The Center for Economic Research
Columbian College of Arts & Sciences

WORKING PAPER SERIES

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Working Paper No. 2026-015

Oct, 2026

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October 15, 2026

Abstract

A global digital liability is a contract with three clauses that can be written separately at issuance: denomination, which governs what the liability pays and how well it insures its holders; the unit of account, which governs pass-through into prices; and backing, the government bonds held against it, which governs the yield privilege and the issuer's currency exposure. Dollar stablecoins set all three to the dollar; multilateral proposals would move them to a basket. In a three-country New Keynesian model with currency-disaster risk in the periphery, a convenience yield on reserve assets, and original sin, we show that, however it is written, the contract cannot remove peripheral currency risk, only decide who bears it. Insuring against the peripherys currency creates an open position for the issuer. With fixed capital, that exposure places a limit on the scale at which the liability can be supplied credibly: a new Triffin dilemma that can be overcome by capital that grows with the open position. The mechanism behind the old dilemma reappears in general equilibrium: the country whose bonds back the liability runs a permanent trade deficit financed by the world's demand for them, and the yield privilege it draws exceeds the interest it saves, as its terms of trade improve.

JEL: F31, F33, F41, E42, E44, G23

Keywords: stablecoins, digital dollarization, unit of account, convenience yield, original sin, portfolio balance, international monetary system, Triffin dilemma

*I thank Sebastian Carranza, Graciela Kaminsky, Rafael Lopez-Monti and my discussant at the 3rd SAFE-CEPR Frankfurt Hub Conference, Remo Isch-Taudien, for helpful discussions and comments on a previous draft of this paper. The views expressed and all remaining errors are my own.

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1 Introduction

When Triffin warned in 1960 that the Bretton Woods system carried the seed of its own instability, the problem was not the quantity of reserve assets but the tension between the world's demand for a monetary liability and the capacity of the institution that supplied it to honor it. Dollar stablecoins revive that tension in a new, digital form. They are dollar liabilities, held increasingly outside the United States, backed by U.S. Treasuries, and used to price and settle transactions. Proposals for a multilateral alternative—an e-SDR—would replace the dollar with a basket. This paper asks what such a redesign would achieve, and for whom.

A global digital monetary liability is a contract with three clauses, each of which can be written independently at issuance. Its *denomination* states what one unit is: a bundle of currencies, delivered on redemption. Its *unit of account* states the bundle in which merchants who accept it quote sticky prices. Its *backing* states the currencies of the government bonds held against it. A dollar stablecoin sets all three to the dollar; the IMF's Special Drawing Right bundles the first two on one basket and has no third. Treating the clauses separately turns the design of a global digital currency into a question a macroeconomic model can answer: what does each clause govern, and what happens when each is moved from the dollar to a basket?

We answer it in a three-country New Keynesian model with two advanced economies and a periphery whose currency is exposed to disasters and whose external debt is partly in dollars. Households hold the liability for transactions and, in the periphery, as insurance against a collapse of their own currency; producers quote sticky prices in its unit of account; the issuer's reserve demand creates a convenience yield; and a financier with limited risk-bearing capacity absorbs currency imbalances.

The first result is that the clauses *separate*. Denomination governs what the liability pays its holders and, through its currency content, how well it insures them. The unit of account governs how exchange rates pass into the prices quoted in it and enters no financial equation. Backing governs who receives the yield privilege of issuing reserve assets, how much of the periphery's original sin is relieved, and how exposed the issuer is. When the issuer's reserves mirror the geography of holdings, the unit of account changes every country's response to a shock by the same amount and leaves exchange rates untouched; denomination does the same except through the periphery's demand for insurance.

The second result is that a stablecoin is valuable to the periphery because it is *not* the periphery's currency. Households hedge against a currency disaster with a claim that pays off when their currency collapses, and the hedge is proportional to the share of foreign currencies in denomination. Putting the periphery's own currency into a multilateral basket makes the liability a worse safe haven for the periphery, not a better one.

The third result is an accounting of who bears peripheral currency risk. The exposures of the periphery's dollar borrowers, the issuer, the financier, and holders of the liability to a depreciation of the periphery's currency add up to the same total under every contract; the contract decides who

bears that risk. The same accounting has a sharp corollary. A design in which every clause mirrors the geography of holdings—the *consistent design*—gives the issuer no open currency position and the currency markets no imbalance, but puts the periphery's currency into denomination and weakens the liability as insurance. Every unit of insurance the contract gives the periphery beyond the consistent design is, one for one, an open position of the issuer in the periphery's currency. Insurance against the periphery's currency is a short position in that currency for whoever supplies it.

An open position is not insolvency: an issuer with capital, or with a fiscal backstop, can carry it. The fourth result is that it can carry it only up to a scale. With fixed capital and a backstop whose use is costly, the promise to redeem at par after a disaster is credible for every belief of holders only while the liability is small relative to that loss-absorbing capacity; beyond a threshold, there is a range in which both a safe liability and a run on it are equilibria, and beyond a second threshold only the run remains. As the world's demand for the liability grows against fixed capital, the insurance it offers the periphery comes to rest on confidence. This is the sense in which the paper speaks of a new Triffin dilemma, in the spirit of Farhi and Maggiori (2018): the more the liability succeeds, the less credible the insurance that made it valuable to the periphery.

Whether the issuer can carry the position is a matter of governance, which we treat as a design margin alongside the three clauses (Section 3): how much capital the issuer holds, how quickly it recovers its losses and at what fiscal cost, how those losses are shared among members, whether it hedges, and how it manages the duration of its reserves. Capital that grows with the liability, in proportion to the open position times the size of a disaster, removes the dilemma.

The quantitative analysis measures the welfare of households in the three countries from a second-order solution of the model, in which defaults on the periphery's dollar debt and the taxes that finance transfers to the issuer carry deadweight costs, and measures every contract, the consistent design included, against an economy without the liability. Welfare counts the value in a disaster of every position households hold in the periphery's currency; the issuer, designed to absorb losses, is kept apart, and its loss in a disaster is reported as the bill its capital and backstop must cover. Over fluctuations in disaster risk, a dollar stablecoin raises the cost of an episode that depreciates the periphery's currency by 10 percent by 14.3 basis points of a year's consumption for the periphery: dollar pricing imports the depreciation, and the flight into the liability widens the swings in consumption and employment. Every single redesign of the dollar stablecoin's clauses lowers that cost. The periphery loses under every contract except those whose reserves are held in its own currency, negligibly under the consistent design, which is close to neutral for every country; and backing the dollar liability with the periphery's bonds makes every country better off than without the liability. The value of the liability in the disaster state is worth about 14.8 basis points per episode to the periphery under dollar denomination and 5.7 under the consistent design; we report how much of each welfare number rests on it.

Setting households' gains against the issuer's bill shows the new Triffin dilemma in welfare terms.

Over the business cycle, backing in the periphery's currency is the only design under which every country's households gain relative to no liability and their gains exceed the issuer's expected losses, so that some sharing of those losses makes every country better off. In a risk-off episode it is the design whose issuer loses most, more than households gain. In such episodes the market compensates holders of the periphery's currency for only about a third of the expected disaster loss, so whoever holds that currency bears the rest: the issuer, under the designs that back a dollar liability with it, and the advanced economies' households, under the designs that put it into denomination. The stakes grow with adoption, but for the contracts we examine the sign of each country's gain does not depend on it, except in the periphery under the two designs with a neutral position.

Per year, the yield privilege attached to backing outweighs the gains from stabilization unless crises are frequent, so backing decides the permanent distribution of the liability's benefits. In general equilibrium the privilege is worth about twice the interest it saves: the country whose bonds back the liability runs a permanent trade deficit, financed by the world's demand for its bonds, and its terms of trade improve. This is Triffin's dilemma in its original form, and the backing clause decides who bears it. Governance matters as much as the clauses: when only recapitalizations carry a fiscal cost, or when the issuer's income and losses go to the members whose bonds it holds, the ranking of the contracts that carry an open position changes.

Relation to the Literature

The paper connects five strands of research.

The literature on the *international monetary system and safe assets* asks who supplies the world's reserve assets, at what cost, and with what consequences for stability and risk sharing (Caballero et al., 2008; Gourinchas and Rey, 2007a; Obstfeld, 2011; He et al., 2019; Kekre and Lenel, 2024; Gourinchas et al., 2019). The exorbitant privilege of the reserve-currency country (Gourinchas and Rey, 2007b; Eichengreen, 2011) and the convenience yield on its debt (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang et al., 2021; Krishnamurthy and Lustig, 2019) attach, for a contract-based liability, to the backing clause and can be reallocated by design; stablecoin reserve demand already moves short-term Treasury yields (Ahmed and Aldasoro, 2025). Proposals to reform the system around the SDR (Farhi et al., 2011) and to create a global safe asset from emerging-market debt (Brunnermeier and Huang, 2019) correspond, in our terms, to bundling denomination and to backing in the periphery's currency; we show that the two have opposite effects on the periphery's insurance. Farhi and Maggiori (2018) model the Triffin dilemma as a confidence problem of a hegemon whose debt outgrows its fiscal capacity; our Triffin result applies that logic to an issuer whose open position outgrows its capital; Bordo and McCauley (2019) review whether the original dilemma was one. Central bank swap lines are the official backstop for dollar liabilities held abroad (Bahaj and Reis, 2022).

The literature on *dominant currencies and the unit of account* documents that trade is invoiced

disproportionately in dollars and that dollar prices are sticky (Gopinath et al., 2020; Boz et al., 2022), explains how a dominant currency emerges and persists (Matsuyama et al., 1993; Gopinath and Stein, 2021; Chahrour and Valchev, 2022; Mukhin, 2022) and why debt is denominated in it (Eren and Malamud, 2022; Coppola et al., 2023), studies optimal policy under dollar pricing (Egorov and Mukhin, 2023), and explains why a common unit of account economizes on relative-price risk (Doepke and Schneider, 2017); Ikeda (2020) studies digital money used as a unit of account in open economies. The unit-of-account clause is a designed counterpart of invoicing, which we separate from the denomination of the liability.

The literature on *digital money* studies how central bank digital currencies, cryptocurrencies and stablecoins affect monetary transmission, currency competition and banking (Brunnermeier and Niepelt, 2019; Ferrari Minesso et al., 2022; Benigno et al., 2022; Barrdear and Kumhof, 2022; Andolfatto, 2021; Keister and Sanches, 2023; Liao and Caramichael, 2022), and how they bear on monetary sovereignty and regulation (Mancini-Griffoli et al., 2018; Adrian and Mancini-Griffoli, 2019; Brunnermeier et al., 2019; Auer et al., 2023; Gorton and Zhang, 2023), and why stablecoins run (Uhlig, 2022; Ma et al., 2025). These papers treat the digital currency as a single object; we unbundle it into three clauses.

The *currency-substitution and dollarization* literature separates transactions and store-of-value motives for holding foreign currency (Calvo and Végh, 1992; Feige, 2003; Végh, 2013), explains financial dollarization as a hedging choice (Ize and Levy Yeyati, 2003) and documents its consequences (Arteta, 2003; Levy Yeyati, 2006); Bocola and Lorenzoni (2020) show how dollar savings interact with crises and lending of last resort. We derive the store-of-value motive as a hedge against time-varying currency disasters (Barro, 2006; Gourio, 2012; Farhi and Gabaix, 2016), proportional to the foreign-currency content of the liability.

The literature on *original sin and balance-sheet effects* shows that borrowing in foreign currency turns depreciations into balance-sheet crises (Eichengreen and Hausmann, 1999; Céspedes et al., 2004), raises the risk premia on emerging-market currencies (Hofmann et al., 2020), and can be remedied by creating foreign demand for emerging-market currency debt (Eichengreen and Hausmann, 2005; Onen et al., 2023). Backing a multilateral liability with peripheral bonds is such a remedy, and it relocates rather than removes the underlying risk. Risk premia follow the intermediary-based portfolio-balance literature (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021; Camanho et al., 2022; He and Krishnamurthy, 2013), and the multi-country New Keynesian framework follows Clarida et al. (2002), Obstfeld and Rogoff (2002), Devereux and Engel (2003), Galí and Monacelli (2005), Corsetti et al. (2010), and Engel (2011). We use three countries because the transmission of a peripheral shock to the exchange rate between the two advanced economies through the liability requires at least three currencies.

2 Model

Three economies, $j \in \{H, F, R\}$, issue distinct currencies and share a common digital liability. Home (H) and Foreign (F) are advanced economies; the Rest of the World (R) is a periphery whose currency is exposed to disasters and whose external debt is partly in dollars. Each economy has a representative household, monopolistically competitive firms with sticky prices, a government that issues bonds, and a central bank that follows an inertial Taylor rule. A financier with limited risk-bearing capacity intermediates cross-border positions. The liability's contract is chosen by a governance authority and implemented by an institutional issuer. Derivations are in the Appendix.

2.1 The Contract: Three Clauses

The contract is

$$\Theta = (\theta^d, \theta^u, \theta^a), \qquad \theta^x = (\theta^x_H, \theta^x_F, \theta^x_R), \quad \textstyle\sum_j \theta^x_j = 1, \quad x \in \{d, u, a\}, \tag{1}$$

and each clause is a bundle of *currencies*, not of price levels. *Denomination* θ^d defines what one unit of the liability is: a fixed amount of each currency, with value shares θ^d at the steady state, delivered on redemption. *Unit of account* θ^u defines a separate accounting unit U, also a fixed amount of each currency, in which producers who accept the liability quote sticky prices. *Backing* θ^a is the currency composition of the government bonds the issuer holds. The *dollar corner*, $\theta^d = \theta^u = \theta^a = (1, 0, 0)$, describes existing dollar stablecoins; *bundling* a clause moves it from the dollar corner to a basket.

Values. Let $e_{jk,t}$ be the log price of currency k in units of currency j (a fall in $e_{HR,t}$ is a depreciation of R's currency) and lower-case letters log deviations from the steady state. One unit of the liability is worth, in country j's currency,

$$d_{j,t} = \sum_k \theta^d_k \, e_{Hk,t} - e_{Hj,t}, \qquad e_{HH,t} \equiv 0, \tag{2}$$

and earns the expected appreciation

$$g_{j,t} \equiv \mathbb{E}_t \, \Delta d_{j,t+1} = \mathbb{E}_t\big[\theta^d_F \dot{e}_{HF,t+1} + \theta^d_R \dot{e}_{HR,t+1}\big] - \mathbb{E}_t \dot{e}_{Hj,t+1}, \qquad \dot{e}_{jk,t} \equiv e_{jk,t} - e_{jk,t-1}. \tag{3}$$

The same steps give the value of one U in country j's currency and the number of U-priced goods one unit of the liability buys,

$$u_{j,t} = \sum_k \theta^u_k \, e_{Hk,t} - e_{Hj,t}, \qquad d_{j,t} - \big(p^U_{j,t} + u_{j,t}\big) = \sum_k \big(\theta^d_k - \theta^u_k\big) e_{Hk,t} - p^U_{j,t}, \tag{4}$$

where $p^U_{j,t}$ is the index of prices quoted in U. When $\theta^u = \theta^d$ the liability's purchasing power over U-priced goods is insulated from exchange rates in every country at once. The unit of account must be a currency bundle for quoted prices to be sticky: a price quoted in a basket of price levels

would be indexed.

Holdings. Steady-state holdings are tied to adoption ω_j, $\bar{s}_j = \omega_j$, in units of steady-state quarterly consumption, with population shares λ_j (1/3 in the benchmark), world holdings $\bar{s} = \sum_j \lambda_j \bar{s}_j$, holding shares $\bar{h}_j \equiv \lambda_j \bar{s}_j / \bar{s}$, and

$$dS_t = \sum_j \lambda_j \bar{s}_j \, s_{j,t} \tag{5}$$

the change in the total value of holdings (Appendix D.2).

2.2 Firms and Prices Quoted in the Unit of Account

Firms produce with labor, face CES demand with elasticity μ, and reset prices with probability $1-\alpha$. Domestic-goods inflation follows

$$\pi_{jd,t} = \beta \, \mathbb{E}_t \pi_{jd,t+1} + \kappa \, mc_{j,t}, \qquad \kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}. \tag{6}$$

A share $\Upsilon_j = \upsilon \, \omega_j$ of producers quote their prices to domestic customers in U, so that pricing in U spreads with use of the liability, as prices set in a vehicle currency are sticky in that currency (Gopinath et al., 2020; Boz et al., 2022). Their prices satisfy

$$\pi^U_{j,t} = \beta \, \mathbb{E}_t \pi^U_{j,t+1} + \kappa\big(mc_{j,t} + z_{j,t}\big), \qquad z_{j,t} = z_{j,t-1} + \pi_{jd,t} - \Delta u_{j,t} - \pi^U_{j,t}, \tag{7}$$

where the price gap $z_{j,t} \equiv p^d_{j,t} - u_{j,t} - p^U_{j,t}$ anchors U prices to firms' costs. No central bank targets $\pi^U_{j,t}$.

2.3 Households and Liquidity Demand

The household of country j maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma} - \frac{\varphi_l}{2}\big(C_{j,t} - L_{j,t}\big)^2 - \frac{\varphi_n}{1+\nu} N_{j,t}^{1+\nu} + \chi_L \log L_{j,t} \Big], \tag{8}$$

where liquidity services combine normalized real money $\tilde{m}_{j,t}$ and holdings of the liability $\tilde{s}_{j,t}$,

$$L_{j,t} = \Big[(1-\omega_j) \tilde{m}_{j,t}^{\rho_L} + \omega_j \tilde{s}_{j,t}^{\rho_L} \Big]^{1/\rho_L}, \qquad \rho_L < 1, \qquad \Rightarrow \qquad l_{j,t} = (1-\omega_j) mb_{j,t} + \omega_j s_{j,t}, \tag{9}$$

normalized so that $\bar{L}_j = 1$ for every ω_j (Appendix D.2). At the steady state households hold liquid assets worth κ_L of a quarter's consumption, real money $\kappa_L(1-\omega_j)\tilde{m}_{j,t}$ and the liability $\kappa_L \omega_j \tilde{s}_{j,t}$, with $\tilde{m}_j = \tilde{s}_j = 1$; the same holdings enter the external balance sheets of Section 2.4. With $\chi_L = \kappa_L(1-\beta)$, both liquid assets have user cost $1-\beta$ at the steady state. With an employment subsidy that makes the steady-state allocation of labor efficient, real marginal cost and the Euler

equation are

$$mc_{j,t} = \nu y_{j,t} + (\sigma + \varphi_l) c_{j,t} - \varphi_l l_{j,t} + \mathrm{rp}_{j,t}, \tag{10}$$

$$(\sigma + \varphi_l) c_{j,t} - \varphi_l l_{j,t} = \mathbb{E}_t\big[(\sigma + \varphi_l) c_{j,t+1} - \varphi_l l_{j,t+1}\big] - \big(i_{j,t} - \mathbb{E}_t \pi_{j,t+1}\big), \tag{11}$$

where $\mathrm{rp}_{j,t} \equiv p_{j,t} - p^d_{j,t}$ is the price of consumption relative to domestic goods.

Portfolio choice. Money pays nothing; one unit of the liability earns its revaluation $X_{j,t+1}/X_{j,t}$ in j's currency. Their user costs relative to the safe asset are $\mathcal{U}^m_{j,t} = 1 - 1/R_{j,t}$ and $\mathcal{U}^{sc}_{j,t} = 1 - \mathbb{E}_t[X_{j,t+1}/X_{j,t}]/R_{j,t}$, and the first-order conditions are

$$\frac{U_{L,j,t}}{U_{C,j,t}} \frac{\partial L_{j,t}}{\partial m^R_{j,t}} = \mathcal{U}^m_{j,t}, \qquad \frac{U_{L,j,t}}{U_{C,j,t}} \frac{\partial L_{j,t}}{\partial s^R_{j,t}} + h_{j,t} = \mathcal{U}^{sc}_{j,t}, \tag{12}$$

where $h_{j,t}$ is the value of the liability as insurance, the covariance of the household's marginal utility with the liability's return. Covariances generated by the model's own fluctuations are second order; the one generated by a currency disaster is not, because the disaster is a large event whose probability moves at first order. Subtracting the two conditions, with the composition semi-elasticity

$$\eta \equiv \eta_L \frac{\beta}{1 - \beta}, \qquad \eta_L \equiv \frac{1}{1 - \rho_L}, \tag{13}$$

gives the portfolio-composition condition (Appendix D.5)

$$mb_{j,t} - s_{j,t} = -\eta\, g_{j,t} - H_{j,t}, \qquad H_{j,t} \equiv \frac{\eta}{\beta}\, h_{j,t}. \tag{14}$$

Households substitute toward the liability when it is expected to appreciate in their currency (the transactions motive) and when it insures them against a disaster in their own currency (the insurance motive). The money condition, with the aggregator, pins down the scale of liquidity; the insurance motive acts like an additional expected return $H_{j,t}/\eta$ on the liability and raises total liquidity (equation (D.39)).

Currency disasters and hedging demand. The two motives respond to different wedges (Calvo and Végh, 1992). In peripheral economies foreign-currency deposits serve as a store of value (Feige, 2003), and financial dollarization can be read as a hedging choice: in the minimum-variance portfolio of Ize and Levy Yeyati (2003), dollarization rises with the volatility of inflation relative to that of the real exchange rate. We model the store of value as a hedge against currency disasters. Each period, country j's currency loses a fraction J of its value against the other two with probability $p_{j,t}$, and the marginal utility of j's households rises by the factor $B_j > 1$. As in the literature on time-varying disaster risk (Barro, 2006; Gourio, 2012; Farhi and Gabaix, 2016), the dynamics are those of the economy in which a disaster has not yet occurred. Only the periphery is exposed, $p_{H,t} = p_{F,t} = 0$; a risk-off episode is a rise in $p_{R,t}$, and the financier prices the expected

loss

$$\varpi_{R,t} \equiv J\, p_{R,t}, \tag{15}$$

which is the risk-off disturbance in R's premium (29). In a disaster one unit of the liability changes value, in j's currency, by $\pi_j J$, with $\pi_R = 1 - \theta_R^d$ (it rises with the foreign currencies in it) and $\pi_H = \pi_F = -\theta_R^d$ (it falls with R's currency in it), while j's marginal utility is multiplied by B_j, with $B_R > 1$ and $B_H = B_F = 1$. Because the dynamics are those of the no-disaster path, the expected return $g_{j,t}$ in the condition (12) already carries the drift with which the financier prices the expected loss; the disaster-state payoff, weighted by marginal utility in that state, is the remaining term, counted once: $h_{j,t} = \beta B_j \pi_j \varpi_{R,t}$ to first order, and

$$H_{j,t} = \eta\, B_j\, \pi_j\, \varpi_{R,t}, \qquad \pi_R = 1 - \theta_R^d, \quad \pi_H = \pi_F = -\theta_R^d. \tag{16}$$

Net of the drift, the excess return R's households expect from the liability is $(B_R - 1)(1 - \theta_R^d)\varpi_{R,t}$; for H and F, whose marginal utility does not jump, the drift and the disaster payoff cancel and the liability is fairly priced. Hedging demand is driven by the risk of a disaster, not by the endogenous components of the premium; it is proportional to the share of foreign currencies in denomination; and its strength is governed by B_R. Evaluated on the no-disaster path, the disaster-state payoff adds to period utility the term

$$\beta\, B_j\, \pi_j\, \varpi_{R,t}\, \kappa_L \omega_j \tilde{s}_{j,t}, \tag{17}$$

per unit of real holdings, which enters the welfare criterion (Appendix C); Section 4.6 reports how much of each welfare number comes from it.

2.4 International Asset Markets, the Convenience Yield, and the Issuer

External positions. Households hold bonds only in their own currency; a financier takes the other side of each country's position (Gabaix and Maggiori, 2015). The liability adds residents' holdings of it and the issuer's holdings of each government's bonds. In units of H's goods, $\text{NFA}_{j,t} = S_{j,t} - \theta_j^a A_{sc,t} + F_{j,t}$, where $F_{j,t}$ is the position with the financier. Its deviation $b_{j,t}$ evolves, to first order, as

$$\begin{aligned} b_{j,t} = \beta^{-1} b_{j,t-1} + nx_{j,t} + \lambda_j \bar{s}_j \big(\Delta x_t^H - \pi_{Hd,t}\big) - \frac{\theta_j^a \bar{s}}{\beta}\big(r_{j,t} - \pi_{Hd,t}\big) + \frac{\bar{F}_j}{\beta}\big(i_{j,t-1} + \dot{e}_{Hj,t} - \pi_{Hd,t}\big) \\ - (\beta^{-1} - 1)\, dS_{j,t-1} + \alpha_j^T \,\text{div}_{sc,t} + n_{j,t}^{cy} + \text{os}_{j,t}, \end{aligned} \tag{18}$$

with $\sum_j b_{j,t} + E_{sc,t} = 0$: the liability revalues with the bundle, $\Delta x_t^H = \theta_F^d \dot{e}_{HF,t} + \theta_R^d \dot{e}_{HR,t}$; each country pays the holding-period return $r_{j,t}$ on its bonds held by the issuer; the issuer's payout $\text{div}_{sc,t}$ is returned in shares α_j^T, population shares in the baseline (Section 3); $n_{j,t}^{cy}$ is the convenience-yield transfer; and $\text{os}_{j,t}$ is the default transfer on R's dollar debt (Section 2.5; derivation in Appendix A.3).

The financier and the risk premium. In each bilateral market the financier absorbs the net stock $x_{jk,t}$ of currency-k claims at a quadratic cost ψ_b (Gabaix and Maggiori, 2015), so that

$$e_{jk,t} = \mathbb{E}_t e_{jk,t+1} + i_{k,t} - i_{j,t} - \psi_b\, x_{jk,t}. \tag{19}$$

The pricing rule applies to any currency exposure, whatever its source.

The convenience yield. Government bonds provide collateral and settlement services (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang et al., 2021), modeled as a saving of resources for the households that hold them (Feenstra, 1986). The convenience yield $\mathrm{CY}_{k,t} \equiv 1 - R^g_{k,t}/R_{k,t}$ falls with the stock of bonds in private hands, $B^p_{k,t} = \bar{B}^p - \theta^a_k A_{sc,t}$, and, as in the specification of Krishnamurthy and Vissing-Jorgensen (2012), it is linear in the log of that stock (equation (D.45)):

$$\mathrm{CY}_{k,t} = a_{cy} - b_{cy} \log B^p_{k,t}. \tag{20}$$

The issuer's steady-state holdings $\theta^a_k \bar{s}$ raise the convenience yield on country k's bonds by $\Delta\overline{\mathrm{CY}}_k = b_{cy} \log\big[\bar{B}^p/(\bar{B}^p - \theta^a_k \bar{s})\big]$, and, to first order around that scale,

$$\mathrm{cy}_{k,t} = \zeta_{y,k}\, \theta^a_k \big(dS_t + E_{sc,t}\big), \qquad \zeta_{y,k} \equiv \frac{b_{cy}}{\bar{B}^p - \theta^a_k \bar{s}}: \tag{21}$$

the effect of a further purchase depends on how much of the stock the issuer already holds. By the envelope theorem the welfare effect is the pecuniary transfer on the debt held abroad. Country k receives

$$\bar{N}_k = \varphi^f B^g \Big(\Delta\overline{\mathrm{CY}}_k - \lambda_k \sum_m \Delta\overline{\mathrm{CY}}_m\Big), \qquad n^{cy}_{k,t} = \varphi^f B^g \Big(\zeta_{y,k}\theta^a_k - \lambda_k \sum_m \zeta_{y,m}\theta^a_m\Big)\big(dS_{t-1} + E_{sc,t-1}\big), \tag{22}$$

where φ^f is the share of each government's debt held abroad and B^g the common debt stock (Appendix A.5). The yield privilege follows backing, not the issuer's currency exposure. It is a transfer, but its welfare value is not: the country that receives $\bar{N}_k$ runs a permanent trade deficit of the same size, demand shifts toward its goods, and its terms of trade improve. Section 4.6 computes the yield privilege in general equilibrium.

