measured the relative size of revisions caused by the arrival of new information over time and found that those from four-variate models were generally smaller than 0.5 and robust to the use of different output decompositions. In contrast, the three trivariate models, on average, produced the largest relative revisions among all the models analysed. The focus of the second exercise was to evaluate the remaining errors in the final output gap estimates by taking into account parameter uncertainty. The results clearly indicate that the use of a second-order stochastic cycle yields considerably higher final estimation errors than the first-order cycle specification. Finally, we found that the two four-variate models (Models 7 and 8) using industrial production, with the output cycle specified as a first-order cycle, gave the smallest final estimation errors.

Figure 6: Final estimation errors and parameter uncertainty



Notes: Standard errors of the final output gap estimates obtained using MLE from Models 1, 4, 7 and 10 are plotted in solid lines, those from Models 2, 5, 8 and 11 are plotted in dotted lines, and those yielding from Model 3, 6, 9 and 12 are in dashed lines. Standard errors of final estimation errors allowing for parameter uncertainty are plotted using lines with symbols. In order to distinguish those produced in Models 2, 5, 8 and 11 from those in Models 1, 4, 7 and 10, we plot the former using lines with crosses, while the latter are lines with circles.

6 Conclusions

In this paper we have estimated and analysed output gap estimates obtained from twelve multivariate UC models using different variables and output decompositions. Since previous research does not provide any formal analysis of the relationships existing between the estimated output gap and the cyclical components of any additional variables used to identify the gap, we performed this analysis by measuring phase shifts and contemporaneous correlations using the parameter estimates from the models. This provides a better understanding of the different output gap estimates. In addition, we compared differences in output gap estimates attributable to using alternative output decompositions. Finally, we performed two standard tests to assess how reliable our estimates are in terms of the size of revisions and the scale of the errors remaining in the final estimates.

The results can be summarised as follows. First, we conclude that four-variate models with industrial production and a first-order stochastic cycle provide the most reliable output gap estimates. Second, although the use of a second-order cycle in the trivariate model significantly reduces the size of revisions, in general it is found that errors in the final estimates are increased dramatically when second-order cycles are used. It also appears that the second-order cycle is not suitable for four-variate models with investment as both the log-likelihood and the contemporaneous cycle correlations among most cyclical components decrease. Finally, the lead-and-lags between output gap estimates and individual cyclical components are considerably larger in the four-variate models containing investment.

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