Notation

y_t	value of series in period *t*
$Y_t = \{y_1, y_2, \ldots, y_t\}$	sample of size *t*
ε_t	innovation in period *t*
Δ	difference operator
L	lag operator
$\phi(L), \theta(L), \psi(L)$	polynomial lag functions
ϕ	damping factor
σ^2	variance
τ_t	level of trend in period *t*
c_t	level of transitory component in period *t*

Innovations State Space Model

Measurement equation

$$y_t = z' x_{t-1} + \varepsilon_t$$

Transition equations

$$x_t = F x_{t-1} + g\varepsilon_t$$

Stochastic Assumptions

$$\varepsilon_t \sim \text{NID}\left(0, \sigma^2\right)$$

The vectors z, g and the matrix F potentially depend on an unknown vector of parameters θ.

Exponential Smoothing

The one-step ahead prediction errors needed for likelihood evaluation is generated with the following algorithm (originally proposed by Box and Jenkins)

Given trial values of the seed state x_0 and the parameter vectors θ to start the algorithm, repeat the following steps in each period *t*:

1. Prediction step
   $\hat{y}_t = z' x_{t-1}$
2. Error step
   $e_t = y_t - \hat{y}_t$
3. Revision step
   $x_t = F x_{t-1} + g e_t$

This is used as a simple alternative to the Kalman filter, which performs essentially the same function in traditional approaches.

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