The issuer. The issuer holds reserves $A_{sc,j,t} = \theta^a_j A_{sc,t}$ and may hedge a share ς of its currency exposure into the denomination currencies, which gives the *effective* position

$$\theta^e \equiv (1-\varsigma)\,\theta^a + \varsigma\,\theta^d. \tag{23}$$

It rebalances to the value shares θ^a every quarter (Section 4.9 relaxes this). Reserves are bonds with coupon decay δ_j, Macaulay duration $D_j = 1/(1-\beta\delta_j)$, price $q_{j,t} = -D_j i_{j,t}$, and holding-period return $r_{j,t} = i_{j,t-1} + \dot{e}_{Hj,t} + \beta\delta_j \Delta q_{j,t}$ in H's currency; the baseline holds one-period bills.

Steady-state seigniorage $(\beta^{-1}-1)\bar{s}$ is paid out, so steady-state equity is zero, and payout beyond seigniorage is a fraction ϕ_{div} of last period's equity, distributed in shares α_j^T. Equity evolves as

$$E_{sc,t} = \big(\beta^{-1} - \phi_{\text{div}}\big)E_{sc,t-1} + \pi_t^A + \text{val}_{q,t} + \text{val}_{fx,t}, \tag{24}$$

the carry on yields fixed a period earlier, π_t^A, the capital gain on long-duration reserves, $\text{val}_{q,t} = \bar{s}\sum_j \theta_j^a \delta_j \Delta q_{j,t}$, and the revaluation of the open currency position,

$$\text{val}_{fx,t} = \bar{s}\Big[(\theta_F^e - \theta_F^d)\dot{e}_{HF,t} + (\theta_R^e - \theta_R^d)\dot{e}_{HR,t}\Big] + (\beta^{-1}-1)\,\bar{s}\big(\theta_F^e\dot{e}_{HF,t} + \theta_R^e\dot{e}_{HR,t}\big). \tag{25}$$

The unit of account does not appear: the issuer owes currency amounts, not a price index.

Proposition 2.1 (Open Currency Position)**.** *Neglecting the interest on revalued reserves, of order* $\beta^{-1}-1$, $\text{val}_{fx,t} \equiv 0$ *for every exchange-rate realization if and only if* $\theta^e = \theta^d$*: reserves match denomination or the position is fully hedged. If* $\theta_R^e > \theta_R^d$*, a depreciation of R's currency lowers equity, precisely when the peripheral shock is largest. No choice of* θ^u *affects* $\text{val}_{fx,t}$*.*

An open position is a source of equity risk, not insolvency: whether the issuer can carry it depends on its capital and on who stands behind it (Section 2.9). With long-duration reserves, a rise in country k's interest rate lowers the value of the issuer's holdings of k's bonds, which is a gain for k's government, and the reserve rule then requires the issuer to buy k's bonds, which supports k's currency (Section 4.9).

2.5 Peripheral External Debt and Original Sin

R has gross external debt $\bar{B}_R$, in units of steady-state output, of which a share χ_{os} is in dollars. The issuer's steady-state holdings of R-currency bonds, $\theta_R^a\bar{s}$, are foreign lending to R in R's currency—the demand that original sin denies the periphery (Eichengreen and Hausmann, 1999, 2005). The dollar share of R's debt is

$$\chi_{os}(\theta_R^a) = \chi_{os}^0 - \frac{\theta_R^a\,\bar{s}}{\bar{B}_R}, \qquad 0 \le \chi_{os}(\theta_R^a) \le \chi_{os}^0. \tag{26}$$

The next result derives (26) from the borrowers' choice of the currency of their debt.

Lemma 2.2 (The Currency of R's Debt)**.** *Suppose that (a) R's borrowers choose the currency composition of* $\bar{B}_R$ *to maximize an objective that is increasing in the expected value and strictly decreasing in the variance of their net worth in units of R's output; (b) at the steady state the expected cost of borrowing is the same in both currencies (the steady-state premium is zero); and (c) investors other than the issuer hold at most* $(1-\chi_{os}^0)\bar{B}_R$ *of R-currency debt, while the issuer holds* $\theta_R^a\bar{s}$ *by its reserve rule. Then the borrowers' optimal dollar share is* $\max\{\chi_{os}^0 - \theta_R^a\bar{s}/\bar{B}_R,\,0\}$*, which is* (26)*.*

The proof is in Appendix E.5. Dollar debt revalues with R's terms of trade, R-currency debt does

not, so borrowers take as much R-currency debt as investors will hold; the issuer's demand raises that limit one for one. The dollar share is a slow-moving feature of R's balance sheet, set by the steady-state architecture and held fixed over the cycle.

Default and the premium. The value of the dollar debt in units of R's output moves with the terms of trade, $\hat{\Gamma}_{R,t} = -\text{tot}_{HR,t}$. R's borrowers default when their assets fall short of their debt, and creditors lose a fraction ℓ of their claims (Appendix D.8). A real depreciation pushes more borrowers below the threshold; with ζ_{os} the response of creditors' expected loss to a one-percent rise in R's leverage, the premium on R's currency acquires the term

$$-\kappa_{os}\,\text{tot}_{HR,t}, \qquad \kappa_{os}(\theta_R^a) \equiv \zeta_{os}\,\chi_{os}(\theta_R^a), \tag{27}$$

which scales the impact of a risk-off disturbance on R's currency by $1/(1-\kappa_{os})$. Defaults in excess of the steady state cost R's creditors $-\kappa_{os}\bar{B}_R\text{tot}_{HR,t}$, because the debt is long-term with a coupon that prices the steady-state default rate. Recovering the assets of defaulting borrowers is costly (Bernanke et al., 1999): a share τ_T of the creditors' loss is a transfer to R's borrowers, and the rest is spent on bankruptcy, absorbing R's output. Both τ_T and the dispersion of borrowers' assets follow from the default rate, the loss given default and κ_{os} (Appendix D.8). A bounded equilibrium exists for feedbacks far above the calibrated value (Section 4.10).

Proposition 2.3 (Backing and Original Sin)**.** (i) *Original-sin amplification falls with the weight of R's currency in backing, $\partial\kappa_{os}/\partial\theta_R^a = -\zeta_{os}\bar{s}/\bar{B}_R$; denomination, the unit of account, and hedging do not affect κ_{os}.*

(ii) *(Accounting of peripheral currency risk.) Measured in H's currency, a one-percent depreciation of R's currency at given goods prices changes the positions of R's borrowers, the issuer, the financier, and holders of the liability by, respectively, $\chi_{os}(\theta_R^a)\bar{B}_R$, $(\theta_R^e - \theta_R^d)\bar{s}$, $(\theta_R^a - \theta_R^e)\bar{s}$, and $\theta_R^d\bar{s}$, which sum to $\chi_{os}^0\bar{B}_R$ for every contract.*

Part (ii) is an accounting decomposition, not a behavioral result: given the borrowers' choice of Lemma 2.2, every unit of R-currency debt the issuer holds is a unit no longer owed in dollars by R's borrowers. Backing moves exposure from R's borrowers to the issuer, hedging moves it to the financier, denomination moves it to holders. What the contract changes is how the exposure is priced: on R's borrowers it amplifies depreciation through κ_{os}; on the issuer it moves equity.

2.6 Currency-Market Clearing and the Financial Wedge

Households pay for the liability with assets in their own currency and convert redemption proceeds back; the issuer keeps its effective position at θ^e; central banks supply money elastically at their policy rates; and the financier absorbs the residual imbalance between any two currencies. The liability's net addition to the supply of currency-k assets is then $Z_{k,t} = S_{k,t} - \theta_k^e A_{sc,t}$, and the financier holds $x_{jk,t}^{sc} = Z_{k,t} - Z_{j,t}$, with steady-state value $\bar{x}_{jk}^{sc} = \bar{s}[(\bar{h}_k - \theta_k^e) - (\bar{h}_j - \theta_j^e)]$. To first

order, the liability's contribution to the bilateral premium is

$$\psi^{sc}_{Hk,t} = \psi_b\Big[dS_{k,t} - dS_{H,t} - (\theta^e_k - \theta^e_H)\big(dS_t + E_{sc,t}\big)\Big], \qquad k \in \{F, R\}, \tag{28}$$

with $dS_{k,t} = \lambda_k \bar{s}_k(s_{k,t} + \mathrm{rp}_{k,t} + \mathrm{tot}_{Hk,t})$. Denomination enters only through θ^e. The complete premia are

$$\psi_{HF,t} = \psi_b b_{HF,t} + \psi^{sc}_{HF,t} - \chi_E E_{sc,t}, \qquad \psi_{HR,t} = \psi_b b_{HR,t} + \psi^{sc}_{HR,t} - \chi_E E_{sc,t} - \kappa_{os}\,\mathrm{tot}_{HR,t} + \varpi_{R,t}, \tag{29}$$

where $b_{jk,t} \equiv b_{j,t} - b_{k,t}$, $\chi_E \geq 0$ lets issuer equity ease intermediation capacity (Gabaix and Maggiori, 2015; He and Krishnamurthy, 2013), and the risk-off disturbance follows $\varpi_{R,t} = \rho_\varpi \varpi_{R,t-1} + \varepsilon_{\varpi_R,t}$. The exchange rates satisfy

$$e_{Hk,t} = \mathbb{E}_t e_{Hk,t+1} + i_{k,t} - i_{H,t} - \psi_{Hk,t}, \qquad k \in \{F, R\}, \qquad e_{FR,t} = e_{HR,t} - e_{HF,t}. \tag{30}$$

Proposition 2.4 (Flight into the Liability)**.** *In country R, holdings of the liability relative to money satisfy*

$$s_{R,t} - m b_{R,t} = \big(1 - \theta^d_R\big)\Big[\eta B_R\, \varpi_{R,t} - \eta\big(\psi_{HR,t} - (i_{R,t} - i_{H,t})\big)\Big] + \eta\, \theta^d_F\big(\psi_{HF,t} - (i_{F,t} - i_{H,t})\big).$$

For a risk-off shock that raises $\varpi_{R,t}$ and $\psi_{HR,t}$ one for one, holding $\psi_{HF,t}$ and interest differentials fixed: (i) R's holdings rise relative to money if and only if $B_R > 1$, whatever the contract; (ii) denomination scales the flight, in either direction, by $1 - \theta^d_R$.

Proposition 2.5 (Neutrality, Privilege, and the Unit of Account)**.** (i) *(Neutrality.) The liability creates no steady-state imbalance in any currency market if and only if $\theta^e = \bar{h}$—under symmetric adoption, $\theta^e = (1/3, 1/3, 1/3)$. At $\theta^e = \bar{h}$ the liability's premium depends only on shifts in demand* across *countries.*

(ii) *(Privilege.) Country k receives the steady-state yield privilege $\bar{N}_k$ of (22), which follows backing and is independent of θ^d, θ^u and hedging. A country whose weight in the effective position exceeds its residents' share of holdings borrows from the issuer without paying a risk premium on that debt, $\bar{b}_{jk} = -\bar{x}^{sc}_{jk}$; an increase in holdings originating in country k raises the premium on k's currency by less the larger k's weight in θ^e.*

(iii) *(Orthogonality.) θ^u does not enter the issuer's balance sheet, the transfers, the convenience yield, original sin, or the premia; it enters the equilibrium only through the prices of U-priced goods.*

Invariance at the neutral position. Denomination and the unit of account enter every country's equations through terms common to all countries: by (3) and (4), $g_{j,t} = G_t - \mathbb{E}_t \dot{e}_{Hj,t+1}$ and $\Delta u_{j,t} = \mathcal{U}_t - \dot{e}_{Hj,t}$, with G_t and $\mathcal{U}_t$ the same for every j.

Proposition 2.6 (Invariance at the Neutral Position)**.** *Suppose that (a) the countries share all structural parameters other than those of the risk-off shock, the insurance motive and original sin; (b)* $\theta^a = \theta^e = \lambda$*; and (c)* $\chi_E = 0$*. Then, for any path of the risk-off shock and any duration of reserves: (i) exchange rates, premia, interest-rate differentials, external positions and every cross-country difference of a country variable are independent of* θ^u*, and depend on* θ^d *only through* θ^d_R*; (ii) a change in* θ^u*, or in* θ^d *at given* θ^d_R*, changes the path of each country variable by the same amount in every country; (iii) issuer equity affects no allocation.*

The proof is in Appendix E.3. At the neutral position the unit of account and, except for the periphery's weight, denomination act only on the common component of the world economy's response: no country can use them to shift exchange-rate adjustment onto another.

2.7 Market Clearing and Monetary Policy

CPI inflation combines local-priced domestic goods, U-priced domestic goods converted into local currency, and imports at the exporters' prices; for R,

$$\pi_{R,t} = (1-\gamma)\big[(1-\Upsilon_R)\pi_{Rd,t} + \Upsilon_R(\pi^U_{R,t} + \Delta u_{R,t})\big] + \tfrac{\gamma}{2}\big(\pi_{Hd,t} - \dot{e}_{HR,t}\big) + \tfrac{\gamma}{2}\big(\pi_{Fd,t} - \dot{e}_{FR,t}\big), \quad (31)$$

and analogously for H and F (Appendix D.6). With terms of trade $\mathrm{tot}_{jk,t} \equiv e_{jk,t} + p^d_{k,t} - p^d_{j,t}$, $\mathrm{rp}_{j,t} = \frac{\gamma}{2}(\mathrm{tot}_{jk,t} + \mathrm{tot}_{jl,t}) - (1-\gamma)\Upsilon_j z_{j,t}$. Output is demand determined and net exports are exports less imports,

$$\begin{aligned} y_{j,t} &= (1-\gamma)\big(c_{j,t} + \mathrm{rp}_{j,t} + \Upsilon_j z_{j,t}\big) + \sum_{k\neq j} \tfrac{\gamma}{2}\big(c_{k,t} + \mathrm{rp}_{k,t} + \mathrm{tot}_{jk,t}\big), \\ nx_{j,t} &= \sum_{k\neq j} \tfrac{\gamma}{2}\big(c_{k,t} + \mathrm{rp}_{k,t} + \mathrm{tot}_{jk,t}\big) - \gamma\big(c_{j,t} + \mathrm{rp}_{j,t}\big), \end{aligned} \quad (32)$$

and each central bank follows $i_{j,t} = \rho_i\, i_{j,t-1} + (1-\rho_i)(\phi_\pi \pi_{j,t} + \phi_y y_{j,t})$. The Blanchard–Kahn conditions are verified in Appendix B.

2.8 What Each Clause Governs

Denomination determines holders' return, the liability's value as insurance (proportional to $1-\theta^d_j$) and hence the size of flows in a risk-off episode, what the issuer owes and hence its open position, and the share of peripheral currency risk borne by holders. The *unit of account* determines how exchange rates pass into the prices quoted in it and enters no financial equation. *Backing* determines who receives the yield privilege, how much original sin is relieved, the issuer's open position, the currencies it buys when it rebalances, and, for an unhedged issuer, the liability's footprint in currency markets.

The consistent design. Holders want $\theta^u = \theta^d$, so that the liability keeps its purchasing power over U-priced goods; a zero open position requires $\theta^e = \theta^d$ (Proposition 2.1); neutral currency

markets require $\theta^e = \bar{h}$ (Proposition 2.5). All three hold only in the *consistent design*

$$\theta^u = \theta^a = \theta^d = \bar{h}, \tag{33}$$

which puts the periphery's own currency into denomination and so weakens the liability as insurance.

Proposition 2.7 (Insurance and the Issuer's Open Position)**.** *Let $x_R \equiv \theta^e_R - \theta^d_R$ be the issuer's open position in R's currency. For any contract, the liability's insurance for R satisfies $1-\theta^d_R = (1-\theta^e_R)+x_R$. With a neutral effective position, $\theta^e = \bar{h}$, every unit of insurance beyond the consistent design, $1-\theta^d_R > 1-\bar{h}_R$, is matched one for one by an open position of the issuer in R's currency.*

Proposition 2.7 is an accounting decomposition: it states who carries the risk that the periphery's insurance transfers, not whether the carrier can bear it. The dollar corner gives the periphery the full hedge with no open position, but only by concentrating the issuer's position on the dollar. The decomposition points to a *redesigned dollar coin*: keep denomination on the dollar, so that the liability remains the periphery's hedge, and bundle the other two clauses,

$$\theta^d = (1,0,0), \qquad \theta^u = \theta^a = \bar{h}. \tag{34}$$

The unit of account on the basket removes the dollar's pass-through into U-priced goods; backing on the basket shares the yield privilege, relieves original sin, and makes the issuer's position neutral. The price is an open position $x = \bar{h} - \theta^d$ in the basket currencies.

Proposition 2.8 (The Price of Bundling)**.** *Starting from the dollar corner and holding the other clauses fixed: (i) bundling denomination lowers the liability's value as insurance for the countries added to the bundle by the factor $1-\theta^d_j$ and, with dollar backing, gives the issuer the open position $\bar{s}[-\theta^d_F \dot{e}_{HF,t} - \theta^d_R \dot{e}_{HR,t}]$; (ii) bundling the unit of account reduces the pass-through $\Delta u_{j,t}$ for the countries added to the unit, at the cost of holders' purchasing-power stability, which moves by $\sum_k (\theta^d_k - \theta^u_k)\dot{e}_{Hk,t}$ in every country; (iii) bundling backing transfers the yield privilege to the countries added, relieves R's original sin by $\theta^a_R \bar{s}/\bar{B}_R$, and, for an unhedged issuer, opens the position $\mathrm{val}_{fx,t} = \bar{s}[\theta^a_F \dot{e}_{HF,t} + \theta^a_R \dot{e}_{HR,t}]$.*

2.9 The New Triffin Dilemma: Insurance against Fixed Capital

Proposition 2.7 says who carries the periphery's currency risk; it does not say whether the carrier can. Consider an issuer with capital $K \geq 0$ that has issued S units of the liability, each redeemable at par, and holds the effective position θ^e with open position $x_R > 0$. In a disaster R's currency falls by J, and the issuer's equity falls by $x_R JS$. If its assets then fall short of its liabilities, its members may cover the shortfall Δ; raising the resources costs them $\vartheta\Delta$ in excess burden, and they do so if and only if $\vartheta\Delta \leq \bar{c}$, where $\bar{c}$ is the cost to them of letting the issuer fail. Let $\bar{G} \equiv K + \bar{c}/\vartheta$ be the issuer's loss-absorbing capacity. Holders can redeem as soon as a disaster occurs, and redemptions

are served at par in sequence while the issuer has assets; meeting them early forces the issuer to sell R-currency reserves at a discount $\phi^{fs} \in (0,1)$, because peripheral bonds are illiquid in a crisis.

Proposition 2.9 (The New Triffin Dilemma)**.** *Let* $\bar{S}_1 \equiv \bar{G}/(x_R J)$ *and* $\bar{S}_0 \equiv \bar{G}/(x_R J + \phi^{fs}\theta_R^e) < \bar{S}_1$.

(i) *A* safe *equilibrium, in which holders do not run and the liability is redeemed at par after a disaster, exists if and only if* $S \leq \bar{S}_1$.

(ii) *A* run *equilibrium, in which holders redeem at the onset of a disaster and those who come last bear the shortfall, exists if and only if* $S > \bar{S}_0$.

(iii) *For* $S \leq \bar{S}_0$ *the liability is safe; for* $\bar{S}_0 < S \leq \bar{S}_1$ *both equilibria exist, and whether R's holders receive the insurance they paid for depends on confidence; for* $S > \bar{S}_1$ *only the run equilibrium remains.*

(iv) *With no open position,* $x_R = 0$*, a safe equilibrium exists at every scale.*

The proof is in Appendix E.6. The dilemma is Triffin's: as the world's demand for the liability grows against fixed loss-absorbing capacity, the insurance that made it valuable to the periphery moves from safe, to dependent on confidence, to undone by a run. Expectations matter because a run forces sales of the periphery's bonds into a falling market, which enlarges the shortfall the members would have to cover—the same mechanism by which, in Farhi and Maggiori (2018), investors' fear of a devaluation raises the hegemon's fiscal burden. The consistent design, with $x_R = 0$, is safe at any scale against the disaster, though it gives the periphery less insurance; the dollar corner, with no peripheral reserves, is also immune to the run. The magnitudes are not small: with $x_R = 1/3$, as in the redesigned dollar coin, and a disaster of $J = 0.3$, the issuer needs capital plus credible backstop of $x_R J = 10$ percent of its liabilities for a safe equilibrium to exist. Section 3 turns capital, the backstop and the other rules that stand behind the issuer into design choices.

2.9.1 What the Model Leaves Out

Three simplifications define the scope. *Adoption is given*: ω_j is a state of the monetary environment, the same in every country, so the model says what each contract does if held, not which would be adopted (Gopinath and Stein, 2021; Chahrour and Valchev, 2022). *One disturbance*: the analysis considers a risk-off episode in the periphery, the disturbance in which households seek a hedge, original sin bites, and the liability's footprint moves the periphery's exchange rate; shocks to U.S. monetary policy and global risk appetite would bear on the reserve-currency country's privilege and are left for future work. *The issuer always redeems* in the dynamic model, which is the safe equilibrium of Proposition 2.9; Section 3 states the governance under which that equilibrium exists.

3 Issuer Governance

The three clauses say what the liability is. They do not say who stands behind the issuer. That is settled by a second set of rules, written in the issuer's charter rather than in the liability: how

much capital the issuer holds, how its losses are shared among its members, how its profits are paid out and its losses recovered, whether it hedges its currency exposure, and how it manages its reserves. Proposition 2.7 shows that every unit of insurance the contract gives the periphery beyond the consistent design is an open position of the issuer. Proposition 2.9 shows that whether the issuer can carry that position depends on its capital and on its members' backstop. Governance is therefore a design margin in its own right: the same contract can be safe or fragile, and cheap or costly for the members, depending on the rules behind it.

3.1 The Designer's Problem

A governance authority with welfare weights λ^W on the three countries chooses a contract $\Theta = (\theta^d, \theta^u, \theta^a)$ and a set of governance rules Γ:

$$(\Theta^*, \Gamma^*) = \arg \max_{(\Theta,\Gamma)\in\mathcal{F}} \sum_j \lambda_j^W W_j(\Theta, \Gamma \mid \omega). \tag{35}$$

We do not solve this problem, because its solution depends on weights that members would negotiate. Instead we map the trade-offs that any authority faces, clause by clause and rule by rule. The rules Γ are the five set out below. Each corresponds to a parameter or a block of the model, so it can be varied without changing anything else.

3.2 Five Rules

Capital. Let the issuer hold capital $K = \mathcal{K}S$, a fixed fraction of its liabilities, invested in the denomination bundle, so that a disaster does not reduce it. Substituting $K = \mathcal{K}S$ into Proposition 2.9 turns the scale thresholds into capital requirements.

Proposition 3.1 (Capital and Scale)**.** *Let capital be $K = \mathcal{K}S$ and let $\bar{g} \equiv \bar{c}/\vartheta$ be the members' credible backstop.*

(i) *If $\mathcal{K} \geq x_R J$, a safe equilibrium exists at every scale. Otherwise one exists if and only if $S \leq \bar{g}/(x_R J - \mathcal{K})$.*

(ii) *If $\mathcal{K} \geq x_R J + \phi^{fs}\theta_R^e$, no run equilibrium exists at any scale. Otherwise a run equilibrium exists if and only if $S > \bar{g}/(x_R J + \phi^{fs}\theta_R^e - \mathcal{K})$.*

The proof repeats that of Proposition 2.9 with $K = \mathcal{K}S$ (Appendix E.6). Capital that grows with the liability removes the Triffin dilemma, which arises only when the demand for the liability grows against *fixed* loss-absorbing capacity. The requirement does not scale with the size of the liability: it scales with the open position times the size of a disaster, and, to rule out runs, with the share of peripheral reserves that would have to be sold into a falling market. A consistent design needs no capital against the disaster but does need $\phi^{fs}\theta_R^e$ against a liquidity run. The dollar corner needs neither. In the dynamic model capital enters through the speed at which the issuer recovers its losses, discussed next.

Payout and recapitalization. The issuer pays out seigniorage and a fraction ϕ_{div} of its equity each quarter, equation (24). The net transfer from each resident of country j to the issuer is $T_{j,t} = -\alpha_j^T \text{div}_{sc,t}$, in units of quarterly output: positive when the members recapitalize it, negative when it pays them. With $\phi_{\text{div}} = 1$, an issuer without capital passes every loss to its members in the following quarter. With $\phi_{\text{div}} < 1$, a capitalized issuer absorbs the loss in its equity and rebuilds it from retained earnings over about $1/\phi_{\text{div}}$ quarters.

Transfers between the issuer and its members are raised or returned through distortionary taxes. They cost each resident

$$\Omega(T_{j,t}) = \xi_1 T_{j,t} + \tfrac{\xi_2}{2} T_{j,t}^2 \tag{36}$$

in excess burden, so that the marginal cost of public funds is $1 + \xi_1 + \xi_2 T$. We set $\xi_1 = 0.365$, the midpoint of the range of marginal excess burdens in Ballard et al. (1985), 17 to 56 cents per dollar of revenue. In the baseline, members finance a recapitalization by issuing debt and smooth the taxes that service it (Barro, 1979), so the marginal cost of public funds is constant and $\xi_2 = 0$. If instead the taxes were raised within the quarter, their curvature would matter: for a proportional tax at rate τ^T on a base, equal to output, with constant elasticity e^T, the marginal cost of public funds is $1/\big(1 - \tau^T e^T/(1-\tau^T)\big)$ and its slope in revenue is $\xi_2 = (1+\xi_1)^3 e^T/(1-\tau^T)^2$; with e^T chosen so that the marginal cost equals $1 + \xi_1$ at $\tau^T = 0.3$, $\xi_2 = 3.24$, which Section 4.10 reports. Lump-sum taxes would make the transfers costless, $\Omega = 0$, which is also reported.

Under the baseline rule the cost is *symmetric*: profits paid to members lower distortionary taxes by as much as recapitalizations raise them. Under the alternative, *one-sided* rule, only recapitalizations are costly, and payouts are worth their face value. With smoothed taxes a loss costs its members the same whether it is recovered at once or over time, up to discounting; the speed of recapitalization matters instead through the issuer's balance sheet, since a loss that is not recovered keeps the issuer's reserves smaller while it lasts.

Loss allocation. Members bear the issuer's transfers in shares α_j^T, with $\sum_j \alpha_j^T = 1$, which replace the population shares λ_j in the payout term of equation (18), and each country bears the excess burden of its own transfers. Under the baseline, $\alpha_j^T = \lambda_j$: members own the issuer in proportion to population, and every resident bears the same burden. Under the alternative, $\alpha_j^T = \theta_j^a$: the members whose bonds the issuer holds bear its losses in proportion to those holdings. This rule ties the cost of the open position to the backing clause, and it has a sharp implication for the periphery. When the issuer holds R's currency against dollar liabilities, a depreciation of R's currency inflicts a loss that falls partly on R's own government, when its fiscal capacity is weakest. Part of the insurance R's households receive is then paid back by R's taxpayers.

Loss allocation does not change what the liability pays its holders. Holders are redeemed at par in every state: a rule that charged losses to holders would change denomination after the fact, and the issuer's insurance would be a different contract.

Hedging. The issuer may hedge a share ς of its currency exposure into the denomination currencies, equation (23). Hedging scales the open position to $x_R = (1-\varsigma)(\theta_R^a - \theta_R^d)$, and with it the capital requirement of Proposition 3.1. It does not change what the liability pays, so holders' insurance is unchanged. What changes is who supplies it: the counterparty to the hedge is the financier, and the exposure the issuer sheds becomes a currency imbalance priced by its limited risk-bearing capacity, equation (19). Hedging moves peripheral currency risk from the members' budgets to the currency market.

Reserve management. The issuer chooses the Macaulay duration of its reserves, D_j, and the speed $\varphi^{reb} \in (0, 1]$ at which it closes the gap between its holdings and the value shares θ^a. Reserve regulation typically limits duration. Duration and rebalancing jointly decide whether the reserve rule makes the issuer a buyer of the periphery's bonds when their price falls. They also decide how much interest-rate risk the issuer, and through it the members, carry. An issuer that never rebalances, $\varphi^{reb} = 0$, has no bounded equilibrium (Section 4.9).

3.3 The Governance Grid

Table 1 lists the rules and the values we consider. Each rule is varied one at a time from the baseline governance, for every contract, with all calibrated parameters held at their baseline values. The values are round numbers that span the range of practice and of the literature, and none is chosen with reference to the results. Capital enters the dynamic model through the payout rule; for each contract we report the open position x_R, which by Proposition 3.1 sets the capital that makes the issuer safe at every scale. Section 4.6 sets the issuer's loss in a disaster against households' gains, and Section 4.8 reports the welfare of each rule.

4 Quantitative Analysis

This section traces a risk-off episode at the dollar corner and then measures every contract against the economy without the liability, Regime 1, as in Figure 3, so that the costs and gains of the consistent design are measured like those of every other contract. Impulse responses are computed from the first-order solution of the log-linear model (Appendix A). Welfare is computed from the second-order solution of the nonlinear model of which that system is the exact first-order approximation (Appendix C). Every architecture satisfies the Blanchard–Kahn conditions at the baseline, and every model-generated number quoted in the text is written by the replication code.

4.1 Calibration and Regimes

Table 2 lists the calibration at a quarterly frequency. Preferences, technology and monetary policy take conventional values. The portfolio-balance coefficient $\psi_b = 0.30$ prices the financier's limited risk-bearing capacity, and the risk-off disturbance has persistence $\rho_\varpi = 0.85$. Where a parameter

is calibrated to a published range, we use the midpoint of the range and report its endpoints in Section 4.10.

Substitution between money and the liability. The composition semi-elasticity $\eta = 2$ says that an expected appreciation of the liability of one percent over a quarter raises holdings relative to money by two percent. Because the user cost of both liquid assets is $1-\beta \approx 1$ percent a quarter, this semi-elasticity corresponds to a low elasticity of substitution between normalized holdings, $\eta_L = \eta(1-\beta)/\beta \approx 0.02$ ($\rho_L \approx -48$): money and the liability serve largely different transactions, but the composition still responds to return differentials of the size observed in a risk-off episode. No direct estimate exists for a digital liability; Table C.1 reports the results for $\eta = 1$ and $\eta = 5$.

The liability. Liquid assets are a third of a quarter's consumption, $\kappa_L = 1/3$, and the weight of the liquidity gap is $\varphi_l = \kappa_L/2$. Adoption is $\omega_j = 0.5$ in every country, so that each household holds the liability worth a sixth of a quarter's consumption, and half of the goods of a country that uses the liability are priced in its unit, $\upsilon = 0.5$. This is a scenario, not a measurement: current stablecoin holdings are a small fraction of broad money, and a quarter of domestic goods priced in a foreign unit is observed today only in highly dollarized economies. Section 4.6 reports how the results scale with adoption, and Table C.1 reports $\upsilon = 0.2$, which prices a tenth of domestic goods in U. Population weights are equal, so the neutral position is $\bar{h} = \lambda = (1/3, 1/3, 1/3)$. In the baseline the issuer holds one-period bills, rebalances every quarter, pays out its equity each period ($\phi_{\text{div}} = 1$), does not hedge ($\varsigma = 0$), shares its transfers by population ($\alpha^T = \lambda$), and its equity does not enter intermediation capacity ($\chi_E = 0$); Section 3 varies these rules.

Insurance motive. The hedging demand of R households is governed by one structural parameter, the jump $B_R = (1-\Delta c)^{-\sigma}$ in R's marginal utility in a currency disaster. We set $B_R = 1.375$, a fall in consumption of 10.1 percent with $\sigma = 3$. In the emerging-market crises of Barro and Ursúa (2008, Table 8), consumption fell by 11.3 percent in Mexico in 1995, 14.3 percent in Korea in 1998, 30.0 percent in Peru in 1992 and 32.7 percent in Chile in 1985. Our value is just below the smallest of these; Section 4.10 reports a disaster of 30 percent ($B_R = 2.92$). By Proposition 2.4, any $B_R > 1$ produces a flight into the liability.

Convenience yield. The convenience yield on U.S. Treasuries averaged 73 basis points a year between 1926 and 2008 (Krishnamurthy and Vissing-Jorgensen, 2012). Their Table 1 regresses yield spreads on the log of the debt-to-GDP ratio, which is the form (20): the coefficients range from 55 basis points a year per log point, for the P2 commercial paper–bill spread, to 196, depending on the spread; at the sample-mean debt ratio, 0.439, they imply that a reduction in Treasury supply of one percent of GDP raises the spread by 1.3 to 4.5 basis points a year. We use the midpoint of the coefficients, $b_{cy} = 126$ basis points a year per log point. With privately held debt of $\bar{B}^p = 90$ percent of annual GDP, a reduction of one percent of GDP raises the convenience yield by 1.40 basis points a year before the issuer buys anything, and by 1.62 at the dollar corner, where the

issuer holds 12.5 percent of the reserve country's annual GDP. There the yield on H's bonds falls by 18.8 basis points a year, against 35.8 under a linear schedule with the slope at the sample-mean debt ratio. The log form keeps the slope that the data identify at the debt levels at which they are measured. With a non-resident share of government debt $\varphi^f = 0.3$ and debt of 100 percent of annual GDP ($B^g = 4$ in quarterly units), the yield privilege of a country holding the whole of backing is 3.8 basis points of annual consumption every year as a transfer, and 8.1 in general equilibrium (Section 4.6).

Original sin. R's gross external debt is 30 percent of annual GDP ($\bar{B}_R = 1.2$). The dollar share absent the liability is $\chi^0_{os} = 0.52$, its value at the end of 2022 in the IMF's sovereign investor base for emerging markets: the local-currency share of foreign-held government securities rose from 26 percent in the first quarter of 2004 (Arslanalp and Tsuda, 2014) to 48 percent by the end of 2022 (Pedrono and Ishii, 2024), a decline of original sin documented in detail by Onen et al. (2023); Section 4.10 reports the 2004 value. The feedback of the dollar burden onto R's premium is $\kappa_{os} = 0.025$ at the dollar corner, so that a one-percent real depreciation raises R's premium by 10 basis points a year: in Hofmann et al. (2020), a one-percent depreciation of an emerging-market currency against the dollar on monetary-policy news days raises local-currency sovereign spreads by up to 9–10 basis points within 50 days (Figures 3–4 of the working-paper version); the authors note that effects estimated from all exchange-rate changes are smaller. Defaults follow the costly-state-verification model of Bernanke et al. (1999) (Appendix D.8). With an annual default rate of 3 percent, their business failure rate, and a loss given default of 62 percent, one minus the 38.4 percent average recovery on senior unsecured corporate bonds in Hamilton et al. (2007, Exhibit 6), the feedback κ_{os} implies a dispersion of borrowers' assets $\sigma_\omega = 0.27$ and a bankruptcy cost of 0.58 of the assets of a defaulting borrower; the share of creditors' losses that reaches R's borrowers is then $\tau_T = 0.143$. With the bankruptcy cost of 0.12 in Bernanke et al. (1999) instead, $\tau_T = 0.153$; we use the midpoint, $\tau_T = 0.148$. Through (26), the dollar share falls to 0.38 when backing is bundled and to 0.10 when backing is held entirely in R's currency.

Transfers to the issuer. The excess burden of the taxes that finance transfers to the issuer has the slope $\xi_1 = 0.365$ of Section 3.2; members smooth the taxes, so $\xi_2 = 0$.

The episode. Every experiment is a single risk-off episode: one innovation to ϖ_R, from the steady state, sized so that R's currency depreciates by 10 percent on impact against the dollar without the liability. Welfare is that of households in the three countries, measured as the one-time loss of consumption, in basis points of one year's consumption, that a household would accept to avoid the episode. Our main measure is the welfare effect of fluctuations in disaster risk: the average of a risk-off and an equally large risk-on episode, which is exactly quadratic in the size of the episode. The second measure is a risk-off episode alone. The dynamics are those of the path on which no disaster occurs, so both measures add the value in the disaster state of every position households hold in R's currency: the liability, through the term (17); their share of the financier's

position; and R's dollar debt and its defaults (Appendix C.2). The issuer's loss in the disaster state is not part of households' welfare: the issuer is designed to absorb losses, and we report its loss separately, as the bill its capital and backstop must cover. The yield privilege is a permanent flow; to compare the two, we report welfare per year with one episode every five or every ten years.

Regimes. Table 3 defines the architectures. Regime 1 has no liability and Regime 2 is the dollar corner. Regimes 3D, 3U and 3A bundle one clause on the basket λ, holding the other two on the dollar; Regime 3R holds reserves entirely in R's currency against a dollar liability, the largest relief of original sin the model admits. Regime 4 is the consistent design and Regime 5 the redesigned dollar coin. Regimes 3A, 4 and 5 share the neutral effective position $\theta^e = \lambda$; they differ in R's insurance, full in 3A and 5 and two thirds in 4, and in the issuer's open position, zero only in 4. Every gain is measured relative to Regime 1, the economy without the liability.

4.2 A Risk-Off Episode at the Dollar Corner

Figure 1 shows the episode at the dollar corner. R's currency depreciates by 10.6 percent on impact; in real terms it recovers within five quarters, while in nominal terms it is still 7.6 percent weaker after ten years because R's price level rises permanently relative to H's. R's premium rises by 5.82 percentage points; the liability's footprint on it is negligible on impact, 0.01 points, because the dollar value of R's holdings falls with its currency, and it builds up as R's currency recovers. R's households flee into the liability, whose real holdings rise by 5.47 percent: it pays off in exactly the disaster they fear. R's central bank raises its rate against imported inflation, consumption falls by 2.8 percent, and the real depreciation turns R's trade balance into surplus. In H and F the appreciation against R's currency is deflationary; their central banks ease, and consumption rises.

The dollar liability gives R insurance and takes more than that back through prices and activity (Table 4). Relative to Regime 1, the dollar corner raises R's inflation on impact from 3.23 to 4.81 percent, because a quarter of R's goods are now priced in dollars, and deepens the fall in R's consumption. The dollar corner raises the welfare cost of the episode to R by 14.3 basis points of a year's consumption relative to no liability (Table 7). The liability's value in the disaster state lowers R's cost by 14.8 basis points, because holdings are high exactly when the risk of a disaster is high; the larger swings in R's consumption and employment raise it by 24.2.

4.3 What Each Clause Does

Figure 2 isolates each clause: it plots the difference between two regimes that differ in that clause alone.

Denomination. Regimes 5 and 4 differ only in denomination. Moving denomination from the basket to the dollar makes the liability a full hedge for R: the flight into the liability rises from 2.80 to 6.70 percent on impact. Exchange rates barely move: R's depreciation on impact differs by

0.08 points, as Proposition 2.6 implies. For R the larger hedge is worth 9.7 basis points per episode through the disaster-state term, and the larger swings in consumption and employment that come with the larger flight cost 9.6; on balance R's cost of the episode is 1.7 basis points higher under the dollar denomination. With dollar backing (Regime 3D), bundling denomination lowers R's cost by 6.4 basis points relative to the dollar corner, but R still loses 8.0 relative to no liability; R's holdings rise by 1.70 percent on impact, against 5.47 at the dollar corner, and the issuer, holding dollars against a liability that contains R's and F's currencies, gains 1.63 percent of output as they depreciate.

Unit of account. Regimes 5 and 3A differ only in the unit of account. Quoting prices in the basket lowers R's inflation on impact from 4.48 to 3.96 percent and the cost of the episode to R by 6.3 basis points; H and F pay part of the bill, 0.9 and 0.7 basis points, because their U-priced goods now import part of R's depreciation. With dollar backing the same move (3U against 2) is worth 7.7 basis points to R, and the exchange rate does not move (Proposition 2.5).

Backing. Moving reserves from dollar bonds to the basket (3A against 2) lowers the cost of the episode by 6.0 basis points for R and 13.3 for F, and raises it by 4.6 for H. The issuer takes the currency risk instead, losing 1.6 percent of output on impact; the dollar share of R's debt falls from 0.52 to 0.38, and H's yield privilege of 8.1 basis points a year is shared. Backing entirely in R's currency (3R against 2) magnifies each effect: R's depreciation on impact is 8.9 percent instead of 10.6, R gains 17.3 basis points per episode, and the issuer loses 4.5 percent of output on impact.

The dollar corner and the consistent design. Relative to no liability, the dollar corner gives R the full hedge but costs R 14.3 basis points per episode; H gains 9.7 and F loses 6.4. F loses in every architecture whose effective position is the dollar and gains in every architecture with a neutral or peripheral position. Each single move away from the dollar corner lowers R's cost of the episode. The consistent design is close to neutral for every country: H gains 1.3 basis points and F 2.2, and R loses 0.4. Its liability insures R less, so the flight into it is smaller (2.80 percent on impact, against 5.47 at the dollar corner), and its neutral position adds no imbalance to currency markets.

4.4 The Redesigned Dollar Coin

The redesigned dollar coin keeps the dollar's insurance and the consistent design's neutral position (Figure 3). Relative to no liability, it gains 4.3 basis points for H and 6.2 for F and costs R 2.1; world welfare rises by 2.8 basis points per episode. Against the consistent design, from which it differs only in denomination, it gains 3.0 for H and 4.0 for F and costs R 1.7. Against the dollar corner it is worth 12.2 basis points to R and 12.6 to F, while H gives up 5.5 per episode and the yield privilege. As Proposition 2.7 requires, the insurance R keeps is matched by an open position of the issuer in the basket currencies, which loses 1.6 percent of output on impact.

Table 5 shows how the redesigned coin trades R's relief against the issuer's exposure, relative to no liability. Hedging the open position makes the effective position the dollar again: R's loss rises to 7.6 basis points and F's gain turns into a loss of 6.7. Moving backing entirely into R's currency (Regime 5R) turns R's loss into a gain of 7.5 basis points and raises the world's gain to 8.4, the largest of any contract we consider, at the cost of an issuer loss of 4.5 percent of output on impact; hedged, the same contract costs R 9.9.

Holding $\theta^a = \lambda$ and varying θ^u over a grid of currencies and baskets, and θ^d over a grid that keeps $\theta^d_R = 1/3$, the paths of exchange rates, premia and external positions differ from those of the consistent design, and the changes in country variables differ across countries, by less than 6e-12 with one-period bills and by at most 2e-11 with reserves of fifteen years' duration, as Proposition 2.6 states.

4.5 Channels

Table 6 switches off the model's channels one at a time.

The insurance motive drives the flight. Without it ($B_R = 1$), R's holdings under the consistent design fall by 0.4 percent on impact, and at the dollar corner they rise by only 1.06. The redesigned dollar coin and the consistent design then do almost equally well for R, 1.0 and 0.9 basis points relative to no liability, and the dollar corner costs R 7.5. With the insurance motive, the larger flight raises the disaster-state term for R under every contract with dollar denomination, and it also widens the swings in R's consumption and employment; the second effect dominates for every such contract except those whose reserves are held in R's currency.

The convenience yield is distributive. Removing it changes R's episode gains by at most 0.9 basis points; what disappears is the yield privilege.

Original sin cuts both ways. It amplifies the depreciation through R's premium, but defaults on R's dollar debt move R's net foreign assets in its favor when its currency falls, and only part of that is lost in bankruptcy. Removing original sin raises the cost of the episode to R under the consistent design from -7.2 to -4.6 basis points and changes R's gains by at most 3.2. The amplification itself is small: $1/(1-\kappa_{os}) \approx 1.026$ at the dollar corner.

Buffers are second order in an episode. Retained earnings and an equity feedback on intermediation capacity change R's gains by at most 0.6 basis points.

4.6 Welfare

Table 7 reports, in Panel A, the welfare effect of fluctuations in disaster risk: the cost of one episode, its reduction relative to no liability (Regime 1), and three components of that reduction—the value of the liability in the disaster state, the value of households' share of the financier's position in that state, and the excess burden of transfers to the issuer. Panel B reports risk-off episodes alone.

Panel C reports welfare per year: the yield privilege, a permanent flow, plus the episode gains of Panel A spread over five or ten years. Figure 4 plots the gains.

First, over fluctuations in disaster risk, a widely held liability is costly for the periphery unless its issuer holds the periphery's currency. Relative to no liability, R loses 14.3 basis points at the dollar corner, 8.4 with bundled backing, 8.0 with bundled denomination, 6.7 with the bundled unit of account and 2.1 under the redesigned dollar coin. The consistent design is close to neutral for R (-0.4). Only backing in R's currency makes R better off: 3.0 basis points with dollar prices and 7.5 with basket prices (Table 5).

Second, the value of the liability in the disaster state is large relative to these differences. It is worth 14.8 to 15.9 basis points per episode to R under every contract with dollar denomination, and 5.7 under the consistent design, whose denomination contains R's own currency. Without it, R would lose under every contract, by 6.1 basis points under the consistent design, 17.5 under the redesigned dollar coin and 12.9 with backing in R's currency. The periphery's case for holding the liability, and for dollar denomination in particular, rests on the insurance it buys.

Third, households' share of the financier's position and the excess burden of transfers to the issuer are small. The financier is short R's currency when disaster risk is high, so its position gains in the disaster state; relative to no liability it changes R's gain by at most 1.8 basis points. With smoothed taxes, the excess burden lowers every country's gain by 1.6 basis points at the dollar corner and 1.7 under the bundled unit of account, where members recapitalize the issuer on average over the two episodes, and raises it by 1.8 with backing in R's currency and 0.5 under the consistent design, whose issuers pay out more on average than they call in.

Fourth, backing in R's currency makes every country better off than no liability, 8.8, 10.2 and 3.0 basis points for H, F and R, and the contracts that keep the issuer's position on the dollar—the dollar corner and the bundled unit of account—benefit H at the expense of F and R. The consistent design raises world welfare by 1.0 basis points per episode. World welfare moves by at most 7.3 basis points per episode among the contracts of Table 7 (Regime 3R), and by 8.4 under Regime 5R (Table 5), while individual countries gain or lose up to 14.3.

Fifth, per year, the yield privilege dominates. It moves by up to 8.1 basis points a year whenever backing moves away from the basket, against episode gains of at most a few basis points a year. R's ranking per year is led by backing in R's currency (8.4 to 8.7 basis points); among the designs with a neutral position, R does best under the consistent design. Every contract that holds reserves in dollars costs R per year and benefits H, by 9.0 to 10.0 basis points at the dollar corner. For every country and the world the ranking of contracts per year is the same with one episode every five or every ten years. The world gains little per year: at most 1.5 basis points (Regime 3R). The contract is primarily a distributive instrument per year.

Risk-off episodes. In a risk-off episode alone (Panel B) the welfare effects are an order of magnitude larger, because the first-order effect of the episode no longer cancels against that of a risk-on episode, and they are dominated by one mechanism. On the path without a disaster the premium on R's currency compensates only about 35 percent of the expected disaster loss: in a risk-off episode the financier is short R's currency, and the portfolio-balance term lowers the premium it requires. Whoever holds R's currency through the episode therefore bears an expected loss that the market does not compensate. Every contract gives R's households a large gain relative to no liability, because they hold a claim that pays off in the disaster: 181.0 basis points at the dollar corner and under the redesigned dollar coin, of which the disaster-state value of their positions is 258.1, and 104.9 under the consistent design, whose liability falls with R's currency by a third. The consistent design is also the design that makes the advanced economies' households lose: H and F lose 25.7 and 25.8 basis points, and the disaster-state value of their positions costs them 50.7 and 59.3, more than their whole loss, because a third of every unit they hold is R's currency, held without compensation. Under the redesigned dollar coin, which differs from the consistent design only in denomination, they gain 36.2 and 37.3. Putting the periphery's currency into denomination makes the advanced economies' households bear the periphery's crash risk without compensation.

The issuer's bill. The issuer's own loss in the disaster state is the counterpart of the insurance its contract supplies, and it is not part of households' welfare: the issuer is designed to absorb losses. Table 8 sets it against households' gains. If the average household gain $\bar{G}$ exceeds the issuer's loss $\bar{L}$, including the excess burden of covering it, some sharing of that loss among members makes every country gain relative to no liability; if in addition every country's household gain is non-negative, no member needs to be compensated to join (equation (C.3)).

Over fluctuations in disaster risk, the consistent design's issuer has no bill, so the world gains what its households gain, 1.0 basis points, but R's households do not gain. Backing in R's currency creates the most for the world net of the issuer's expected loss: 1.8 basis points with dollar prices and 3.1 with basket prices. They are the only designs under which every country's households gain, so both can be made to improve on no liability for every country by sharing the issuer's loss. The redesigned dollar coin nets 0.9, about as much as the consistent design, and the designs that keep the issuer's position on the dollar lose for the world, 4.0 basis points at the dollar corner. In risk-off episodes the ranking reverses. The designs in which the issuer is long R's currency—bundled backing, backing in R's currency and the redesigned coin—cost the issuer 93.1 to 279.2 basis points, more than households gain, and the world loses 8.2 to 178.4. The dollar corner and the bundled unit of account, whose issuer holds no open position in R's currency, gain 71.7 and 73.5 for the world, and the consistent design 17.8, R's gain net of H's and F's losses. Backing in R's currency is the best design over the business cycle and the worst in a crisis unless its capital and backstop cover the loss: the new Triffin dilemma of Proposition 2.9, in welfare terms.

The yield privilege in general equilibrium. The yield privilege is a transfer, $\bar{N}_k$ in (22), but its welfare value is larger. The country whose bonds back the liability pays less on its debt held abroad and, in the new steady state, runs a permanent trade deficit of the same size; with home bias, demand shifts toward its goods and its terms of trade improve, and the wealth effect lowers its labor supply (equation (C.5)). At the dollar corner, H's interest saving is 3.8 basis points of consumption a year; H runs a trade deficit of 3.8 basis points of output, consumes 3.1 basis points more and produces 5.0 less, and its terms of trade against R improve by 14.3 basis points. The terms-of-trade gain adds 4.3 basis points, so the privilege is worth 8.1 to H, and F and R each lose 4.0 instead of 1.9. Backing in R's currency gives R the same 8.1. This is the original form of Triffin's dilemma: the country that supplies the world's reserve asset runs a permanent external deficit, financed by the world's demand for its bonds, and the privilege it draws is the counterpart of that deficit.

4.7 Adoption

Adoption is a state of the monetary environment in the model: the contract determines what the liability does once held, not how widely. Table 9 repeats the comparison at four levels of adoption, from a tenth of liquid assets held in the liability to three quarters.

Three things grow with adoption besides the yield privilege, which rises from 1.5 basis points a year at $\omega = 0.1$ to 12.6 at $\omega = 0.75$. First, the cost of a crisis to the periphery at the dollar corner grows faster than adoption up to half of liquid assets, and keeps growing beyond: relative to no liability, R loses 0.8 basis points per episode at $\omega = 0.1$, 4.6 at $\omega = 0.25$, 14.3 at $\omega = 0.5$ and 19.6 at $\omega = 0.75$. More of R's goods are priced in dollars, so R's inflation on impact rises from 3.54 to 5.63 percent, and the flight into the liability applies to a larger stock. F's loss grows the same way, from nothing at $\omega = 0.1$ to 10.7 basis points. Second, the two designs with a neutral position turn from a gain into a loss for R between a quarter and a half of liquid assets: the consistent design from 0.9 basis points at $\omega = 0.25$ to -0.4 at $\omega = 0.5$, and the redesigned dollar coin from 1.4 to -2.1, while H and F gain under both at every level. Third, backing in R's currency improves on no liability for every country at every level of adoption, and the world's gain rises from 1.8 to 8.2 basis points.

For the dollar corner and backing in R's currency, the sign of every country's gain is the same at every level of adoption, and for the two designs with a neutral position it is the same except in the periphery; only the magnitudes grow. The answers the paper gives do not depend on which level of adoption a contract would reach. What adoption decides is how much is at stake: at today's levels the choice of contract matters little, and it becomes first order once the liability is a significant part of the monetary system.

4.8 Governance

Table 10 reports the governance grid of Section 3.3: each rule, for every contract, against no liability. The economy without the liability has no issuer, so the rules do not change it.

Capital. The designs that give R full insurance with reserves outside the dollar need capital against the disaster: 10 percent of liabilities for the redesigned dollar coin and bundled backing, 30 percent with backing in R's currency. The dollar corner, the bundled unit of account and the consistent design need none, and bundled denomination leaves the issuer short R's currency. These are the designs whose issuer's bill in Table 8 is large in a risk-off episode.

Recapitalization. With smoothed taxes the speed of recovery works through the issuer's balance sheet, whose reserves stay smaller while a loss is not recovered. Spreading the recovery of losses over a year or five years shifts the gains from the designs with an open position in R's currency toward R: under backing in R's currency H's gain falls from 8.8 to 7.6 and 5.9 basis points while R's stays at about 3.0, and under the redesigned dollar coin R's loss falls from 2.1 to 1.5 and 1.1.

The cost of transfers. When only recapitalizations carry an excess burden, the cost of the issuer's losses becomes first order in the size of the episode, and backing in R's currency and the redesigned coin, whose issuers are long R's currency, lose for every country relative to no liability: backing in R's currency costs H, F and R 11.1, 9.7 and 16.9 basis points, and the redesigned dollar coin 1.9, 0.1 and 8.3. Whether payouts to members relieve their taxes as much as recapitalizations raise them decides whether backing in R's currency is the best design in an episode or the worst.

Loss allocation. Allocating the issuer's transfers by backing weight moves them to the countries whose bonds are held. At the dollar corner and under the bundled unit of account H bears them alone, and H's gains turn into losses of 19.6 and 23.1 basis points. With backing in R's currency R recapitalizes the issuer for the losses on its own currency but also receives all of its payouts, which on average exceed its losses: R's gain rises from 3.0 to 14.5 basis points, while H and F lose 2.0 and 0.9. Who owns the issuer's average income matters more than who recapitalizes it in a crisis. With bundled backing the rule coincides with population shares.

Hedging. Hedging moves the effective position toward the dollar. It leaves denomination, and so the insurance each unit gives R, unchanged, shifts the exposure to the currency market, and reproduces the distribution of the dollar corner: fully hedged, backing in R's currency gains H 9.2 basis points and costs F and R 7.4 and 17.9.

4.9 Reserve Duration and Rebalancing

Reserve regulation typically limits the duration of reserves. In the model, duration matters through the reserve rule: when a risk-off episode raises R's interest rate, R's bonds lose value, and the issuer must buy them to restore R's share of its reserves. That purchase of R-currency assets lowers the premium on R's currency in the middle of the episode.

Figure 5 varies the Macaulay duration of reserves from one quarter to the perpetuity (values in Table C.2). Longer duration lowers R's depreciation in every architecture: with fifteen years' duration, by 5.6 percent under the redesigned dollar coin, against 9.9 with bills; by 5.2 with backing in R's currency, against 8.9; and by 8.5 at the dollar corner, against 10.6. Under a neutral position the issuer's equity barely responds to duration (standard deviation 0.9 percent of output at fifteen years under the redesigned coin); under a concentrated position it rises steeply, to 13.5 at the dollar corner and 37.4 with backing in R.

The stabilization comes from the reserve rule, not from duration alone. If the issuer closes only part of the gap to its value shares each quarter, the stabilization shrinks (Figure C.1). Under the redesigned dollar coin, comparing fifteen years' duration with one-period bills at the same speed of rebalancing, R's depreciation on impact is 5.6 against 9.9 percent with full rebalancing, 7.6 against 10.2 if a quarter of the gap is closed each quarter, 8.8 against 10.3 if a tenth is, and 10.1 against 10.3 if one fiftieth is. An issuer that never rebalances has no bounded equilibrium: its currency positions drift with nominal exchange rates, which have unit roots under inflation targeting.

These are first-order results about exchange rates. We do not report the welfare effect of duration: with long-dated reserves an episode of the size studied here moves bond prices by tens of percent, and the second-order approximation is not reliable for such movements (Appendix C.2). The results therefore show that long-dated reserves under a value-share rule stabilize the periphery's exchange rate in a crisis, not that they raise welfare.

4.10 Robustness

Original sin and determinacy. With one-period reserves, original sin alone eliminates the bounded equilibrium only for a feedback more than 174 times its estimated value (Regime 1); in every other regime, a bounded equilibrium exists up to the upper end of the search grid, 200 times the estimated value.

Endpoints of the calibrated ranges. Table 11 repeats the episode comparison at the endpoints of each sourced range, one at a time. The convenience-yield coefficient at either end of the range in Krishnamurthy and Vissing-Jorgensen (2012) changes R's gains by at most 0.5 basis points, the dollar share of R's debt at its 2004 value by at most 0.8, and the excess burden at either end of the range in Ballard et al. (1985) by at most 0.9. Raising the taxes within the quarter instead of smoothing them adds the curvature $\xi_2 = 3.24$ and changes R's gains by at most 1.4, through backing in R's currency, where R's gain falls to 1.6; with lump-sum taxes, which make the transfers costless, R's gains change by at most 1.8. Counting in the disaster state only the value of the liability, as households' welfare would if it ignored their share of the financier's position, changes R's gains by at most 1.8. A disaster of 30 percent, as in Peru in 1992 ($B_R = 2.92$), magnifies every difference: R's gain from backing in its currency rises to 18.3 basis points, its loss at the dollar corner to 74.2 and under the redesigned dollar coin to 33.6, and the consistent design now costs R

14.8. A larger disaster raises the flight into the liability, and with it both the disaster-state term and the swings in consumption and employment.

Substitution and U-pricing. Table C.1 varies η and υ. R's gain from the redesigned dollar coin relative to no liability is 2.3 basis points with $\eta = 1$ and -14.4 with $\eta = 5$, against -2.1 at the baseline; a stronger response of holdings to returns widens every difference, as a larger disaster does. With a tenth of domestic goods priced in U ($\upsilon = 0.2$), R's gain is 3.2. Backing in R's currency improves on no liability for R in each case, by 2.9 basis points with $\eta = 1$ and 3.4 with $\eta = 5$.

Size and frequency of episodes. The size of the episode scales the gains of Panel A of Table 7 by a common factor and leaves their ranking unchanged; size and frequency change the balance between episodes and the yield privilege. With episodes twice as frequent as in the five-year case, or about 40 percent larger, gains per year from episodes double; R's loss at the dollar corner would then exceed the yield privilege it forgoes.

5 Policy Implications

The results concern what a contract does once the liability is widely held, and they are conditional on the governance behind the issuer.

A dollar stablecoin raises the cost of a crisis for the periphery. In the model, a widely held dollar stablecoin raises the welfare cost of fluctuations in disaster risk for the periphery's households by 14.3 basis points of a year's consumption per episode: dollar pricing imports the depreciation, and the flight into the liability widens the swings in consumption and employment by more than its value in the disaster state. Every single move away from the dollar corner—bundling denomination, quoting prices in a basket, or holding reserves in a basket or in the periphery's currency—lowers that cost.

Keep the periphery's currency out of what the liability is. Putting the periphery's currency into denomination weakens the liability as insurance for the periphery and, in a crisis, makes the advanced economies' households bear the periphery's crash risk without compensation: in a risk-off episode the consistent design costs H's and F's households 25.7 and 25.8 basis points relative to no liability, because the disaster-state value of the periphery's currency they hold through the liability costs them 50.7 and 59.3, while the redesigned dollar coin, which differs from it only in denomination, gains them 36.2 and 37.3.

Backing decides who carries the risk and who earns the privilege. Holding the reserves of a dollar liability in the periphery's currency is the only design that makes every country better off than no liability over fluctuations in disaster risk, with dollar or with basket prices (Regimes 3R and 5R), and the one whose households' gains exceed the issuer's expected losses by most; per year

it gives the periphery the yield privilege. It also gives the issuer the largest open position. In a risk-off episode its issuer loses 279.2 basis points per resident of the world, more than households gain, and by Proposition 3.1 it needs capital of 30 percent of its liabilities against a 30 percent disaster to be safe at every scale. It becomes the worst design for every country if recapitalizations carry a fiscal cost that payouts do not relieve, and it no longer improves on no liability for H and F if the periphery bears the issuer's losses and receives its income.

The privilege is a deficit. Whoever backs the world's digital reserve asset earns more than the interest it saves: it runs a permanent trade deficit financed by the demand for its bonds, and the deficit improves its terms of trade, so the privilege is worth about twice the interest saving (8.1 against 3.8 basis points a year at the dollar corner). This is Triffin's dilemma in its original form, attached to a contract clause: the backing clause decides which country runs the deficit that supplies the reserve asset, and, through the open position it creates when backing differs from denomination, whether the issuer can honor that asset in a crisis.

Write the governance with the contract. Capital requirements for an issuer should scale with its open position times a plausible disaster size—10 percent of liabilities for the redesigned coin and 30 percent with reserves in the periphery's currency against a 30 percent disaster—not with the size of the liability alone. The issuer's bill in a crisis is the price of the insurance its contract supplies, and the designs that are best for households over the business cycle are those whose bill is largest in a crisis. Whether payouts relieve members' taxes as much as recapitalizations raise them, who owns the issuer's income and bears its losses, and whether the issuer hedges change the welfare ranking of contracts by as much as the clauses do.

Reserve regulation and the rebalancing rule. Requiring reserves in the currency of the peg aligns backing with denomination and removes the open position (Proposition 2.1), but delivers the yield privilege and the safe-haven flows to the issuing country and rules out backing in the periphery's currency. Under a rule that rebalances reserves to fixed value shares, long-dated reserves make the issuer a buyer of the periphery's bonds when their price falls and stabilize the periphery's exchange rate in a crisis; the speed of rebalancing, which the contract can specify, decides how much.

Measure the insurance motive. The value of the liability to the periphery rests on households' demand for insurance against their own currency, which is worth 14.8 basis points per episode to the periphery under the dollar corner and 5.7 under the consistent design. How strongly demand for a foreign-currency liability rises with the risk of a currency collapse is the parameter most worth estimating before a multilateral design is chosen.

6 Conclusion

A global digital currency is a contract whose clauses can be written separately. Denomination, the unit of account and backing each govern distinct outcomes, and the contract cannot remove peripheral currency risk, only decide who bears it. Insurance against the periphery's currency is an open position for whoever supplies it, and an issuer with fixed capital can supply it credibly only up to a scale—a new Triffin dilemma that capital growing with the open position removes. The old dilemma reappears in general equilibrium: the country whose bonds back the liability runs a permanent trade deficit financed by the world's demand for them, and the yield privilege it draws is worth about twice the interest it saves.

Quantitatively, a widely held dollar stablecoin raises the cost of crises to the periphery, and every single redesign of its clauses lowers it. A consistent basket design is close to neutral over the business cycle and in a crisis insures the periphery at the expense of the advanced economies' households. Backing a dollar liability with the periphery's bonds makes every country better off than no liability over the business cycle, and creates more welfare than its issuer's expected losses cost; in a crisis, the same design leaves its issuer with the largest bill. The designs that serve households best over the business cycle are those that most need capital in a crisis, and the clause that decides who absorbs crises is not the clause that decides who gains in normal times: per year, the yield privilege attached to backing dominates.

Treating a global digital currency as a contract opens a research agenda that models of the international monetary system built around national currencies cannot address. The first question is what happens when confidence fails. Proposition 2.9 identifies the scale beyond which the issuer's insurance rests on confidence; the next step is to study the run itself, as a crisis with creditor coordination in which the contract's margins shape when holders flee and how far the flight reaches the periphery. The second is who stands behind the issuer. Here the backstop is a parameter; the next step is to make it an institution, a rule-based lender of last resort, contingent on outflows, for the countries outside the network of central bank swap lines. The third is which contract the world would adopt: making adoption endogenous would connect contract design to the emergence of dominant currencies, and ask whether a better-designed liability can displace an incumbent one. The fourth is how strongly demand for a dollar liability responds to the risk of a currency collapse, a parameter that stablecoin flows during episodes of stress are beginning to make measurable. The answers will decide whether a multilateral digital currency becomes a safe asset shared by the world or a new channel through which the periphery's crises are passed on.

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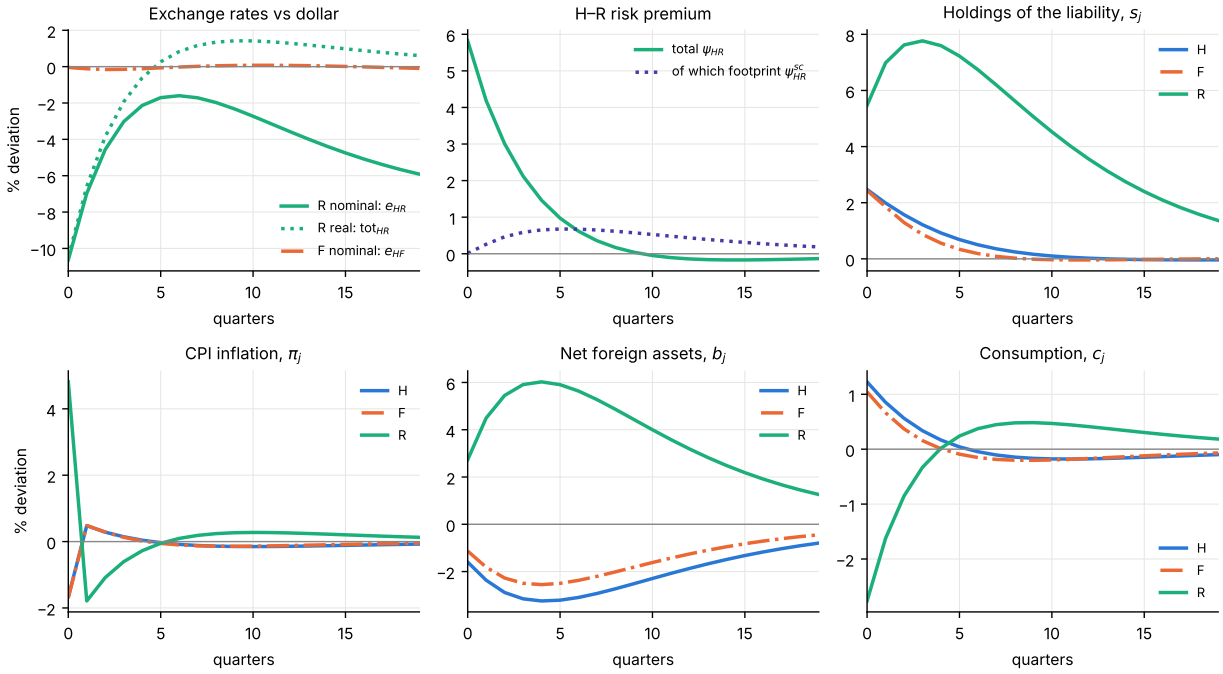
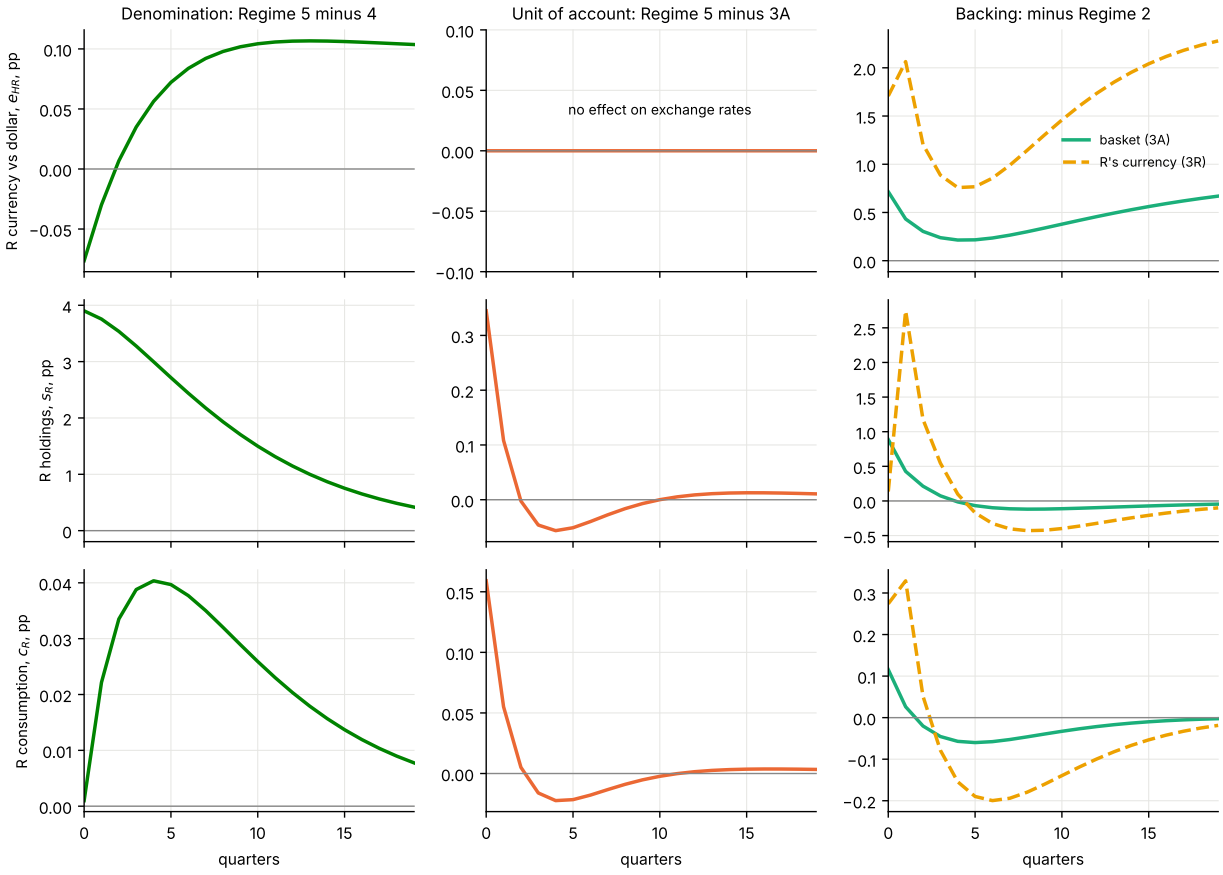
Figure 1: A risk-off episode at the dollar corner (Regime 2)



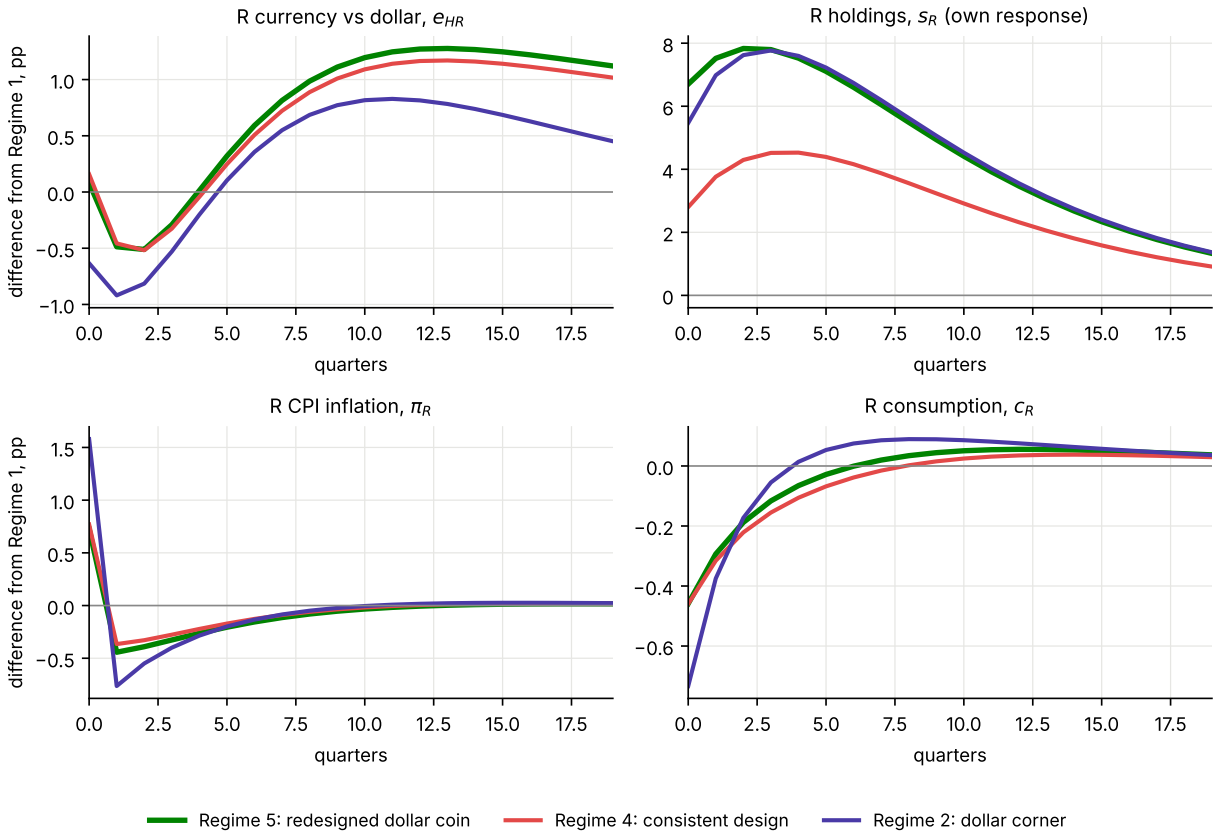
Note. Percent deviations from the steady state; premia in percentage points per quarter; net foreign assets in percent of steady-state quarterly output.

Figure 2: One clause at a time



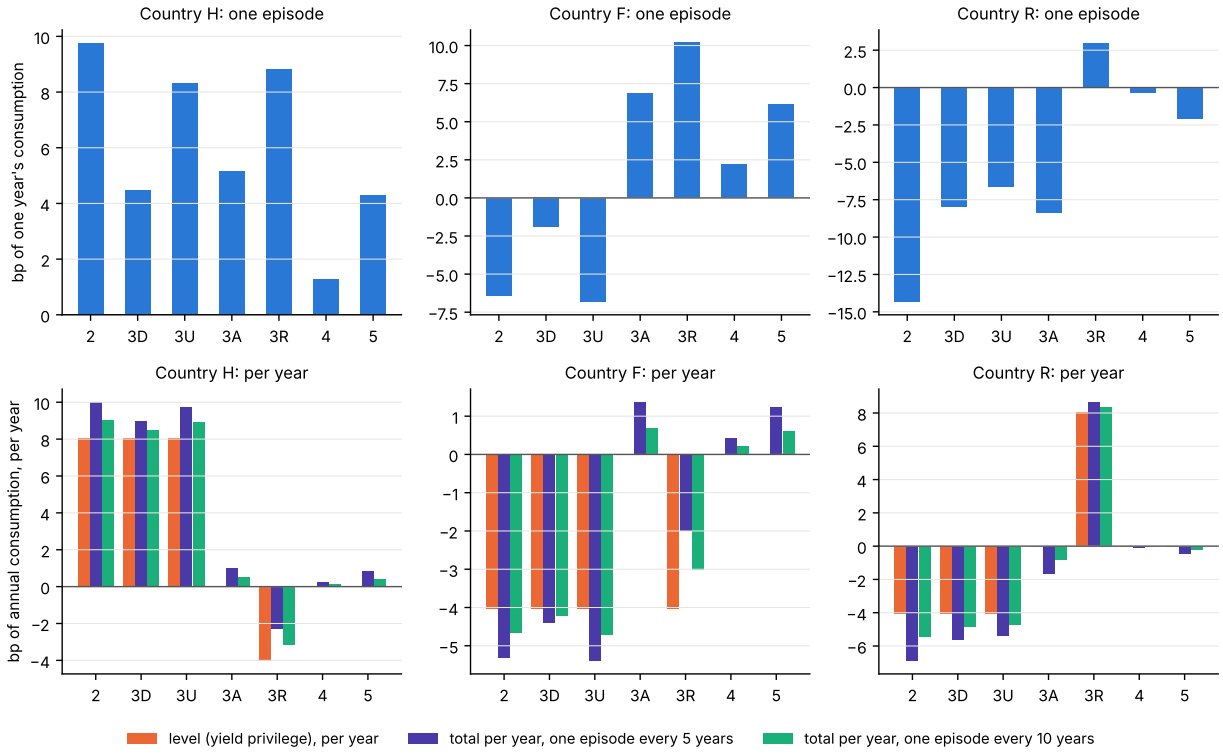
Note. The response under one regime minus the response under a regime that differs from it in that clause only, percentage points. Denomination: Regime 5 minus Regime 4. Unit of account: Regime 5 minus Regime 3A. Backing: Regimes 3A and 3R minus Regime 2.

Figure 3: The redesigned dollar coin (Regime 5)



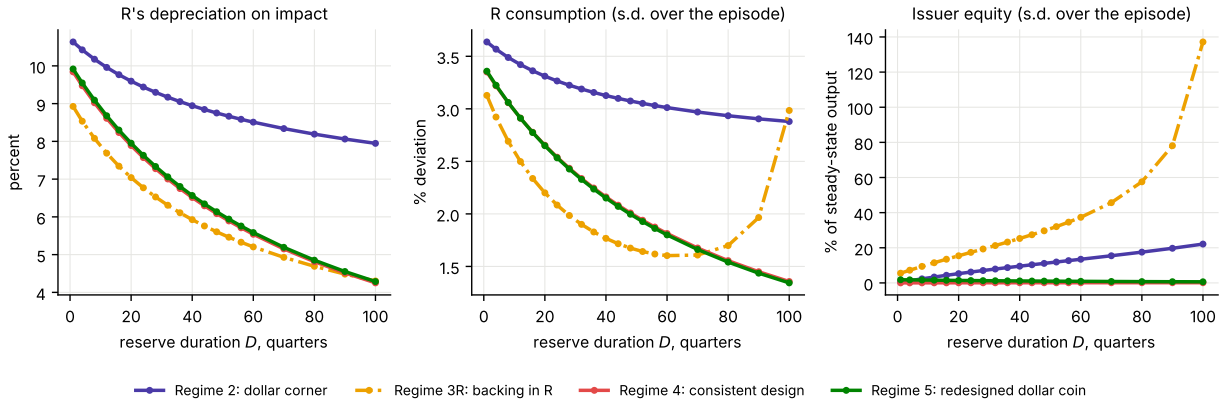
Note. The consistent design (Regime 4) and the dollar corner (Regime 2), minus the response with no liability (Regime 1). Percentage points; the holdings panel shows each regime's own response.

Figure 4: Households' welfare gains relative to no liability (Regime 1)



Note. Fluctuations in disaster risk. Top: one episode, basis points of one year's consumption. Bottom: per year—the yield privilege, and the total with one episode every five and every ten years.

Figure 5: Reserve duration



Note. R's depreciation on impact and the standard deviation over the episode of R's consumption and of issuer equity (percent of output), as a function of the Macaulay duration of reserves, rebalanced to θ^a every quarter.

Table 1: Governance Rules and the Values Considered

Rule	Model object	Baseline	Alternatives
Capital	$\mathcal{K}$	0	5% and 10% of liabilities
Payout and recapitalization	ϕ_{div}	1 (one quarter)	1/4 (one year), 1/20 (five years)
Cost of transfers	$\Omega(T)$	symmetric	one-sided (recapitalizations only)
Loss allocation	α_j^T	population, λ_j	backing weight, θ_j^a
Hedging	ς	0	1/2, 1
Reserve duration	D_j	one quarter (bills)	one to 25 years
Rebalancing	φ^{reb}	1 (every quarter)	1/2, 1/4, 1/10, 1/50

Note. Each rule is varied one at a time from the baseline governance. The payout rule ϕ_{div} implies a mean time to recover a loss of $1/\phi_{\text{div}}$ quarters. Under the one-sided rule only recapitalizations, $T_{j,t} > 0$, carry an excess burden, averaged over the risk-off and the risk-on episode. Duration and rebalancing are reported for their first-order effects (Section 4.9); their welfare is not, because the second-order approximation is not reliable for the bond-price movements they involve (Appendix C.2).

Table 2: Baseline Calibration (Quarterly)

Parameter	Description	Value	Source / target
$\beta,\ \sigma,\ \nu$	Discount factor, inverse IES, inverse Frisch	0.99, 3, 1	Standard
$\kappa_L,\ \varphi_l$	Liquid assets (quarters of consumption); liquidity-gap weight	1/3, 1/6	$\varphi_l = \kappa_L/2$
η	Composition semi-elasticity	2.0	Eq. (14); Table C.1
$\alpha,\ \mu$	Calvo parameter, elasticity of substitution	0.75, 6	4-quarter price duration; 20% markup
γ	Import share	0.30	
ρ_i, ϕ_π, ϕ_y	Taylor rule	0.85, 1.5, 1.0	Standard
$\psi_b,\ \rho_\varpi$	Financier's cost; persistence of risk-off	0.30, 0.85	
$\bar{\varepsilon}_\varpi$	Size of the risk-off episode	–	10% impact depreciation of R, no liability
$\omega_j,\ \upsilon$	Adoption; U-pricing per unit of adoption	0.50, 0.50	Scenario; U-pricing share $\upsilon\omega_j = 0.25$
λ_j	Population weights	1/3	Symmetric
δ_j	Coupon decay (duration)	0	One-period bills
$\phi_{\text{div}},\ \chi_E,\ \varsigma,\ \alpha_j^T$	Payout, equity feedback, hedge, loss allocation	1, 0, 0, λ_j	Baseline governance (Section 3)
B_R	Jump in R's marginal utility in a disaster	1.375	10.1% fall in consumption; Barro and Ursúa (2008)
b_{cy}	Convenience yield per log point of private debt	126 bp/yr	Midpoint; Krishnamurthy and Vissing-Jorgensen (2012, Table 1)
$\bar{B}^p,\ B^g,\ \varphi^f$	Private and total debt (% of annual GDP); foreign-held share	90, 100, 0.3	
χ_{os}^0	Dollar share of R's external debt	0.52	End-2022; Arslanalp and Tsuda (2014), Pedrono and Ishii (2024)
$\bar{B}_R$	R's gross external debt	1.2	30% of annual GDP
$\kappa_{os} = \zeta_{os}\chi_{os}$	Premium feedback at the dollar corner	0.025	Hofmann et al. (2020), Figs. 3–4 (WP)
$\bar{F},\ \ell$	Default rate, loss given default	3%/yr, 0.62	Bernanke et al. (1999); Hamilton et al. (2007)
τ_T	Share of default losses transferred to R	0.148	Midpoint; Appendix D.8
$\xi_1,\ \xi_2$	Excess burden of transfers to the issuer	0.365, 0	Ballard et al. (1985), midpoint; tax smoothing

Table 3: Regimes

	Denomination θ^d	Unit of account θ^u	Backing θ^a	R's insurance $1-\theta_R^d$
1: No liability		$\omega_j = 0$		–
2: Dollar corner	$(1,0,0)$	$(1,0,0)$	$(1,0,0)$	1
3D: Bundled denomination	λ	$(1,0,0)$	$(1,0,0)$	2/3
3U: Bundled unit of account	$(1,0,0)$	λ	$(1,0,0)$	1
3A: Bundled backing	$(1,0,0)$	$(1,0,0)$	λ	1
3R: Backing in R	$(1,0,0)$	$(1,0,0)$	$(0,0,1)$	1
4: Consistent design	λ	λ	λ	2/3
5: Redesigned dollar coin	$(1,0,0)$	λ	λ	1

Table 4: Impact Responses to a Risk-Off Episode

	1	2	3D	3U	3A	3R	4	5
R exchange rate vs dollar, e_{HR}	−10.00	−10.63	−10.13	−10.61	−9.92	−8.93	−9.84	−9.92
R real exchange rate, tot_{HR}	−9.48	−10.49	−9.99	−10.46	−9.82	−9.07	−9.72	−9.82
F exchange rate vs dollar, e_{HF}	0.23	−0.05	0.38	−0.02	0.14	0.04	0.14	0.14
H–R premium, ψ_{HR}	5.81	5.82	5.93	5.80	5.38	5.79	5.32	5.38
of which footprint, ψ^{sc}_{HR}	0.00	0.01	0.06	−0.01	0.01	1.24	−0.06	0.01
R holdings, s_R	–	5.47	1.70	5.88	6.36	5.61	2.80	6.70
Value of holdings, dS	0.00	0.22	−1.00	0.16	0.46	0.36	−1.04	0.38
Issuer FX revaluation, val_{fx}	0.00	0.00	1.63	0.00	−1.65	−4.51	−0.02	−1.65
R net foreign assets, b_R	2.77	2.73	2.09	2.73	4.19	6.71	3.62	4.19
H CPI inflation, π_H	−1.56	−1.66	−1.46	−2.22	−1.54	−1.37	−1.99	−2.06
R CPI inflation, π_R	3.23	4.81	4.72	4.24	4.48	3.94	4.00	3.96
H consumption, c_H	0.95	1.23	1.09	1.39	1.06	1.04	1.20	1.22
R consumption, c_R	−2.03	−2.76	−2.69	−2.59	−2.65	−2.49	−2.49	−2.49
R consumption, sum over 8 quarters	−3.38	−4.49	−4.49	−4.33	−4.64	−4.65	−4.76	−4.52

Note. Impact responses, in percent (premia in percentage points per quarter; dS, val_{fx} and b_R in percent of quarterly output). A fall in e_{Hk} is a depreciation of currency k against the dollar. Regime 1 has no liability, so s_R is not defined.

Table 5: The Redesigned Dollar Coin: Relief and Exposure

	Impact response				Episode gain relative to Regime 1			
	e_{HR}	s_R	ψ^{sc}_{HR}	val_{fx}	H	F	R	World
4: consistent design	−9.84	2.80	−0.06	−0.02	1.3	2.2	−0.4	1.0
5: backing on the basket	−9.92	6.70	0.01	−1.65	4.3	6.2	−2.1	2.8
5, hedged issuer	−10.68	5.94	−0.01	0.00	7.8	−6.7	−7.6	−2.2
5R: backing in R	−8.95	5.88	1.27	−4.52	8.2	9.6	7.5	8.4
5R, hedged issuer	−10.81	6.07	−0.02	0.00	8.0	−7.7	−9.9	−3.2

Note. Unit of account and backing as stated; denomination on the dollar in every row but the first. Impact responses in percent (ψ^{sc}_{HR} in percentage points per quarter, val_{fx} in percent of quarterly output). Episode gains relative to Regime 1 (no liability), basis points of one year's consumption. Hedged: $\varsigma = 1$. The first row is the consistent design, with denomination on the basket.

Table 6: Channels

	Regime 4, R			Episode gain for R relative to Regime 1							World
	e_{HR}	s_R	cost	2	3D	3U	3A	3R	4	5	5
Baseline	−9.84	2.80	−7.2	−14.3	−8.0	−6.7	−8.4	3.0	−0.4	−2.1	2.8
No convenience yield ($b_{cy} = 0$)	−9.84	2.80	−7.2	−14.0	−7.8	−6.3	−8.4	2.1	−0.4	−2.1	2.8
No insurance motive ($B_R = 1$)	−9.69	−0.39	−3.1	−7.5	−4.8	−0.8	−5.0	0.5	0.9	1.0	1.2
No original sin ($\zeta_{os} = 0$)	−10.02	2.92	−4.6	−17.6	−9.3	−9.4	−9.3	4.9	−0.3	−2.7	3.2
Issuer buffers ($\phi_{\text{div}} = 0.5$, $\chi_E = 0.025$)	−9.84	2.80	−7.2	−13.8	−8.1	−6.2	−8.1	3.0	−0.4	−1.8	2.8

Note. Columns 1–2: impact responses under the consistent design, percent. Column 3: welfare cost of the episode to R under the consistent design. Columns 4–10: gain for R relative to no liability (Regime 1) in the same variant; column 11: the same for the world, Regime 5. Basis points of one year's consumption; the yield privilege is not included.

Table 7: Households' Welfare

	1	2	3D	3U	3A	3R	4	5
A. Fluctuations in disaster risk (basis points of one year's consumption)								
Welfare cost of one episode								
H	−5.3	−15.1	−9.8	−13.6	−10.4	−14.1	−6.6	−9.6
F	−3.9	2.5	−2.0	2.9	−10.8	−14.1	−6.1	−10.1
R	−7.6	6.7	0.4	−0.9	0.8	−10.6	−7.2	−5.5
World	−5.6	−1.9	−3.8	−3.9	−6.8	−12.9	−6.6	−8.4
Gain relative to no liability (Regime 1)								
H	0.0	9.7	4.5	8.3	5.1	8.8	1.3	4.3
F	0.0	−6.4	−1.9	−6.8	6.9	10.2	2.2	6.2
R	0.0	−14.3	−8.0	−6.7	−8.4	3.0	−0.4	−2.1
World	0.0	−3.7	−1.8	−1.7	1.2	7.3	1.0	2.8
of which: disaster-state value of the liability								
H	0.0	0.0	0.5	0.0	0.0	0.0	0.5	0.0
F	0.0	0.0	0.5	0.0	0.0	0.0	0.5	0.0
R	0.0	14.8	5.5	15.0	15.2	15.9	5.7	15.4
of which: disaster-state value of the financier's position (each country, per resident)								
R	0.0	−1.8	−1.1	−1.7	−0.8	0.8	−0.5	−0.8
of which: excess burden of transfers to the issuer (each country)								
H, F, R	0.0	−1.6	0.0	−1.7	0.6	1.8	0.5	0.6
B. Risk-off episodes (basis points of one year's consumption)								
Gain relative to no liability (Regime 1)								
H	0.0	43.4	−21.7	40.4	37.2	83.0	−25.7	36.2
F	0.0	−8.1	−59.8	−7.9	38.1	60.1	−25.8	37.3
R	0.0	181.0	110.9	189.2	174.9	159.3	104.9	181.0
World	0.0	72.1	9.8	73.9	83.4	100.8	17.8	84.8
of which: disaster-state value of households' positions in R's currency								
H	0.0	−1.3	−59.7	−1.3	8.0	26.3	−50.7	8.0
F	0.0	−1.3	−59.7	−1.3	−0.6	0.6	−59.3	−0.6
R	0.0	258.1	167.9	258.3	257.8	256.6	166.9	257.9
C. Per year (basis points of annual consumption, every year; fluctuations)								
Level: yield privilege, relative to Regime 1								
H	0.0	8.1	8.1	8.1	0.0	−4.0	0.0	0.0
F	0.0	−4.0	−4.0	−4.0	0.0	−4.0	0.0	0.0
R	0.0	−4.0	−4.0	−4.0	0.0	8.1	0.0	0.0
Total gain relative to Regime 1, one episode every 5 years								
H	0.0	10.0	9.0	9.7	1.0	−2.3	0.3	0.9
F	0.0	−5.3	−4.4	−5.4	1.4	−2.0	0.4	1.2
R	0.0	−6.9	−5.6	−5.4	−1.7	8.7	−0.1	−0.4
World	0.0	−0.7	−0.4	−0.3	0.2	1.5	0.2	0.6
Total gain relative to Regime 1, one episode every 10 years								
H	0.0	9.0	8.5	8.9	0.5	−3.2	0.1	0.4
F	0.0	−4.7	−4.2	−4.7	0.7	−3.0	0.2	0.6
R	0.0	−5.5	−4.8	−4.7	−0.8	8.4	0.0	−0.2
World	0.0	−0.4	−0.2	−0.2	0.1	0.7	0.1	0.3

Note. Households' welfare in each country, in basis points of one year's consumption; negative costs are gains. Panel A: the average of a risk-off and a risk-on episode of equal size (second-order solution), including the disaster-state value of households' positions in R's currency and the excess burden of transfers to the issuer. The disaster-state value of R's dollar debt is zero over fluctuations and is not shown. Panel B: a risk-off episode alone (first- and second-order terms); the component is the disaster-state value of households' positions in R's currency (the liability, the financier's position and R's dollar debt). Panel C: basis points of annual consumption, every year; the level component is the yield privilege in general equilibrium (equation (C.5)). World is the population-weighted average. The issuer's loss in the disaster state is reported in Table 8.

Table 8: The Issuer's Bill

	Fluctuations in disaster risk				Risk-off episodes			
	Households $\bar{G}$	Issuer $\bar{L}$	Net $\bar{G}-\bar{L}$	Every $G_j \geq 0$	Households $\bar{G}$	Issuer $\bar{L}$	Net $\bar{G}-\bar{L}$	Every $G_j \geq 0$
2: Dollar corner	−3.7	0.4	−4.0	no	72.1	0.4	71.7	no
3D: Bundled denom.	−1.8	0.2	−2.0	no	9.8	−91.0	100.8	no
3U: Bundled unit	−1.7	0.4	−2.1	no	73.9	0.4	73.5	no
3A: Bundled backing	1.2	1.9	−0.7	no	83.4	93.1	−9.7	yes
3R: Backing in R	7.3	5.5	1.8	yes	100.8	279.2	−178.4	yes
4: Consistent design	1.0	0.0	1.0	no	17.8	0.0	17.8	no
5: Redesigned coin	2.8	1.8	0.9	no	84.8	93.1	−8.2	yes
5R: Coin, backing in R	8.4	5.3	3.1	yes	101.9	279.0	−177.1	yes

Note. Per resident of the world, basis points of one year's consumption, relative to no liability (Regime 1). $\bar{G}$: average gain of households (Table 7). $\bar{L}$: the issuer's loss in the disaster state over the episode, including the excess burden ξ_1 of covering it; a negative value is a gain. Every $G_j \geq 0$: no country's households lose relative to no liability.

Table 9: Adoption

	Impact, 2		2: Dollar corner			4: Consistent			5: Redesigned			Privilege
Adoption ω	s_R	π_R	H	F	R	H	F	R	H	F	R	H, per year
0.1	6.2	3.54	0.9	0.0	−0.8	0.2	0.4	0.7	0.6	1.0	1.4	1.5
0.25	5.9	4.01	3.6	−1.4	−4.6	0.6	1.1	0.9	2.0	2.9	1.4	3.9
0.5	5.5	4.81	9.7	−6.4	−14.3	1.3	2.2	−0.4	4.3	6.2	−2.1	8.1
0.75	4.9	5.63	10.1	−10.7	−19.6	0.8	1.9	−1.7	4.1	6.1	−3.8	12.6

Note. Adoption $\omega_j = \omega$ in every country; the U-pricing share is $\upsilon\omega$. Columns 2–3: impact responses at the dollar corner (Regime 2), percent. Columns 4–12: households' episode gains under Regimes 2, 4 and 5 relative to no liability, fluctuations in disaster risk, basis points of one year's consumption. Column 13: H's yield privilege at the dollar corner relative to no liability, in general equilibrium, basis points of annual consumption every year.

Table 10: Governance

		2	3D	3U	3A	3R	4	5
Open position and capital (Proposition 3.1), unhedged issuer								
Open position x_R		0.00	−0.33	0.00	0.33	1.00	0.00	0.33
Capital for safety at every scale, $x_R J$, $J = 0.3$ (%)		0	0	0	10	30	0	10
Episode gain relative to Regime 1 (no liability)								
Baseline governance	H	9.7	4.5	8.3	5.1	8.8	1.3	4.3
	F	−6.4	−1.9	−6.8	6.9	10.2	2.2	6.2
	R	−14.3	−8.0	−6.7	−8.4	3.0	−0.4	−2.1
Recapitalization over a year, $\phi_{\text{div}} = 1/4$	H	9.7	4.6	8.3	4.8	7.6	1.3	3.9
	F	−5.7	−1.8	−6.2	6.5	8.9	2.2	5.8
	R	−13.1	−8.4	−5.5	−7.8	3.0	−0.4	−1.5
Recapitalization over five years, $\phi_{\text{div}} = 1/20$	H	9.8	4.7	8.2	4.6	5.9	1.3	3.7
	F	−4.7	−1.8	−5.1	6.2	7.2	2.2	5.5
	R	−12.2	−9.1	−4.6	−7.4	2.8	−0.4	−1.1
One-sided cost of transfers	H	8.2	−2.5	6.8	−1.1	−11.1	0.5	−1.9
	F	−8.0	−8.9	−8.4	0.6	−9.7	1.5	−0.1
	R	−15.9	−15.0	−8.2	−14.7	−16.9	−1.1	−8.3
Losses allocated by backing weight, $\alpha_j^T = \theta_j^a$	H	−19.6	3.7	−23.1	5.1	−2.0	1.3	4.3
	F	7.1	−1.4	8.8	6.9	−0.9	2.2	6.2
	R	−5.7	−10.4	3.9	−8.4	14.5	−0.4	−2.1
Half hedged, $\varsigma = 1/2$	H	9.7	3.4	8.3	6.7	9.4	1.3	5.7
	F	−6.4	0.7	−6.8	0.6	4.2	2.2	0.1
	R	−14.3	−6.7	−6.7	−11.6	−8.8	−0.4	−4.5
Fully hedged, $\varsigma = 1$	H	9.7	2.8	8.3	9.2	9.2	1.3	7.8
	F	−6.4	2.9	−6.8	−6.3	−7.4	2.2	−6.7
	R	−14.3	−5.7	−6.7	−15.5	−17.9	−0.4	−7.6

Note. Top panel: the issuer's open position in R's currency and the capital, in percent of liabilities, at which by Proposition 3.1 a safe equilibrium exists at every scale, for a disaster that depreciates R's currency by 30 percent. Bottom panel: episode gain relative to Regime 1 (no liability), basis points of one year's consumption. Each rule is varied alone from the baseline governance.

Table 11: Robustness: Endpoints of the Calibrated Ranges

		2	3D	3U	3A	3R	4	5
Baseline	R	−14.3	−8.0	−6.7	−8.4	3.0	−0.4	−2.1
	World	−3.7	−1.8	−1.7	1.2	7.3	1.0	2.8
Disaster: 30% fall in consumption	R	−74.2	−35.8	−61.3	−41.0	18.3	−14.8	−33.6
	World	−5.4	−2.3	−2.3	14.8	32.9	6.7	16.5
Bond-supply slope: low end	R	−14.2	−7.9	−6.5	−8.4	2.5	−0.4	−2.1
	World	−3.7	−1.8	−1.7	1.2	7.4	1.0	2.8
Bond-supply slope: high end	R	−14.5	−8.1	−6.8	−8.4	3.5	−0.4	−2.1
	World	−3.7	−1.8	−1.7	1.2	7.3	1.0	2.8
Dollar share of R's debt: 0.74 (2004)	R	−14.3	−8.0	−6.6	−8.0	3.8	−0.2	−1.8
	World	−3.7	−1.8	−1.7	1.3	7.6	1.1	2.9
Excess burden: 17 cents	R	−13.5	−8.0	−5.8	−8.7	2.1	−0.7	−2.4
	World	−2.8	−1.8	−0.8	0.9	6.4	0.7	2.5
Excess burden: 56 cents	R	−15.2	−8.0	−7.5	−8.1	3.9	−0.1	−1.8
	World	−4.5	−1.8	−2.6	1.5	8.3	1.3	3.1
Excess burden: taxes within the quarter	R	−14.4	−8.2	−6.7	−8.5	1.6	−0.4	−2.3
	World	−3.7	−2.0	−1.7	1.1	6.0	1.0	2.6
No excess burden (lump-sum taxes)	R	−12.7	−8.0	−5.0	−9.0	1.2	−0.9	−2.7
	World	−2.1	−1.8	−0.1	0.6	5.6	0.5	2.2
Disaster state: the liability only	R	−12.6	−6.9	−4.9	−7.6	2.2	0.1	−1.3
	World	−2.2	−0.9	−0.3	1.8	6.6	1.4	3.4

Note. Episode gains relative to no liability (Regime 1), basis points of one year's consumption, with one parameter at a time at the end of its range. Disaster: $B_R = 2.92$ (Barro and Ursúa, 2008, Table 8). Bond-supply slope: 1.3 and 4.5 basis points a year per percent of GDP (Krishnamurthy and Vissing-Jorgensen, 2012, Table 1). Dollar share: $\chi^0_{os} = 0.74$, $\tau_T = 0.206$. Excess burden: $\xi_1 = 0.17$ and $\xi_1 = 0.56$ (Ballard et al., 1985); taxes within the quarter, $\xi_2 = 3.24$; lump-sum taxes, $\xi_1 = \xi_2 = 0$. Disaster state, the liability only: households' welfare without the disaster-state value of their share of the financier's position.

Appendix

Appendix A derives the log-linear system, Appendix B verifies the Blanchard–Kahn conditions, Appendix C defines the welfare measure and reports additional results, Appendix D gives derivations, Appendix E the proofs, and Appendix F states the model in full.

A Log-Linearization from the Theoretical Model

This appendix derives the log-linear system from the nonlinear primitives of Section 2. Sections A.1–A.2 restate the firm and household blocks, whose derivations are carried out largely in the main text, and complete the steps the main text abbreviates; Sections A.3–A.9 derive the international, convenience-yield, issuer, original-sin, currency-market, and aggregation blocks.

A.1 Firms

Nonlinear primitives: production $Y_{j,t}(f) = N_{j,t}(f)$, CES demand (D.36), and the Calvo reset-price problem (D.1), whose first-order condition (D.2) and price index (D.3) are nonlinear in levels. Log-linearizing both around the symmetric zero-inflation steady state and combining gives the New Keynesian Phillips curve (6),

$$\pi_{jd,t} = \beta\,\mathbb{E}_t\pi_{jd,t+1} + \kappa\,mc_{j,t}, \qquad \kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha},$$

with $mc_{j,t} = w_{j,t} - p^d_{j,t}$ left open pending the labor-market equations of Section A.2. The full step-by-step derivation is Section 2.2, equations (D.1)–(F.2).

U-priced goods. A firm that resets its U price at t maximizes $\mathbb{E}_t\sum_k(\alpha\beta)^k\Lambda_{t,t+k}[P^{U*}_{j,t}U_{j,t+k} - (1-\tau)W_{j,t+k}](P^{U*}_{j,t}U_{j,t+k}/P^{dom}_{j,t+k})^{-\mu}Y_{j,t+k}$. The first-order condition sets $P^{U*}_{j,t}$ equal to the markup times a weighted average of expected $(1-\tau)W_{j,t+k}/U_{j,t+k}$; since the bracket is zero at the steady state, the log-linear condition is $p^{U*}_{j,t} = (1-\alpha\beta)\sum_k(\alpha\beta)^k\mathbb{E}_t[mc_{j,t+k} + p^d_{j,t+k} - u_{j,t+k}]$. Writing it recursively, $p^{U*}_{j,t} = (1-\alpha\beta)(mc_{j,t} + p^d_{j,t} - u_{j,t}) + \alpha\beta\mathbb{E}_t p^{U*}_{j,t+1}$, and combining with $p^U_{j,t} = \alpha p^U_{j,t-1} + (1-\alpha)p^{U*}_{j,t}$ exactly as for the Phillips curve above, with $mc_{j,t} + p^d_{j,t} - u_{j,t}$ in place of nominal marginal cost $mc_{j,t} + p^d_{j,t}$, gives $\pi^U_{j,t} = \beta\mathbb{E}_t\pi^U_{j,t+1} + \kappa(mc_{j,t} + p^d_{j,t} - u_{j,t} - p^U_{j,t})$, which is (7) with the gap (F.14). First-differencing the definition of $z_{j,t}$ gives its law of motion.

A.2 Households

Nonlinear primitive: the utility function (8), in expectation over the disaster state of Section 2.3, maximized subject to the liquidity aggregator (9) and the nominal budget constraint of Section 2.3. Log-linearizing the marginal utility of consumption (F.16) gives (D.19); combined with the labor-supply condition (D.14) and the production normalization, this closes $mc_{j,t}$ from Section A.1:

$$mc_{j,t} = \nu\,y_{j,t} + (\sigma + \varphi_l)\,c_{j,t} - \varphi_l\,l_{j,t} + \mathrm{rp}_{j,t},$$

where $\mathrm{rp}_{j,t} = p_{j,t} - p^d_{j,t}$ enters because the household deflates the wage by the CPI and the firm by the price of domestic goods (Section D.3). Log-linearizing the Euler equation (11) gives the consumption path (11).

The insurance value. In the condition for the liability, the covariance term is

$$h_{j,t} = \beta \operatorname{Cov}_t\big(\lambda_{j,t+1}/\lambda_{j,t},\, X_{j,t+1}/X_{j,t}\big).$$

Write the next-period state as a disaster in R's currency, with probability $p_{R,t}$, or no disaster. In a disaster the liability's value in j's currency changes by the factor $1+\pi_j J$, with $\pi_R = 1-\theta^d_R$ and $\pi_H = \pi_F = -\theta^d_R$, and $\lambda_{j,t+1}$ is multiplied by B_j, with $B_H = B_F = 1$. The dynamics are those of the no-disaster path, so the expected return $g_{j,t}$ in the condition for the liability is the no-disaster return; it already carries the drift with which the financier compensates holders of R's currency for the expected loss $\varpi_{R,t} = Jp_{R,t}$. The full expectation adds the disaster-state payoff, weighted by marginal utility in that state: to first order in $p_{R,t}$, and with the no-disaster value of $\lambda_{j,t+1}/\lambda_{j,t}$ equal to $1/(\beta R_{j,t}) \approx 1$ at the steady state, $h_{j,t} = \beta B_j \pi_j \varpi_{R,t}$. The expected loss is counted once, in the drift; net of it, R's households expect the excess return $(B_R - 1)(1-\theta^d_R)\varpi_{R,t}$, and for H and F the drift and the disaster payoff cancel. Covariances arising from the no-disaster fluctuations of the model are second order and are dropped.

Composition. At the steady state both sides of (F.25) and (12) equal $1-\beta$ and $\bar\rho_j = 0$. Taking relative deviations, the money condition gives $\widehat{\mathcal{P}}^L_{j,t} + (\rho_L - 1)mb_{j,t} + (1-\rho_L)l_{j,t} = \widehat{\mathcal{U}}^m_{j,t}$. In the condition for the liability, the insurance value is $h_{j,t}/(1-\beta)$ relative to the steady-state value $1-\beta$ of both sides, so $\widehat{\mathcal{P}}^L_{j,t} + (\rho_L - 1)s_{j,t} + (1-\rho_L)l_{j,t} + h_{j,t}/(1-\beta) = \widehat{\mathcal{U}}^{sc}_{j,t}$. Subtracting gives (D.26); substituting the user costs (D.27) and dividing by $-(1-\rho_L) = -1/\eta_L$,

$$mb_{j,t} - s_{j,t} = -\eta\, g_{j,t} - \frac{\eta_L}{1-\beta} h_{j,t} = -\eta\, g_{j,t} - \eta\, B_j \pi_j\, \varpi_{R,t}, \qquad \eta \equiv \eta_L \frac{\beta}{1-\beta},$$

which is (14) with $H_{j,t}$ from (16).

The composition condition alone leaves $mb_{j,t}$ and $s_{j,t}$ individually undetermined. Closing this requires the scale condition (D.21) used on its own rather than in ratio. Log-linearizing $U_{L,j,t}$ from (F.17) around $\bar C_j = \bar L_j = 1$, using $\chi_L = \kappa_L(1-\beta)$, gives the absolute deviation

$$u_{L,j,t} = \varphi_l\, c_{j,t} - \big(\kappa_L(1-\beta) + \varphi_l\big) l_{j,t}. \tag{A.1}$$

Combined with $u_{C,j,t}$ from (D.19), the shadow price of liquidity (D.20) log-linearizes to

$$\widehat{\mathcal{P}}^L_{j,t} = \frac{u_{L,j,t}}{\kappa_L(1-\beta)} - u_{C,j,t} = \left(\sigma + \varphi_l + \frac{\varphi_l}{\kappa_L(1-\beta)}\right) c_{j,t} - \left(1 + \varphi_l + \frac{\varphi_l}{\kappa_L(1-\beta)}\right) l_{j,t}, \tag{A.2}$$

Log-linearizing the money user cost (F.23) around $R_{j,t} = \beta^{-1} e^{i_{j,t}}$ gives the relative deviation $\widehat{\mathcal{U}}^m_{j,t} = \beta i_{j,t}/(1-\beta)$, while the marginal contribution (D.24) log-linearizes to $(\rho_L - 1)mb_{j,t} + (1-\rho_L)l_{j,t}$,

so that (D.21) becomes

$$\widehat{\mathcal{P}}^{L}_{j,t} = \frac{\beta}{1-\beta} i_{j,t} - (\rho_L - 1) mb_{j,t} - (1-\rho_L) l_{j,t}. \tag{A.3}$$

Equations (A.2) and (A.3) both equal $\widehat{\mathcal{P}}^{L}_{j,t}$. The aggregator (F.20) and the composition condition give $mb_{j,t} - l_{j,t} = -\omega_j(\eta g_{j,t} + H_{j,t})$, so the right-hand side of (A.3) is $\frac{\beta}{1-\beta} i_{j,t} + (1-\rho_L)(mb_{j,t} - l_{j,t}) = \frac{\beta}{1-\beta}\big[i_{j,t} - \omega_j(g_{j,t} + H_{j,t}/\eta)\big]$, using $(1-\rho_L)\eta = \beta/(1-\beta)$; the substitution parameter ρ_L has canceled. Equating with (A.2) and multiplying by $1-\beta$: the coefficient on $c_{j,t}$ is $\varphi_l/\kappa_L + (1-\beta)(\sigma + \varphi_l)$ and the coefficient on $l_{j,t}$ is $-[(1-\beta)(1+\varphi_l) + \varphi_l/\kappa_L] = -\Lambda$; the shadow price of liquidity is measured per unit of liquid assets, $U_{L,j,t}/(\kappa_L U_{C,j,t})$, with κ_L the steady-state value of liquid assets in units of consumption. Solving,

$$\begin{aligned} l_{j,t} &= -\eta_\ell \Big[i_{j,t} - \omega_j \Big(g_{j,t} + \frac{H_{j,t}}{\eta} \Big) \Big] + \eta_c \, c_{j,t}, \\ \eta_\ell &= \frac{\beta}{\Lambda}, \qquad \eta_c = \frac{\varphi_l/\kappa_L + (1-\beta)(\sigma + \varphi_l)}{\Lambda}, \qquad \Lambda = (1-\beta)(1+\varphi_l) + \varphi_l/\kappa_L, \end{aligned}$$

with $mb_{j,t} = l_{j,t} - \omega_j(\eta g_{j,t} + H_{j,t})$ and $s_{j,t} = l_{j,t} + (1-\omega_j)(\eta g_{j,t} + H_{j,t})$ recovered from the aggregator and composition condition. That η_ℓ and η_c do not involve ρ_L—so that $\eta_\ell \neq \eta$ in general—and that the $c_{j,t}$-coefficients of $mb_{j,t}$ and $s_{j,t}$ are identical and hence cancel from their difference, reproducing exactly (14), are both features of the closed-form solution rather than assumptions used to obtain it. Substituting the closed form for $l_{j,t}$ into $mc_{j,t} = \nu y_{j,t} + (\sigma + \varphi_l) c_{j,t} - \varphi_l l_{j,t} + \mathrm{rp}_{j,t}$ above gives (D.42), stated in full in Section D.4.

A.3 International Asset Markets

Nonlinear primitives: the financier's problem (F.35), with first-order condition (F.36), and the external balance sheet (F.33). Log-linearizing the expected excess return around the zero-inflation steady state, $\mathbb{E}_t[R_{k,t} E_{jk,t+1}/E_{jk,t} - R_{j,t}] \approx i_{k,t} + \mathbb{E}_t[e_{jk,t+1} - e_{jk,t}] - i_{j,t}$ (up to the constant $1/\beta$, absorbed into ψ_b), and solving for the level gives

$$e_{jk,t} = \mathbb{E}_t e_{jk,t+1} + i_{k,t} - i_{j,t} - \psi_b \, x_{jk,t}.$$

External balance sheets. In units of H's goods, country j's position evolves as

$$\mathrm{NFA}_{j,t} = S_{j,t-1} G^S_t - \theta^a_j A_{sc,t-1} G^b_{j,t} + F_{j,t-1} G^f_{j,t} + NX_{j,t} + \mathrm{Tot}_{Hj,t} + \lambda_j \mathrm{Div}_t + N_{j,t} + \mathrm{OS}_{j,t} + \tau_j,$$

with $F_{j,t-1} = \mathrm{NFA}_{j,t-1} - S_{j,t-1} + \theta^a_j A_{sc,t-1}$. The gross returns in H's goods are $G^S_t = (X^H_t/X^H_{t-1})(P^d_{H,t-1}/P^d_{H,t})$ on the liability, $G^b_{j,t} = [(1+\delta_j Q_{j,t})/Q_{j,t-1}](E_{Hj,t}/E_{Hj,t-1})(P^d_{H,t-1}/P^d_{H,t})$ on country j's bonds, and $G^f_{j,t} = R_{j,t-1}(E_{Hj,t}/E_{Hj,t-1})(P^d_{H,t-1}/P^d_{H,t})$ on positions with the financier. Purchases of the liability and of reserves do not change $\mathrm{NFA}_{j,t}$: a household that buys the liability, or a government whose bonds the issuer buys, settles through its position with the

financier. The constant τ_j offsets the steady-state flows of interest on the steady-state positions and of seigniorage, which have zero present value for each country (Section C.3). At the steady state the financier's positions are zero, so that $\overline{\mathrm{NFA}}_j = \lambda_j \bar{s}_j - \theta^e_j \bar{s}$ and $\bar{F}_j = (\theta^a_j - \theta^e_j)\bar{s}$. Linearizing, with $\bar{G}^S = 1$ and $\bar{G}^b = \bar{G}^f = \beta^{-1}$, and using the issuer's dividends of Section A.6, gives (18). The liability pays no interest, which gives the term $-(\beta^{-1} - 1)dS_{j,t-1}$; the log deviations of the returns are $\Delta x^H_t - \pi_{Hd,t}$, $r_{j,t} - \pi_{Hd,t}$ with $r_{j,t} = i_{j,t-1} + \dot{e}_{Hj,t} + \beta\delta_j \Delta q_{j,t}$ by (D.48), and $i_{j,t-1} + \dot{e}_{Hj,t} - \pi_{Hd,t}$. Net exports enter at their deviation, since $\overline{NX}_j = 0$. Summing over countries, the payments on reserves and the revaluation of the liability equal the issuer's income, which is either paid out or added to its equity; hence $\sum_j b_{j,t} + E_{sc,t} = 0$ at every date.

A.4 The Contract

Denomination. One unit contains c_k units of currency k, with $c_k \bar{E}_{Hk} = \theta^d_k \bar{X}^H$. Its value in H's currency is $X^H_t = \sum_k c_k E_{Hk,t}$; by the rule for sums, $x^H_t = \sum_k \theta^d_k e_{Hk,t}$. Dividing by $E_{Hj,t}$ gives (2), $d_{j,t} = \sum_k \theta^d_k e_{Hk,t} - e_{Hj,t}$, and the expected one-period appreciation (3), $g_{j,t} = \mathbb{E}_t \Delta d_{j,t+1}$.

Unit of account. The same steps applied to the U give (4), $u_{j,t} = \sum_k \theta^u_k e_{Hk,t} - e_{Hj,t}$, and the coin-per-U rate (F.5).

A.5 The Convenience Yield

Household problem. A household of country j chooses bond holdings $b_{jk,t}$, $k \in \{H, F, R\}$, in addition to the choices of Section A.2. A bond of country k, hedged into j's currency at covered interest parity on policy rates, pays $R^g_{k,t} R_{j,t}/R_{k,t}$ in j's currency per unit of j's currency invested; a deposit pays $R_{j,t}$. Holding $b_{jk,t}$ instead of deposits changes next period's real income by $-(R_{j,t} - R^g_{k,t} R_{j,t}/R_{k,t})\, b_{jk,t} = -R_{j,t}\, \mathrm{CY}_{k,t}\, b_{jk,t}$ and changes this period's resource cost $v_{jk}\Gamma(b_{jk,t})$, which enters the budget constraint as goods. With the budget-constraint multiplier $\Lambda_{j,t}$ and the Euler equation for deposits, $\Lambda_{j,t} = \beta \mathbb{E}_t \Lambda_{j,t+1} R_{j,t}/\Pi_{j,t+1}$, the first-order condition for $b_{jk,t}$ is

$$-v_{jk}\Gamma'(b_{jk,t})\, \Lambda_{j,t} = \beta\, \mathbb{E}_t \Big[\Lambda_{j,t+1} \frac{R_{j,t}}{\Pi_{j,t+1}}\Big] \mathrm{CY}_{k,t} = \Lambda_{j,t}\, \mathrm{CY}_{k,t},$$

which is (D.44): marginal utility cancels, because both the resource saving and the interest forgone are in goods.

Demand and market clearing. With $-\Gamma'(b) = -b_{cy} \log b$, (D.44) gives $b_{jk,t} = \exp(-\mathrm{CY}_{k,t}/b_{cy})$, the same for every holder. Summing over holders and imposing $\sum_j \lambda_j b_{jk,t} = B^p_{k,t} \equiv \bar{B}^p - \theta^a_k A_{sc,t}$ gives (D.45).

Linearization. $d\mathrm{CY}_{k,t} = -b_{cy}\, dB^p_{k,t}/B^p_k$ with $dB^p_{k,t} = -\theta^a_k (dS_t + E_{sc,t})$ and $B^p_k = \bar{B}^p - \theta^a_k \bar{s}$ at the adopted scale, which is (21).

Welfare incidence. A household's net income from its holdings of country k's bonds, relative to deposits, is $\mathcal{V}_{jk,t} = -R_{j,t} \mathrm{CY}_{k,t} b_{jk,t} - v_{jk}\Gamma(b_{jk,t})$, up to a constant. Its total derivative is $d\mathcal{V}_{jk,t} =$

$-R_{j,t}b_{jk,t}\,d\mathrm{CY}_{k,t} - \big[R_{j,t}\mathrm{CY}_{k,t} + v_{jk}\Gamma'(b_{jk,t})\big]db_{jk,t}$, and the term in brackets is zero by (D.44) (with $\beta\bar{R}=1$, to first order). The change in resources spent on collateral services is offset by the change in interest forgone, and holders lose only $\bar{b}_{jk}\,d\mathrm{CY}_{k,t}$ per unit of the policy return. The government saves $B^g_k\,d\mathrm{CY}_{k,t}$ and rebates it to domestic households. Adding the issuer, which holds $\theta^a_k\bar{s}$ and passes its income to countries in the shares λ_j with its payout, the world gains and losses sum to zero, since $B^g_k = \sum_j \lambda_j \bar{b}_{jk} + \theta^a_k\bar{s}$. The part paid by domestic holders cancels nationally, leaving the part paid by non-resident holders, $\varphi^f B^g_k\,d\mathrm{CY}_{k,t}$. Non-resident holders, including the issuer, are distributed in shares λ_j, so country k pays $\lambda_k\sum_m \varphi^f B^g_m\,d\mathrm{CY}_{m,t}$. Combining, with payment one period after purchase and a common debt stock B^g,

$$n^{cy}_{k,t} = \varphi^f B^g\Big[\zeta_{y,k}\theta^a_k - \lambda_k\sum_m \zeta_{y,m}\theta^a_m\Big]\big(dS_{t-1} + E_{sc,t-1}\big),$$

which is (22). The same steps applied to the steady-state increment $\Delta\overline{\mathrm{CY}}_k$ give $\bar{N}_k$. The issuer's reserve income is recorded at policy rates in Section 2.4; the convenience component it forgoes is recorded as part of the transfer above instead, which changes no aggregate.

A.6 The Institutional Issuer

Bond returns. The price (D.46) follows from summing the coupon stream, $Q_{j,t} = \sum_{n\geq 1}\delta_j^{n-1}(1+I_{j,t})^{-n} = 1/(1+I_{j,t}-\delta_j)$, with $I_{j,t}\equiv R_{j,t}-1$. Around $1+\bar{I}=1/\beta$, $dQ/Q = -\beta\,dI/(1-\beta\delta_j)$ and $i_{j,t} = \beta\,dI_{j,t}$, giving (D.47). The gross one-period return of a bond bought at $t-1$ is $(1+\delta_j Q_{j,t})/Q_{j,t-1}$; with $1+\delta_j\bar{Q}_j = \bar{Q}_j/\beta$, its log deviation is $\beta\delta_j q_{j,t} - q_{j,t-1} = i_{j,t-1} + \beta\delta_j\Delta q_{j,t}$, which is (D.48).

Equity. In units of H's goods, $E_{sc,t} = G^A_t A_{sc,t-1} - G^S_t L_{t-1} - \mathrm{Div}_t$, with $A_{sc,t-1} = L_{t-1} + E_{sc,t-1}$, where $G^A_t = \sum_j \theta^a_j G^b_{j,t} + \varsigma\sum_j(\theta^d_j - \theta^a_j)G^f_{j,t}$ is the gross return on reserves rebalanced to the shares θ^a, including the hedge, and G^S_t the revaluation of the liability (Section A.3). At the steady state $\bar{G}^A = 1/\beta$ and the issuer earns the seigniorage $(\beta^{-1}-1)\bar{s}$, which is paid out, so that $\bar{E}_{sc}=0$. Dividends in deviation are $\mathrm{div}_{sc,t} = \phi_{\mathrm{div}}E_{sc,t-1} + (\beta^{-1}-1)dS_{t-1}$. Subtracting the steady state and dropping products of deviations,

$$E_{sc,t} = (\beta^{-1}-\phi_{\mathrm{div}})E_{sc,t-1} + \frac{\bar{s}}{\beta}\big(r^A_t - \pi_{Hd,t}\big) - \bar{s}\big(\Delta x^H_t - \pi_{Hd,t}\big),$$

with $r^A_t = \sum_j \theta^e_j(i_{j,t-1} + \dot{e}_{Hj,t}) + \sum_j \theta^a_j\beta\delta_j\Delta q_{j,t}$ and $\Delta x^H_t = \theta^d_F\dot{e}_{HF,t} + \theta^d_R\dot{e}_{HR,t}$. Collecting terms gives (24)–(25).

Transfers. The issuer's payout is received in shares λ_j, giving (F.48); its income is paid by the countries whose bonds it holds, as part of their external positions (Section A.3). Reserves earn the policy rate; the convenience yield forgone on the issuer's own holdings is included in the transfer of Section A.5, which is distributed in the same shares λ_j as dividends.

A.7 Peripheral External Debt and Original Sin

Nonlinear primitives: the dollar share of R's external debt (26), $\chi_{os}(\theta_R^a) = \chi_{os}^0 - \theta_R^a \bar{s}/\bar{B}_R$, which follows from the one-for-one refinancing of dollar debt with the issuer's steady-state holdings of R-currency bonds; the value of the dollar debt in R's output (F.50), $\Gamma_{R,t} = \chi_{os}\bar{B}_R/\mathrm{Tot}_{HR,t}$; and the default threshold $\bar{\omega}_t = [(1-\chi_{os})\bar{B}_R + \Gamma_{R,t}]/\bar{A}_R$ and the creditors' loss $d_t = F(\bar{\omega}_t) - (1-\mu^{bk})G(\bar{\omega}_t)/\bar{\omega}_t$ of (D.49), with $\log \omega_i \sim N(-\sigma_\omega^2/2, \sigma_\omega^2)$; and the financier's problem in the H–R market, (D.50). The first-order condition is

$$\mathbb{E}_t\left[R_{R,t}\big(1-d_t\big)\frac{E_{HR,t+1}}{E_{HR,t}} - R_{H,t}\right] = \psi_b\, x_{HR,t}. \tag{A.4}$$

Burden. Taking logs of (F.50) around $\bar{\Gamma}_R = \chi_{os}\bar{B}_R$, with $\overline{\mathrm{Tot}}_{HR} = 1$, gives $\hat{\Gamma}_{R,t} = -\mathrm{tot}_{HR,t}$ exactly, since $\chi_{os}\bar{B}_R$ is a constant.

Default threshold. Since $D_{R,t} = (1-\chi_{os})\bar{B}_R + \chi_{os}\bar{B}_R/\mathrm{Tot}_{HR,t}$ and $\bar{D}_R = \bar{B}_R$, a first-order expansion gives $d\log\bar{\omega}_t = d\log D_{R,t} = \chi_{os}\hat{\Gamma}_{R,t} = -\chi_{os}\mathrm{tot}_{HR,t}$: only the dollar share of the debt is revalued.

Expected loss. With $F(\omega) = \Phi\big((\log\omega + \sigma_\omega^2/2)/\sigma_\omega\big)$ and $z = (\log\bar{\omega} + \sigma_\omega^2/2)/\sigma_\omega$, $\bar{\omega}F'(\bar{\omega}) = \phi(z)/\sigma_\omega$ and $G(\bar{\omega}) = \Phi(z - \sigma_\omega)$; since $G'(\omega) = \omega F'(\omega)$,

$$d_t - \bar{d} \approx \left[\mu^{bk}\bar{\omega}F'(\bar{\omega}) + (1-\mu^{bk})\frac{G(\bar{\omega})}{\bar{\omega}}\right] d\log\bar{\omega}_t = -\chi_{os}\,\zeta_{os}\,\mathrm{tot}_{HR,t}, \tag{A.5}$$

which is (D.51). Repayment capacity is a stock of assets valued in R's output, so the threshold moves only with the terms of trade. If it were instead R's real income, the threshold would also fall with R's output, adding a term in it to the premium with the same coefficient ζ_{os}; we abstract from that channel, which would strengthen the amplification in recessions.

Transfers. Default losses $d_t\bar{B}_R$ fall on creditors, the financier in H. The fixed coupon prices the steady-state loss $\bar{d}\bar{B}_R$, so what enters the external accounts is the deviation, $(d_t - \bar{d})\bar{B}_R = -\kappa_{os}\bar{B}_R\,\mathrm{tot}_{HR,t}$. A share τ_T of it (D.52) reduces H's net claim on R in (18); the rest is spent on bankruptcy and subtracted from R's output.

Excess return. Log-linearizing the gross expected excess return in (A.4) around the zero-inflation steady state, as in Section A.3, and using $\log(1-d_t) \approx -d_t$,

$$\mathbb{E}_t\left[R_{R,t}(1-d_t)\frac{E_{HR,t+1}}{E_{HR,t}} - R_{H,t}\right] \approx i_{R,t} + \mathbb{E}_t e_{HR,t+1} - e_{HR,t} - i_{H,t} - \big(d_t - \bar{d}\big),$$

where the steady-state loss $\bar{d}$ is absorbed into the steady-state premium, which is normalized to zero in deviations as for every other component of $\psi_{HR,t}$. With the financier's position from Section A.8,

and adding the equity term and the risk-off disturbance, rearranging in the exchange-rate level gives

$$e_{HR,t} = \mathbb{E}_t e_{HR,t+1} + i_{R,t} - i_{H,t} - \psi_{HR,t}, \qquad \psi_{HR,t} = \psi_b b_{HR,t} + \psi^{sc}_{HR,t} - \chi_E E_{sc,t} - \kappa_{os}\,\mathrm{tot}_{HR,t} + \varpi_{R,t}, \tag{A.6}$$

with $\kappa_{os} = \zeta_{os}\chi_{os}(\theta^a_R)$, which is (30) with (29) and (27). The H–F condition (F.59) and the premium $\psi_{HF,t}$ contain no such term, and $\psi_{FR,t} = \psi_{HR,t} - \psi_{HF,t}$ inherits it through triangular consistency. Because χ_{os} is set by the steady-state architecture, κ_{os} is a constant in the log-linear system; the architecture affects it, the business cycle does not.

Why the endogenous position cannot generate the term. Suppose instead that a share χ_{os} of the endogenous bilateral position were dollar-denominated, so that its R-currency value were $\chi_{os}B_{HR,t}\bar{E}_{HR}/E_{HR,t}$. Its deviation from steady state is, to second order,

$$\chi_{os}\Big[\big(B_{HR,t} - \bar{B}_{HR}\big) - \bar{B}_{HR}\, e_{HR,t} - \big(B_{HR,t} - \bar{B}_{HR}\big)e_{HR,t}\Big].$$

At $\bar{B}_{HR} = 0$, the term linear in $e_{HR,t}$ vanishes and the valuation effect reduces to the product $-\chi_{os}b_{HR,t}e_{HR,t}$, which is second order and drops out of the log-linear system. A first-order balance-sheet channel therefore requires a stock that is non-zero at the steady state, which is $\bar{B}_R$ in (F.50).

Accounting of peripheral currency risk. Measured in H's currency, a one-percent depreciation of R's currency at given goods prices reduces the dollar value of R's borrowers' R-currency income relative to their dollar debt by $\chi_{os}\bar{B}_R$; changes the issuer's equity by $(\theta^e_R - \theta^d_R)\bar{s}$, from (25); changes the financier's hedge position by $(\theta^a_R - \theta^e_R)\bar{s}$, from (23); and changes the value of holders' R-currency claims by $\theta^d_R\bar{s}$. The sum is $\chi^0_{os}\bar{B}_R - \theta^a_R\bar{s} + \theta^a_R\bar{s} = \chi^0_{os}\bar{B}_R$, which is Proposition 2.3(ii).

The block adds no variable and no lead or lag. Its effects on the log-linear system are the coefficient κ_{os} on $\mathrm{tot}_{HR,t}$ in the H–R premium, which within the period acts as a coefficient $1 - \kappa_{os}$ on $e_{HR,t}$ in the parity condition, and the default transfer $\kappa_{os}\bar{B}_R\mathrm{tot}_{HR,t}$ in the H–R external position.

A.8 Currency-Market Clearing and the Financial Wedge

A financier holding positions q_k in each currency, with $\sum_k q_k = 0$ and cost $\frac{\psi_b}{2}\sum_k q_k^2$, chooses q to maximize expected excess returns; with multiplier μ^q on the adding-up constraint, its first-order conditions are $\mathbb{E}_t \mathrm{xr}_{k,t} - \mu^q = \psi_b q_k$, so the excess return of currency k over currency j is $\psi_b(q_k - q_j)$. Under (A1)–(A4) of Section 2.6, the liability's net addition to the supply of currency-k assets is $Z_{k,t} = S_{k,t} - \theta^e_k A_{sc,t}$, with $A_{sc,t} = L_t + E_{sc,t}$ and $L_t = \sum_k S_{k,t}$, and the financier holds $x^{sc}_{jk,t} = Z_{k,t} - Z_{j,t}$. At the steady state $\bar{S}_k = \lambda_k\bar{s}_k$ and $\bar{A} = \bar{s}$, giving (F.53). The value of residents' holdings in H's goods is $S_{k,t} = \lambda_k\bar{s}_k\,(s^R_{k,t}/\bar{s}_k)P_{k,t}E_{Hk,t}/P^d_{H,t}$, so that $dS_{k,t} = \lambda_k\bar{s}_k(s_{k,t} + \mathrm{rp}_{k,t} + \mathrm{tot}_{Hk,t})$, and $dA_{sc,t} = dS_t + E_{sc,t}$; hence

$$x^{sc}_{jk,t} = dS_{k,t} - dS_{j,t} - (\theta^e_k - \theta^e_j)(dS_t + E_{sc,t}).$$

In terms of external positions, the financier's position in currency k is $q_{k,t} = -F_{k,t} + \varsigma(\theta^a_k - \theta^d_k)A_{sc,t} = S_{k,t} - \theta^e_k A_{sc,t} - \mathrm{NFA}_{k,t}$ by (F.33), so that $x_{jk,t} = q_{k,t} - q_{j,t} = x^{sc}_{jk,t} + b_{jk,t}$ in deviations, and $\bar{q}_k = 0$. Applying the financier's pricing rule $\psi = \psi_b x$ gives (F.54)–(28). The capacity term $-\chi_E E_{sc,t}$ follows from replacing ψ_b by $\psi_b/(1 + \chi'_E E_{sc,t}/\bar{E})$ and log-linearizing. Adding the difference in external positions $\psi_b b_{jk,t}$, the original-sin term of Section A.7, and the risk-off disturbance gives (F.56)–(29), which enter the parity conditions (F.59)–(30). Substituting these into holders' return (3) gives $\mathbb{E}_t \dot{e}_{Hk,t+1} = \psi_{Hk,t} - (i_{k,t} - i_{H,t})$, which is used in the proof of Proposition 2.4.

A.9 Market Clearing and Aggregation

Nonlinear primitive: the consumption basket (D.28), a Cobb-Douglas aggregate of the domestic bundle, itself the CES aggregate of local-priced and U-priced varieties, and of the local-priced goods of the two trading partners. Taking logs and first-differencing gives (D.29)–(D.31), exactly for the Cobb-Douglas aggregator and to first order for the CES domestic bundle, and (F.64) for the price of consumption relative to domestic goods. Household j spends $(1-\gamma)P_{j,t}C_{j,t}$ on the domestic bundle and $(\gamma/2)P_{j,t}C_{j,t}$ on the goods of each partner, whose prices are $E_{jk,t}P^d_{k,t}$. Demand for country j's output is therefore $Y_{j,t} = (1-\gamma)C_{j,t}P_{j,t}/P^{dom}_{j,t} + \sum_{k\neq j}(\gamma/2)C_{k,t}P_{k,t}/(E_{kj,t}P^d_{j,t})$, and its net exports, in units of its local-priced goods, are exports less $\gamma C_{j,t}P_{j,t}/P^d_{j,t}$. Since $p_{j,t} - p^{dom}_{j,t} = \mathrm{rp}_{j,t} + \Upsilon_j z_{j,t}$ and $p_{k,t} - e_{kj,t} - p^d_{j,t} = \mathrm{rp}_{k,t} + \mathrm{tot}_{jk,t}$, linearizing around balanced trade gives (32) and (F.66). Valued in a common unit, the exports of each country to another are that country's imports, so $\sum_j nx_{j,t} = 0$. The Taylor rule (F.67) is posited in log-linear form and carries no shock term.

B Blanchard–Kahn Conditions

Because the full equilibrium system contains static relations, its lead matrix is singular and determinacy is assessed using the generalized eigenvalue decomposition. The system has seventeen forward-looking variables: consumption $\{c_H, c_F, c_R\}$ and liquidity $\{l_H, l_F, l_R\}$ from the Euler equations; the two independent exchange rates $\{e_{HF}, e_{HR}\}$, which also enter the expected return on the liability (3); domestic inflation $\{\pi_{Hd}, \pi_{Fd}, \pi_{Rd}\}$ and U-price inflation $\{\pi^U_H, \pi^U_F, \pi^U_R\}$ from the Phillips curves; and CPI inflation $\{\pi_H, \pi_F, \pi_R\}$ through the Euler equations. Predetermined states are the lagged interest rates, the terms of trade tot_{HF} and tot_{HR}, which are defined on price levels (D.32), the external positions b_H and b_F (that of R follows from $\sum_j b_j + E_{sc} = 0$), issuer equity E_{sc}, lagged holdings, which enter the convenience-yield transfer in (18), the price gaps z_j of (F.14), and the risk-off disturbance. The convenience yield, the insurance motive, and original sin add no forward-looking variable. Under inflation targeting, price levels, and with them the levels of the two independent nominal exchange rates, are not stationary: the companion form has two unit eigenvalues, while every real variable, including the terms of trade, is stationary. As is standard, the unit eigenvalues are counted among the stable ones. No real outcome depends on the levels of nominal exchange rates: they enter the log-linear system only through their changes, in the parity conditions, the contract's values (2)–(4), and the terms of trade. At the baseline calibration, the

number of generalized eigenvalues of the companion form on or inside the unit circle equals the number of state variables under each regime of Section 4.1, so the Blanchard–Kahn conditions hold and the equilibrium is unique; the U-price block is anchored by firms' costs through the gaps z_j. Stationarity of external positions follows from $\psi_b > 0$ (Schmitt-Grohé and Uribe, 2003). With original sin, the order condition holds for $\kappa_{os}(\theta_R^a)$ below the architecture-dependent threshold reported in Section 4.10.

C Welfare

This appendix defines the welfare measure of Section 4. Welfare is computed from the second-order solution of the nonlinear model (Section C.1), for a single episode (Section C.2), and complemented by a level component (Section C.3). Section C.4 explains why the measure is not a quadratic loss evaluated on the first-order solution, and Section C.5 states a non-separability result.

C.1 The Nonlinear Model

The replication files contain the nonlinear model of which the log-linear system of Appendix A is the exact first-order approximation: in every architecture of Section 4, the impulse responses of the 86 variables common to both agree to within 10^{-11}. Every block of Section 2 is entered in exact form: period utility,

$$u_{j,t} = \frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma} - \frac{\varphi_l}{2}\left(C_{j,t} - L_{j,t}\right)^2 - \frac{\varphi_n}{1+\nu} N_{j,t}^{1+\nu} + \chi_L \log L_{j,t} + \beta B_j \pi_j \varpi_{R,t}\, \kappa_L \omega_j \tilde{s}_{j,t}, \tag{C.1}$$

whose last term is the disaster-state value of the liability (17); the CES liquidity aggregator (9) with $\rho_L = 1 - 1/\eta_L$ and the exact first-order conditions for money and for the liability; the Euler equation; Calvo price setting in recursive form for local-currency and U-priced producers, with the price dispersion of each group, the CES index of the domestic bundle, and labor $N_{j,t}$ equal to the output of each group weighted by its dispersion; the contract's bundles as sums of currencies, $X_t^H = \sum_k c_k E_{Hk,t}$ and $U_{H,t} = \sum_k \theta_k^u E_{Hk,t}$; the Cobb-Douglas consumption basket, which gives the CPI, the terms of trade, and the demand for each country's output; marginal cost as the real wage in producer prices; the parity conditions in gross returns; bond prices; and the external balance sheets of countries, the issuer, and the financier in units of H's goods, with gross returns on holdings of the liability, on reserves, and on positions with the financier (Section 2.4). The convenience yield and original sin are entered as in Appendix A. Labor is subsidized at rate $\tau = 1/\mu$, so that $\varphi_n = 1$ and the steady-state allocation of labor is efficient; liquidity carries the user cost $1 - \beta$, so the steady state as a whole is not. Welfare is the recursion $\mathcal{W}_{j,t} = u_{j,t} + \beta \mathbb{E}_t \mathcal{W}_{j,t+1}$.

C.2 The Episode and the Measure

The economy starts at the non-stochastic steady state, receives one innovation $\varepsilon_{\varpi_R,0} = \pm\bar{\varepsilon}$, and no further shock. In the second-order decision rule, $\mathcal{W}_{j,0}(\varepsilon) = \bar{\mathcal{W}}_j + \frac{1}{2} g_{\sigma\sigma} + g_u \varepsilon + \frac{1}{2} g_{uu} \varepsilon^2$, so the average

welfare effect of a risk-off episode and a risk-on episode of equal size, relative to no episode—the welfare effect of fluctuations in disaster risk—is

$$\Delta \mathcal{W}_j = \tfrac{1}{2}\, g_{uu}\, \bar{\varepsilon}^2. \tag{C.2}$$

The first-order term cancels between the two episodes, and the risk correction $g_{\sigma\sigma}$, common to all three paths, drops out. This is the standard treatment of welfare at second order (Woodford, 2003; Galí, 2015): (C.2) is also the expected effect of a mean-zero innovation with second moment $\bar{\varepsilon}^2$, and the one-episode number should be read as the average effect of a risk-off and an equally large risk-on episode. With $\bar{C}_j = 1$ per quarter and $\bar{U}_{C,j} = 1$, a one-time transfer of Δ units of consumption at $t = 0$ raises welfare by Δ, so the cost of the episode in units of one year's steady-state consumption is $-\Delta\mathcal{W}_j/4$; Section 4 reports it in basis points. The size $\bar{\varepsilon}$ depreciates R's currency by 10 percent on impact in the absence of the liability. The cost of an episode is exactly quadratic in $\bar{\varepsilon}$.

Households' positions in the disaster state. The dynamics are those of the path on which no disaster occurs, and that path carries the drift with which the financier prices the expected loss on R's currency. A disaster revalues every cross-border position in R's currency. Welfare is that of households, so it counts the disaster-state value of every position that households hold, directly or through the institutions they own, weighted by their marginal utility in that state, B_j:

(i) their holdings of the liability, the term of (C.1), which is part of period utility because it enters households' choice of holdings;

(ii) their share λ_j of the financier's position in R's currency, $q^f_{R,t}$, whose profits and losses reach households in population shares, $\beta B_j \varpi_{R,t}(-\lambda_j q^f_{R,t})$;

(iii) R's dollar debt: in a disaster the excess defaults cost R's creditors in H $\kappa_{os}\bar{B}_R$ per unit of the jump, of which a share τ_T reaches R's borrowers, $-\beta\varpi_{R,t}\kappa_{os}\bar{B}_R$ for H and $\beta B_R \varpi_{R,t}\tau_T\kappa_{os}\bar{B}_R$ for R.

Terms (ii) and (iii) enter no first-order condition, so they are added to welfare as present values computed from the same second-order solution. Term (iii) is a constant times the disaster risk, which has mean zero over an episode; it matters only for risk-off episodes.

The issuer's disaster-state loss. The issuer's open position loses $\theta^e_R A_t - \theta^d_R S_t$ per unit of the jump, with $A_t = \bar{s} + dS_t + E_{sc,t}$ and $S_t = \bar{s} + dS_t$. The issuer is not part of households' welfare: by design it does not maximize profits, and its balance sheet absorbs losses. Its loss is reported separately, as the bill its capital and its members' backstop must cover (Proposition 3.1), including the excess burden ξ_1 of the taxes that would cover it:

$$\bar{L} = (1+\xi_1)\,\mathbb{E}\sum_t \beta^{t+1}\varpi_{R,t}\big(\theta^e_R A_t - \theta^d_R S_t\big), \tag{C.3}$$

per resident of the world. Comparing $\bar{L}$ with the average household gain $\bar{G}$ answers whether

a design creates more welfare than the issuer's losses cost. Both are measured relative to the economy without the liability, whose issuer's loss is zero: if $\bar{G} > \bar{L}$, some sharing of the issuer's loss among the members makes every country gain, and if in addition every country's household gain is non-negative, no member needs to be compensated to join.

Risk-off episodes. The second measure in Section 4.6 is the effect of a risk-off episode alone, $g_u\bar{\varepsilon} + \frac{1}{2}g_{uu}\bar{\varepsilon}^2$. Its first-order term does not cancel. It is dominated by the disaster-state value of positions in R's currency held without compensation: on the no-disaster path the premium on R's currency rises by less than the expected disaster loss, because in a risk-off episode the financier is short R's currency, so whoever holds R's currency in the episode bears an expected loss that the premium does not cover.

Section 4.6 reports these contributions separately for every welfare number. The replication code also reports the contributions of consumption, the liquidity gap, labor and liquidity services, which with the disaster-state terms add up to the welfare cost of the episode.

Episodes and the yield privilege. The level component of Section C.3 is a permanent flow, while (C.2) is the effect of one episode. Suppose that episodes of size $\bar{\varepsilon}$ arrive independently with probability p per quarter, each risk-off or risk-on with equal probability. To second order the expected discounted effect is the sum of the effects of individual episodes, whose cross terms have zero expectation, so the equivalent permanent change in quarterly consumption is p times the one-episode effect. Writing $N = 1/(4p)$ for the average number of years between episodes, the gain per year, as a share of annual consumption, is

$$\Delta CE_j^{\text{total}}(N) = \frac{\Delta EC_j}{N} + \Delta CE_j^{\text{level}}, \tag{C.4}$$

where ΔEC_j is the one-episode gain. Section 4.6 reports $N = 5$ and $N = 10$.

Accuracy. The second-order approximation is accurate when the episode moves the variables that enter utility and the budget constraints by moderate amounts. That holds for every architecture with the baseline one-period reserves. With long-duration reserves, an episode of the size studied here moves bond prices by tens of percent, and their convexity dominates the second-order terms; Section 4.9 therefore reports first-order responses, not welfare, for the duration of reserves.

C.3 The Level Component

The architecture also changes steady-state flows between countries. The convenience yield gives country k the permanent transfer $\bar{N}_k$, equation (22). A permanent transfer is not consumed one for one. In the new steady state the external positions are unchanged, so country k runs the trade balance $nx_k = -\bar{N}_k$; with home bias in consumption, demand shifts toward the goods of the country that receives the transfer, its terms of trade improve, and, through the wealth effect on labor supply, it produces less. With an efficient steady-state allocation of labor ($u_C\bar{C} = v'(\bar{N})\bar{N} = 1$)

and $\chi_L = \kappa_L(1-\beta)$, the first-order change in period utility of country k is

$$\Delta CE_k^{\text{level}} = c_k - y_k + \chi_L l_k = \bar{N}_k - \text{rp}_k + \chi_L \eta_c c_k, \tag{C.5}$$

where c_k, y_k, rp_k and $l_k = \eta_c c_k$ are the steady-state deviations that solve goods-market clearing (32), $mc_k = 0$, and $nx_k = -\bar{N}_k$ for $k = H, F$; the third trade balance follows. The first term is the transfer, the second the terms-of-trade gain, the price of domestic output relative to the consumption bundle, and the third the value of the extra liquidity, which is small. The terms-of-trade terms sum to zero across countries, as do the transfers: the yield privilege redistributes, it does not create. Neglected are the steady-state positions with the financier that clear the currency markets when holdings of the liability scale with consumption; their income is of order $\beta^{-1} - 1$ times those holdings (`ge_privilege_1013_8.py`).

This is Triffin's original observation in a different form. The country whose bonds back the world's reserve asset runs a permanent external deficit, $nx_k = -\bar{N}_k$, financed by the demand for its bonds, and consumes more than it produces; the privilege it draws from issuing the reserve asset is the counterpart of that deficit, and is larger than the interest saving alone because the deficit also improves its terms of trade.

Two other steady-state flows depend on the architecture but have zero present value for each country, and are not part of the level component. First, a country whose bonds the issuer holds pays interest on them, at the policy rate net of the convenience yield. It received the proceeds when the issuer bought the bonds, and with $\beta\bar{R} = 1$ the present value of the interest equals those proceeds: the flow is the service of a debt, not a transfer. Second, the issuer's steady-state income is rebated to countries in the population shares λ_j, which returns to each country, in present value, what its residents paid for their holdings of the liability; under symmetric adoption no seigniorage is transferred across countries. Both flows would matter under a different ownership of the issuer. An issuer owned by the reserve-currency country would keep the interest on reserves financed by liabilities held abroad, $(\beta^{-1} - 1)\bar{s}(1 - \bar{h}_H)$ per quarter, which is the seigniorage form of the exorbitant privilege; the multilateral issuer studied here rebates it. In the nonlinear model, both steady-state flows are offset by constant lump-sum transfers, so that every architecture shares the same non-stochastic steady state.

The excess burden of transfers to the issuer. The issuer's payout in excess of steady-state seigniorage is returned to members in shares α_j^T, and the transfer from each resident of j to the issuer is $T_{j,t} = -\alpha_j^T \text{div}_{sc,t}$ in units of quarterly output. Its excess burden $\Omega(T_{j,t}) = \xi_1 T_{j,t} + \frac{\xi_2}{2} T_{j,t}^2$ (Section 3.2) is added to the cost of the episode. Under the symmetric rule, its expected present value is $\xi_1 \mathbb{E}\sum_t \beta^t T_{j,t} + \xi_2 \mathbb{E}\sum_t \beta^t T_{j,t}^2/2$, computed, like welfare, from the second-order solution, in which both terms are of second order. Under the one-sided rule only recapitalizations cost, and the average over the risk-off and the risk-on episode is $\frac{1}{2}\xi_1 \sum_t \beta^t |T_{j,t}| + \frac{1}{2}\xi_2 \mathbb{E}\sum_t \beta^t T_{j,t}^2/2$, with $T_{j,t}$ from the first-order solution: the linear term is now of first order in the size of the episode. The

quadratic term is of second order under both rules.

C.4 Why Not a Quadratic Loss

A common alternative evaluates a quadratic approximation of utility on the first-order solution. It is valid when the terms of the expansion of utility that are linear in endogenous variables can be eliminated, typically by an efficient steady state and a goods-market identity (Woodford, 2003; Galí, 2015). With trade derived from preferences, consumption and output differ, to first order, by net exports, the terms of trade, and the gap between the prices of the two pricing groups, and the second-order means of these terms depend on the architecture; so do those of the external accounts. None can be computed from the first-order solution. The second-order solution of the nonlinear model includes them all.

C.5 Welfare Non-Separability

Welfare is not separable in the design margins, even though the unit of account is orthogonal to the issuer's balance sheet and to the digital-liability premia (Proposition 2.5). Orthogonality of the equilibrium system is not separability of the welfare function built from it: denomination and unit of account both move objects that enter welfare—denomination through holders' returns and the path of exchange rates, the unit of account through U-price inflation and the price gap $z_{j,t}$—and the gap $z_{j,t}$ depends on both.

Proposition C.1 (Welfare Non-Separability)**.** *In the quantitative model, welfare is non-separable in denomination and unit-of-account weights: the welfare effect of bundling the unit of account depends on denomination, and the welfare effect of bundling denomination depends on the unit of account. The design margins therefore cannot, in general, be evaluated one at a time.*

The channels behind this result are set out in Appendix E.2. At the calibration of Section 4, with dollar backing, bundling denomination and the unit of account together gives R a gain of 12.58 basis points of a year's consumption relative to the dollar corner, against 6.37 and 7.68 for bundling each alone, whose sum is 14.05: the interaction, -1.47 basis points, is non-zero but small, which is why the comparisons of Section 4.3, which move one clause at a time, are informative.

C.6 Sensitivity to Substitution and to U-Pricing

Table C.1 repeats the welfare comparison with a composition semi-elasticity η of 1 and 5 and with a U-pricing share per unit of adoption $\upsilon = 0.2$ (Section 4.10).

Table C.1: Sensitivity to η and υ

	s_R on impact		Episode gain for R relative to Regime 1							World
	2	5	2	3D	3U	3A	3R	4	5	5
$\eta = 1$	2.9	3.9	−7.0	−5.0	−0.1	−3.7	2.9	1.4	2.3	2.1
$\eta = 2$, $\upsilon = 0.5$ (baseline)	5.5	6.7	−14.3	−8.0	−6.7	−8.4	3.0	−0.4	−2.1	2.8
$\eta = 5$	12.6	14.8	−34.3	−16.3	−24.3	−21.5	3.4	−5.5	−14.4	4.8
$\upsilon = 0.2$	6.0	7.0	−4.0	−0.1	−0.8	0.5	8.5	3.7	3.2	4.8

Note. Columns 1–2: R's holdings on impact, percent, for an episode that depreciates R's currency by 10 percent without the liability in the same variant. Columns 3–9: gain for R relative to no liability (Regime 1) in the same variant; column 10: the same for the world, Regime 5. Basis points of one year's consumption. η varies with $B_R = 1.375$ fixed; with $\upsilon = 0.2$ a tenth of domestic goods is priced in U.

C.7 Reserve Duration: First-Order Values

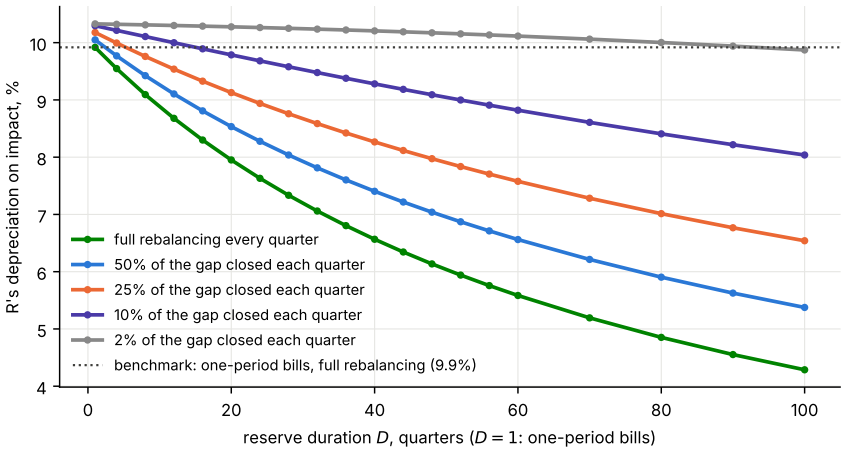
Table C.2 reports the values behind Figure 5, and Figure C.1 the effect of the speed of rebalancing (Section 4.9). They are first-order responses, not welfare (Section C.2).

Table C.2: Reserve Duration

	2	3A	3R	4	5	5R
R's depreciation on impact (%)						
$D = 1$	10.6	9.9	8.9	9.8	9.9	9.0
$D = 20$	9.6	8.0	7.0	7.9	8.0	7.1
$D = 60$	8.5	5.6	5.2	5.5	5.6	5.3
R consumption, s.d. over the episode (%)						
$D = 1$	3.64	3.51	3.13	3.35	3.36	3.01
$D = 20$	3.31	2.77	2.20	2.65	2.65	2.11
$D = 60$	3.01	1.88	1.60	1.81	1.80	1.49
Issuer equity, s.d. over the episode (% of output)						
$D = 1$	0.7	1.8	5.5	0.0	1.8	5.5
$D = 20$	5.3	2.3	15.4	0.1	1.4	14.6
$D = 60$	13.5	2.9	37.4	0.1	0.9	33.7

Note. D is the Macaulay duration of reserves, in quarters. Reserves are rebalanced to the value shares θ^a every quarter. A bounded equilibrium exists up to the perpetuity in every architecture.

Figure C.1: Rebalancing speed



Note. R's depreciation on impact under the redesigned dollar coin (Regime 5), as a function of reserve duration ($D = 1$: one-period bills), when the issuer closes the indicated share of the gap to the value shares θ^a each quarter. Each line should be compared with its own value at $D = 1$.

D Derivations Supporting Section 2

D.1 Optimal Price Setting

A fraction $1 - \alpha$ of firms resets its price each period; the remaining fraction α retains its existing price. A firm resetting in period t chooses $P^*_{j,t}$ to maximize the expected discounted value of profits over the duration of the price:

$$\max_{P^*_{j,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \Lambda_{j,t,t+k} \left[P^*_{j,t} - MC^N_{j,t+k} \right] \left(\frac{P^*_{j,t}}{P^d_{j,t+k}} \right)^{-\mu} Y^d_{j,t+k}, \tag{D.1}$$

where $\Lambda_{j,t,t+k} = \beta^k (C_{j,t+k}/C_{j,t})^{-\sigma} (P^d_{j,t}/P^d_{j,t+k})$ is the household stochastic discount factor between t and $t+k$. The first-order condition gives the standard reset-price formula: the optimal price is a mark-up over a discounted-weighted average of expected future nominal marginal costs,

$$P^*_{j,t} = \frac{\mu}{\mu - 1} \cdot \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \Lambda_{j,t,t+k} Y^d_{j,t+k} \left(P^d_{j,t+k} \right)^{\mu} MC^N_{j,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \Lambda_{j,t,t+k} Y^d_{j,t+k} \left(P^d_{j,t+k} \right)^{\mu}}. \tag{D.2}$$

The elasticity μ governs the desired markup; the Calvo parameter α determines how much weight the firm places on future marginal costs when resetting today.

The domestic price index satisfies

$$\left(P^d_{j,t}\right)^{1-\mu} = \alpha\left(P^d_{j,t-1}\right)^{1-\mu} + (1-\alpha)\left(P^*_{j,t}\right)^{1-\mu}. \tag{D.3}$$

Defining domestic gross inflation $\Pi^d_{j,t} \equiv P^d_{j,t}/P^d_{j,t-1}$, equations (D.2)–(D.3) jointly determine the path of domestic inflation and marginal cost for each country.

D.2 Steady-State Portfolio

We define $\kappa_{S,j}$ as the steady-state scale/normalization parameter that links the institutional adoption weight ω_j to actual steady-state digital-liability balances,

$$\bar{s}_j(\omega_j) = \kappa_{S,j}\,\omega_j, \tag{D.4}$$

together with the money-side convention $\bar{m}_j(\omega_j) = 1-\omega_j$. Thus $\kappa_{S,j}$ is not a behavioral preference parameter and it is not the adoption rate itself: it is the unit-conversion factor between the institutional adoption weight and steady-state digital-liability holdings. In the benchmark case used throughout the remainder of the paper,

$$\kappa_{S,j} = 1 \quad \text{for all } j, \tag{D.5}$$

which gives the particularly clean normalization

$$\bar{m}_j(\omega_j) = 1-\omega_j, \qquad \bar{s}_j(\omega_j) = \omega_j. \tag{D.6}$$

Under this benchmark, ω_j can be read directly as the steady-state digital-liability share of the liquidity portfolio, and the money and digital-liability legs of the aggregator sum to one, $\bar{m}_j(\omega_j) + \bar{s}_j(\omega_j) = 1$. This is the convention that underlies $\bar{L}_j = 1$ in (D.13) above.

An increase in ω_j shifts the steady-state liquid-asset portfolio toward the digital liability while leaving the underlying preferences of the household unchanged.

Aggregate steady-state digital-liability holdings are correspondingly

$$\bar{s} = \sum_{j\in\{H,F,R\}} \lambda_j \bar{s}_j = \sum_{j\in\{H,F,R\}} \lambda_j \kappa_{S,j}\omega_j, \qquad \sum_{j\in\{H,F,R\}} \lambda_j = 1, \tag{D.7}$$

where λ_j denotes country j's exogenous population (or aggregate consumption) share, so that $\bar{s}$ is the population-weighted world average of digital-liability holdings implied by (D.4). This is the single definition of λ_j used throughout the paper; the aggregation rule for the digital liability's architecture margins in Section 2.1 uses this same weight rather than redefining it. Under the

benchmark $\kappa_{S,j} = 1$, this reduces to

$$\bar{s} = \sum_{j \in \{H,F,R\}} \lambda_j \omega_j. \tag{D.8}$$

The steady-state normalization also pins down the liquidity preference parameter. Let $R_{j,t}$ denote the gross nominal return on the domestic safe asset. In the deterministic steady state,

$$\bar{R} = \frac{1}{\beta}. \tag{D.9}$$

At the steady state exchange rates are constant, so the digital liability, like money, has a steady-state gross nominal return of one. Since both liquid assets are then non-interest-bearing in effective terms, their common steady-state user cost relative to the safe asset is

$$\bar{\mathcal{U}}_j^m = \bar{\mathcal{U}}_j^{sc} = 1 - \beta. \tag{D.10}$$

At the steady state, $\bar{C}_j = \bar{L}_j = 1$, so that

$$\bar{U}_{C,j} = 1, \qquad \bar{U}_{L,j} = \chi_L, \tag{D.11}$$

The liquidity-scale condition – formalized below as $\mathcal{P}_{j,t}^L \equiv U_{L,j,t}/U_{C,j,t}$ in (D.20) – per unit of liquid assets, $\bar{U}_{L,j}/\kappa_L = 1 - \beta$, therefore implies

$$\chi_L = \kappa_L(1 - \beta). \tag{D.12}$$

This normalization is invariant across adoption states: ω_j changes the composition of liquid-asset holdings, but not the underlying preference for liquidity services. Equation (D.12) determines χ_L as the value consistent with this steady state; it is not an independent assumption that the liquidity gap must close at any steady state of a liquidity-in-utility model—which is not generally true, since liquidity remains costly even absent inflation—but a calibration target: given $(\sigma, \varphi_l, \beta)$, χ_L is solved so that $\bar{C}_j = \bar{L}_j = 1$ is the model's own steady state, the same normalization technique used for φ_n in (D.18).

Normalization of the aggregator. By construction, $\tilde{m}_{j,t}$ and $\tilde{s}_{j,t}$ equal one at the adoption-specific steady state for *any* choice of $\bar{m}_j(\omega_j), \bar{s}_j(\omega_j)$, so that

$$\bar{L}_j = [(1 - \omega_j) + \omega_j]^{1/\rho_L} = 1 \tag{D.13}$$

holds identically for every value of ω_j. This normalization is therefore a modeling convention rather than an implication of CES symmetry: it fixes the units of $L_{j,t}$ so that adoption changes the *composition* of the portfolio underlying liquidity services without changing the steady-state *level* of those services, which is what permits household preferences to be compared cleanly across adoption

states ω_j.

D.3 Labor Supply and Marginal Cost

The household's intratemporal labor-supply condition equates the real wage, deflated by the consumer price index $P_{j,t}$, to the marginal rate of substitution between labor and consumption:

$$\frac{W_{j,t}}{P_{j,t}} = \frac{\varphi_n N_{j,t}^{\nu}}{U_{C,j,t}}, \tag{D.14}$$

where $P_{j,t}$ is the consumer price index, the price of the bundle the wage buys. This condition links the household block directly to firms' marginal-cost condition.

Consistent with the production normalization in (D.35), $Y_{j,t} = N_{j,t}$. Labor costs are subsidized at a constant rate τ, financed by lump-sum taxes on domestic households, so that real marginal cost is

$$MC_{j,t} = (1-\tau)\frac{W_{j,t}}{P_{j,t}^{d}}, \tag{D.15}$$

where $P_{j,t}^{d}$ is the price of domestic goods, in which firms' revenue is measured. Combining (D.14) and (D.15) gives

$$MC_{j,t} = (1-\tau)\,\frac{P_{j,t}}{P_{j,t}^{d}}\,\frac{\varphi_n N_{j,t}^{\nu}}{U_{C,j,t}}. \tag{D.16}$$

At the symmetric steady state, $\bar{C}_j = \bar{Y}_j = \bar{N}_j = 1$ and $\bar{U}_{C,j} = 1$. The firm's steady-state markup condition,

$$\overline{MC}_j = \frac{\mu - 1}{\mu}, \tag{D.17}$$

where μ is the desired gross price markup defined in Section 2.2 (cross-referenced here for convenience), therefore implies $\varphi_n = (\mu-1)/[\mu(1-\tau)]$. The subsidy is set at $\tau = 1/\mu$, which offsets the monopolistic-competition markup and makes the steady-state allocation of labor efficient, so that

$$\varphi_n = 1. \tag{D.18}$$

Neither τ nor φ_n enters any log-linear equilibrium condition: in logs, τ is a constant that drops out of the deviation of marginal cost. Their only role is in the welfare criterion of Appendix C, where an efficient steady state is what allows the terms linear in endogenous variables to be eliminated.

The liquidity term affects marginal cost through the marginal utility of consumption. Log-linearizing (F.16) around the steady state ($\bar{C}_j = \bar{L}_j = 1$) gives

$$u_{C,j,t} = -(\sigma + \varphi_l)c_{j,t} + \varphi_l l_{j,t}, \tag{D.19}$$

where $u_{C,j,t}$ denotes the log deviation of the marginal utility of consumption. Log-linearizing (D.16)

using $Y_{j,t} = N_{j,t}$ and (D.19), the marginal-cost condition is

$$mc_{j,t} = \nu y_{j,t} + (\sigma + \varphi_l)c_{j,t} - \varphi_l l_{j,t} + \mathrm{rp}_{j,t},$$

with $\mathrm{rp}_{j,t} = p_{j,t} - p^d_{j,t}$, the price of consumption relative to domestic goods. Thus the digital liability affects firms' marginal cost through the equilibrium quantity of liquidity services demanded by households rather than through a separate supply-side restriction. A shift in the composition of the liquid-asset portfolio changes $l_{j,t}$, which changes marginal utility, labor supply, and ultimately real marginal cost.

D.4 The Liquidity-Scale Condition

The two asset-specific first-order conditions also determine the scale of the household's liquidity portfolio. Unlike the composition condition, which is obtained by differencing the two conditions, the scale condition retains the common marginal utility of liquidity.

Define the shadow price of liquidity services as

$$\mathcal{P}^L_{j,t} \equiv \frac{U_{L,j,t}}{U_{C,j,t}}. \tag{D.20}$$

The individual asset FOCs can then be written as

$$\mathcal{P}^L_{j,t} = \frac{\mathcal{U}^m_{j,t}}{\partial L_{j,t}/\partial m^R_{j,t}}, \tag{D.21}$$

$$\mathcal{P}^L_{j,t} = \frac{\mathcal{U}^{sc}_{j,t} - h_{j,t}}{\partial L_{j,t}/\partial s^R_{j,t}}. \tag{D.22}$$

Equations (D.21)–(D.22) make clear that the scale and composition decisions are not separate behavioral assumptions. They are two implications of the same household optimization problem. Their difference determines the composition of the liquid-asset portfolio, while either condition together with the liquidity aggregator determines the desired level of liquidity services.

At the symmetric steady state, the normalized marginal contributions of both liquid assets are equal to one. Hence

$$\bar{\mathcal{P}}^L_j = 1 - \beta, \tag{D.23}$$

which reproduces $\chi_L = \kappa_L(1-\beta)$ from (D.12) above via (D.11), confirming that the liquidity scale is internally consistent with the steady-state portfolio while ω_j remains free to govern its composition.

D.5 Log-Linearizing the Portfolio Conditions

First-order conditions. Both user costs follow from the household's nominal budget constraint,

$$P_{j,t}C_{j,t} + M_{j,t} + X_{j,t}S_{j,t} + B_{j,t} = M_{j,t-1} + X_{j,t}S_{j,t-1} + R_{j,t-1}B_{j,t-1} + W_{j,t}N_{j,t} + \Xi_{j,t},$$

where $S_{j,t}$ is the number of units of the digital liability held, $m^R_{j,t} = M_{j,t}/P_{j,t}$ and $s^R_{j,t} = X_{j,t}S_{j,t}/P_{j,t}$ are real balances, and $\Xi_{j,t}$ collects transfers. With multiplier $\lambda_{j,t} = U_{C,j,t}/P_{j,t}$, the first-order condition for $B_{j,t}$ is the Euler equation, $\lambda_{j,t} = \beta R_{j,t}\mathbb{E}_t\lambda_{j,t+1}$. The condition for $M_{j,t}$ is $U_{L,j,t}(\partial L_{j,t}/\partial m^R_{j,t})/P_{j,t} = \lambda_{j,t} - \beta\mathbb{E}_t\lambda_{j,t+1}$; dividing by $\lambda_{j,t}$ and using the Euler equation gives (F.25) below with $\mathcal{U}^m_{j,t} = 1 - 1/R_{j,t}$. The condition for $S_{j,t}$ is $U_{L,j,t}(\partial L_{j,t}/\partial s^R_{j,t})X_{j,t}/P_{j,t} = \lambda_{j,t}X_{j,t} - \beta\mathbb{E}_t[\lambda_{j,t+1}X_{j,t+1}]$. Dividing by $\lambda_{j,t}X_{j,t}$ and using the Euler equation gives (12) with (F.24), plus the covariance between the household's marginal utility and the liability's return, $h_{j,t} \equiv \beta\,\mathrm{Cov}_t\big(\lambda_{j,t+1}/\lambda_{j,t}, X_{j,t+1}/X_{j,t}\big)$.

For the CES aggregator in (9), the marginal contributions of the two assets are

$$\frac{\partial L_{j,t}}{\partial m^R_{j,t}} = (1-\omega_j)\tilde{m}^{\rho_L-1}_{j,t}\frac{L^{1-\rho_L}_{j,t}}{\bar{m}_j}, \tag{D.24}$$

$$\frac{\partial L_{j,t}}{\partial s^R_{j,t}} = \omega_j\tilde{s}^{\rho_L-1}_{j,t}\frac{L^{1-\rho_L}_{j,t}}{\bar{s}_j}. \tag{D.25}$$

At the steady state, $\bar{\rho}_j = 0$ and both normalized marginal contributions equal one, so (F.25)–(12) hold with $\bar{\mathcal{U}}^m_j = \bar{\mathcal{U}}^{sc}_j = 1-\beta$ for every ω_j (equation (D.10)). Let $\widehat{X} \equiv dX/\bar{X}$ denote a relative deviation. Log-linearizing (F.25) and (12) around this steady state, the marginal contributions deviate by $(\rho_L-1)mb_{j,t} + (1-\rho_L)l_{j,t}$ and $(\rho_L-1)s_{j,t} + (1-\rho_L)l_{j,t}$, the shadow price of liquidity $\mathcal{P}^L_{j,t} \equiv U_{L,j,t}/U_{C,j,t}$ deviates by $\widehat{\mathcal{P}}^L_{j,t}$ in both, and the insurance value, which is zero at the steady state, contributes $h_{j,t}/(1-\beta)$ to the second. Subtracting the two conditions eliminates the common shadow price and gives

$$-(1-\rho_L)\,(mb_{j,t} - s_{j,t}) - \frac{h_{j,t}}{1-\beta} = \widehat{\mathcal{U}}^m_{j,t} - \widehat{\mathcal{U}}^{sc}_{j,t}. \tag{D.26}$$

Using the steady-state return $\bar{R} = 1/\beta$, and writing $1/R_{j,t} \approx \beta(1-i_{j,t})$ and $\mathbb{E}_t[X_{j,t+1}/X_{j,t}] \approx 1+g_{j,t}$, the user costs deviate by $d\mathcal{U}^m_{j,t} = \beta i_{j,t}$ and $d\mathcal{U}^{sc}_{j,t} = \beta(i_{j,t} - g_{j,t})$, so that

$$\widehat{\mathcal{U}}^m_{j,t} - \widehat{\mathcal{U}}^{sc}_{j,t} = \frac{\beta}{1-\beta}\,g_{j,t}. \tag{D.27}$$

D.6 CPI Inflation and Net Exports

The CPI inflation rate $\pi_{j,t}$ used throughout the household block (the Euler equation (11) and the Taylor rule below) has not yet been related to domestic-goods inflation $\pi_{jd,t}$. It is derived here from the household's consumption basket over domestic and imported varieties. Each household consumes a Cobb-Douglas aggregate of the domestic bundle and imports from the other two countries, with a total import share γ split equally between trading partners. Within the domestic bundle, a share Υ_j of goods is priced in U (Section 2.2), at local-currency price $U_{j,t}P^U_{j,t}$. For country H, the

nonlinear CPI is

$$P_{H,t} = \Big[\big(P^d_{H,t}\big)^{1-\Upsilon_H}\big(U_{H,t}P^U_{H,t}\big)^{\Upsilon_H}\Big]^{1-\gamma}\big(E_{HF,t}P^d_{F,t}\big)^{\gamma/2}\big(E_{HR,t}P^d_{R,t}\big)^{\gamma/2}, \tag{D.28}$$

where $E_{HF,t}P^d_{F,t}$ converts F's domestic price into the H numeraire using the same bilateral rate that enters the terms of trade (D.32) below, and symmetrically for R. Because a Cobb-Douglas aggregator makes the log price index an exact expenditure-share-weighted sum of the log sub-prices—the same result that a general CES aggregator delivers only as a first-order approximation around a symmetric steady state—taking logs of (D.28) and first-differencing gives $\pi_{H,t}$ directly, with no separate linearization step required.[1]

$$\pi_{H,t} = (1-\gamma)\Big[(1-\Upsilon_H)\pi_{Hd,t} + \Upsilon_H\big(\pi^U_{H,t} + \Delta u_{H,t}\big)\Big] + \tfrac{\gamma}{2}\big(\pi_{Fd,t} + \dot{e}_{HF,t}\big) + \tfrac{\gamma}{2}\big(\pi_{Rd,t} + \dot{e}_{HR,t}\big). \tag{D.29}$$

The importing country adds the bilateral rate when the rate is quoted with itself as the base currency; the same logic applied from F's and R's perspective, using $\dot{e}_{FR,t} = \dot{e}_{HR,t} - \dot{e}_{HF,t}$ from (F.61), gives

$$\pi_{F,t} = \tfrac{\gamma}{2}\big(\pi_{Hd,t} - \dot{e}_{HF,t}\big) + (1-\gamma)\Big[(1-\Upsilon_F)\pi_{Fd,t} + \Upsilon_F\big(\pi^U_{F,t} + \Delta u_{F,t}\big)\Big] + \tfrac{\gamma}{2}\big(\pi_{Rd,t} + \dot{e}_{FR,t}\big), \tag{D.30}$$

$$\pi_{R,t} = \tfrac{\gamma}{2}\big(\pi_{Hd,t} - \dot{e}_{HR,t}\big) + \tfrac{\gamma}{2}\big(\pi_{Fd,t} - \dot{e}_{FR,t}\big) + (1-\gamma)\Big[(1-\Upsilon_R)\pi_{Rd,t} + \Upsilon_R\big(\pi^U_{R,t} + \Delta u_{R,t}\big)\Big]. \tag{D.31}$$

Writing $\pi^{dom}_{j,t} \equiv (1-\Upsilon_j)\pi_{jd,t} + \Upsilon_j(\pi^U_{j,t} + \Delta u_{j,t})$ for inflation of the domestic bundle, with level $P^{dom}_{j,t}$, the terms $(1-\gamma)\Upsilon_j\Delta u_{j,t}$ are an immediate pass-through of exchange rates into consumer prices through domestically produced goods priced in U; at $\Upsilon_j = 0$ the CPI reduces to producer-currency pricing. This closes the gap between the domestic-goods inflation of Section 2.2 and the CPI inflation used everywhere a household's cost of living, rather than a firm's price, is the relevant object.

Bilateral terms of trade are defined, in levels, as

$$\mathrm{tot}_{jk,t} \equiv e_{jk,t} + p^d_{k,t} - p^d_{j,t}, \tag{D.32}$$

which the model carries as a predetermined state; first-differencing (D.32) gives its law of motion,

$$\dot{\mathrm{tot}}_{jk,t} = \dot{e}_{jk,t} + \pi_{kd,t} - \pi_{jd,t}. \tag{D.33}$$

Triangular consistency implies

$$\mathrm{tot}_{FR,t} = \mathrm{tot}_{HR,t} - \mathrm{tot}_{HF,t}. \tag{D.34}$$

[1] The domestic bundle is the CES aggregate of all domestic varieties, with elasticity μ, as in Section 2.2. Its Cobb-Douglas representation in (D.28), with weights $1-\Upsilon_j$ and Υ_j on the two pricing groups, coincides with it to first order, which is all the log-linear system uses. The welfare calculation of Appendix C uses the CES form, under which the dispersion between the two groups is a second-order cost.

These relationships ensure that exchange-rate and terms-of-trade movements remain mutually consistent across the three-country system.

Net exports and the demand for each country's output follow from the same basket. Household j spends a share $\gamma/2$ of its expenditure on the goods of each trading partner, at the partner's local-currency price converted at the bilateral rate, and a share $1-\gamma$ on the domestic bundle. Exports are local-priced goods: U-priced varieties are sold at home. With the price of consumption relative to domestic goods (F.64), the demand for country j's output (32) and its net exports (F.66) are derived in Section A.9. Consumption and output differ by net exports, the terms of trade, and the price gap between the two pricing groups,

$$y_{j,t} = c_{j,t} + nx_{j,t} + \tfrac{\gamma}{2}\big(\mathrm{tot}_{jk,t} + \mathrm{tot}_{jl,t}\big),$$

because net exports are measured in domestic goods and consumption in the consumption basket.

D.7 Consolidated Balance-Sheet Clearing

Three accounting requirements close the external sector. First, every cross-border payment generated by the liability enters the external positions: the return on the bonds the issuer holds is paid by the issuing country, the revaluation of the liability accrues to its holders, the payout is received in shares λ_j, and the convenience-yield transfer is received in shares θ_j^a and paid in shares λ_j; all enter (18). Second, these payments sum, across countries, to the issuer's income, which is either paid out or added to its equity, so that $\sum_j b_{j,t} + E_{sc,t} = 0$: the issuer neither creates nor destroys resources. Third, the issuer's purchases of reserves and households' purchases of the liability are settled through countries' positions with the financier, whose currency positions are therefore the liability's footprint plus the differences in external positions (Section A.8). In the steady state, country j receives the convenience-yield transfer $\bar{N}_j$, which determines the level component of the architecture; the steady-state flows of interest on its bonds held by the issuer and of seigniorage have zero present value for each country (Section C.3).

D.8 Details Omitted from Section 2

This section collects derivations summarized in the text.

Firms. Production in each country $j \in \{H, F, R\}$ is carried out by a continuum of monopolistically competitive firms indexed by $f \in [0,1]$. Each firm produces a differentiated variety, employs labor as the sole variable input, and sets prices subject to Calvo nominal rigidity. Productivity is common across firms and countries and is normalized to unity throughout:

$$Y_{j,t}(f) = N_{j,t}(f). \tag{D.35}$$

Households and firms in each country allocate expenditure across differentiated varieties according

to a CES aggregator with elasticity $\mu > 1$. Cost minimization implies that the demand facing firm f is

$$Y_{j,t}(f) = \left(\frac{P_{j,t}(f)}{P^d_{j,t}} \right)^{-\mu} Y^d_{j,t}, \tag{D.36}$$

where $P^d_{j,t}$ is the domestic price index and $Y^d_{j,t}$ is aggregate domestic demand. The equilibrium markup is $\mu/(\mu - 1)$.

From the production function (D.35) and the unit cost of labor, nominal marginal cost is

$$MC^N_{j,t}(f) = W_{j,t}, \tag{D.37}$$

and real marginal cost—the object entering the pricing condition—is

$$MC_{j,t} = \frac{W_{j,t}}{P^d_{j,t}}. \tag{D.38}$$

Equation (D.38) records the firm-side relationship only. The equilibrium determination of $W_{j,t}$, and hence of $MC_{j,t}$, follows from the interaction of the labor market with the household block and the remainder of the general equilibrium system; it is not imposed here. Section D.3 closes it using the household's labor-supply condition, (D.16).

Liquidity demand in closed form. The two first-order conditions also determine the scale of the liquid-asset portfolio: their difference gives the composition condition (14), and either one, with the aggregator, gives the level of liquidity services (Section D.4).

Closed-form liquidity and money demand. The composition condition (14) restricts only $mb_{j,t} - s_{j,t}$. The level of liquidity is pinned down by the money condition (D.21) used on its own, together with the aggregator. Because the insurance term enters only the condition for the digital liability, the money condition is the same with or without it. Log-linearizing $U_{L,j,t}$ and $\mathcal{U}^m_{j,t}$, combining with the marginal contribution (D.24), the aggregator (F.20) (which together with (14) gives $mb_{j,t} - l_{j,t} = -\omega_j(\eta g_{j,t} + H_{j,t})$), and using $(1-\rho_L)\eta = \beta/(1-\beta)$, leaves one equation in $l_{j,t}$ in which the substitution parameter ρ_L cancels identically (Section A.2):

$$l_{j,t} = -\eta_\ell \Big[i_{j,t} - \omega_j \Big(g_{j,t} + \frac{H_{j,t}}{\eta} \Big) \Big] + \eta_c \, c_{j,t}, \qquad \eta_\ell \equiv \frac{\beta}{\Lambda}, \quad \eta_c \equiv \frac{\varphi_l/\kappa_L + (1-\beta)(\sigma + \varphi_l)}{\Lambda}, \tag{D.39}$$

where $\Lambda \equiv (1-\beta)(1+\varphi_l) + \varphi_l/\kappa_L > 0$. The interest semi-elasticity η_ℓ governing the *scale* of liquidity is distinct from the semi-elasticity η governing its *composition*: η is driven by ρ_L, whereas η_ℓ depends only on σ, φ_l, κ_L and β. Real money balances and digital-liability holdings follow from

the aggregator and the composition condition,

$$mb_{j,t} = l_{j,t} - \omega_j\big(\eta\, g_{j,t} + H_{j,t}\big), \tag{D.40}$$

$$s_{j,t} = l_{j,t} + (1-\omega_j)\big(\eta\, g_{j,t} + H_{j,t}\big). \tag{D.41}$$

Three features matter. First, money demand responds to consumption as well as to the interest rate; the consumption term enters $mb_{j,t}$ and $s_{j,t}$ identically and cancels from their difference. Second, the insurance motive acts exactly like an additional expected return of $H_{j,t}/\eta$ on the liability: it shifts the composition of liquid assets and, because the money condition is unchanged, also raises total liquidity. Third, holdings respond to denomination through $g_{j,t}$ and, through $H_{j,t}$, to the bilateral premia, which depend on backing (Section 2.6).

Reduced-form marginal cost. Substituting (D.39) into the marginal-cost condition (10) gives

$$mc_{j,t} = \nu y_{j,t} + \big(\sigma + \varphi_l - \varphi_l\eta_c\big)c_{j,t} + \varphi_l\eta_\ell\, i_{j,t} - \varphi_l\eta_\ell\omega_j\Big(g_{j,t} + \frac{H_{j,t}}{\eta}\Big) + \mathrm{rp}_{j,t}. \tag{D.42}$$

A rate increase lowers liquidity and raises the liquidity gap, which raises marginal cost; an expected appreciation of the liability, or a rise in own-currency risk that makes it more valuable as insurance, raises liquidity and lowers marginal cost. This channel is additive to the direct consumption channel in (10).

The convenience yield. Government bonds provide services that other assets do not: they serve as collateral, settle transactions, and store value safely, as documented for U.S. Treasuries by Krishnamurthy and Vissing-Jorgensen (2012) and Jiang et al. (2021). We model these services as a saving of resources, in the spirit of the transaction-cost foundation of money demand (Feenstra, 1986). A household of country j that holds $b_{jk,t}$ units of country k's bonds incurs a resource cost

$$\Gamma_{jk}(b_{jk,t}) = v_{jk}\,\Gamma(b_{jk,t}), \qquad -\Gamma'(b) = -b_{cy}\log b, \quad b_{cy} > 0, \tag{D.43}$$

which falls with its holdings; $v_{jk} > 0$ measures how much country-j households value the services of country-k bonds. Holdings of foreign bonds are hedged against currency risk through swaps with the financier at covered interest parity on policy rates, so they create no open currency position. The alternative to a bond is a deposit paying the policy rate. Choosing $b_{jk,t}$ to maximize the return net of the resource cost, the household equates the return it forgoes by holding a bond to the marginal resource saving:

$$\mathrm{CY}_{k,t} \equiv 1 - \frac{R^g_{k,t}}{R_{k,t}} = -v_{jk}\,\Gamma'(b_{jk,t}) \qquad \text{for every } j, \tag{D.44}$$

where $R^g_{k,t}$ is the gross yield on country k's bonds and $R_{k,t}$ the policy rate. The convenience yield $\mathrm{CY}_{k,t}$ is therefore common to all holders. Inverting (D.44), holdings are

$b_{jk,t} = \exp\big(-\mathrm{CY}_{k,t}/(v_{jk}b_{cy})\big)$; with a common $v_{jk} = 1$ each country holds a constant share of the bonds in private hands, and market clearing, $\sum_j \lambda_j b_{jk,t} = B^p_{k,t} \equiv \bar{B}^p - \theta^a_k A_{sc,t}$, gives

$$\mathrm{CY}_{k,t} = a_{cy} - b_{cy} \log B^p_{k,t}, \tag{D.45}$$

with $a_{cy} = 0$ under this normalization, the specification that Krishnamurthy and Vissing-Jorgensen (2012) estimate. The convenience yield is decreasing in the stock of bonds held privately, and each one-percent fall in that stock raises it by $b_{cy}/100$, whatever the level. The policy rate $R_{k,t}$ is the rate on deposits and central-bank reserves that enters households' Euler equations and the financier's currency positions; the convenience yield is the spread between that rate and the yield on government bonds. Because the services save resources rather than yield utility, the convenience yield depends only on the quantity of bonds in private hands, not on households' marginal utility of consumption.

Reserve bond prices and returns. Reserve assets are bonds paying a coupon of 1 at $t+1$, decaying geometrically at rate δ_j thereafter, valued at the net policy rate $I_{j,t}$. Summing the coupon stream gives the price

$$Q_{j,t} = \frac{1}{1 + I_{j,t} - \delta_j}, \tag{D.46}$$

which is a one-period bill at $\delta_j = 0$ and the pure perpetuity $Q_{j,t} = 1/I_{j,t}$ at $\delta_j = 1$. Around $\bar{Q}_j = \beta/(1-\beta\delta_j)$,

$$q_{j,t} = -D_j\, i_{j,t}, \qquad D_j \equiv \frac{1}{1-\beta\delta_j}. \tag{D.47}$$

A bond bought at $t-1$ pays its coupon at t and is then worth $\delta_j Q_{j,t}$, so its gross one-period return is $(1+\delta_j Q_{j,t})/Q_{j,t-1}$. Using $1 + \delta_j \bar{Q}_j = \bar{Q}_j/\beta$ and $1/D_j = 1 - \beta\delta_j$, its log deviation is

$$r_{j,t} = i_{j,t-1} + \beta\delta_j\, \Delta q_{j,t}\,: \tag{D.48}$$

the yield locked in at purchase plus a capital gain that is zero for a one-period bill.

Original sin: burden, default, and pricing. The burden (F.50) is written on the terms of trade. A burden written on the nominal exchange rate would instead drift with R's price level, which has a unit root under inflation targeting (Appendix B). The burden is generated by a stock that is non-zero at the steady state. The model's own bilateral position $b_{HR,t}$ could not generate it: with $\bar{b}_{HR} = 0$, the valuation of a dollar-denominated share of that position is a product of two first-order deviations and vanishes from the log-linear system (Section A.7).

Default. R's external debt is owed by a continuum of borrowers who differ in their capacity to repay. Borrower i holds assets worth $\omega_i \bar{A}_R$ units of R's output, where ω_i is drawn independently each period from a log-normal distribution with mean one and dispersion σ_ω, and owes, in the same units, $D_{R,t} = (1-\chi_{os})\bar{B}_R + \Gamma_{R,t}$. It defaults when its assets fall short of its debt, $\omega_i < \bar{\omega}_t \equiv D_{R,t}/\bar{A}_R$.

Its creditors then take its assets, but recovering them costs a fraction μ^{bk} of their value (Bernanke et al., 1999). With a continuum of borrowers, the share that defaults is $F(\bar{\omega}_t)$, where F is the distribution of ω_i; with $G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega \, dF(\omega)$, creditors of R lose the fraction

$$d_t = F(\bar{\omega}_t) - (1 - \mu^{bk}) \frac{G(\bar{\omega}_t)}{\bar{\omega}_t} \tag{D.49}$$

of their claims, whatever their currency, and the average loss given default is $\ell = \bar{d}/F(\bar{\omega})$. Of that loss, $F - G/\bar{\omega}$ is the shortfall of defaulting borrowers' assets below their debt, which they keep, and $\mu^{bk} G/\bar{\omega}$ is spent on bankruptcy. A real depreciation raises the value of the dollar debt in R's output and with it the default threshold: by (F.50), $d \log D_{R,t} = -\chi_{os} \text{tot}_{HR,t}$, since $D_{R,t} = \bar{B}_R$ at the steady state. The external debt is long-term with a fixed coupon set at issuance, which prices the steady-state default rate; movements in the premium change the price of new positions, not the service of the existing stock. Defaults in excess of the steady state cost R's creditors $(d_t - \bar{d})\bar{B}_R$. They rise when R's currency depreciates in real terms. The part that R's borrowers keep is a transfer to R, which works as partial insurance for R, paid for by its creditors; the part spent on bankruptcy absorbs R's output. Both enter the external positions (18).

Pricing the burden. In the H–R market the financier's problem becomes

$$\max_{x_{HR,t}} \; \mathbb{E}_t \left[R_{R,t}(1 - d_t) \frac{E_{HR,t+1}}{E_{HR,t}} - R_{H,t} \right] x_{HR,t} \; - \; \frac{\psi_b}{2} \, x_{HR,t}^2. \tag{D.50}$$

Linearizing (D.49) around the steady state, the constant $\bar{d}$ is absorbed into the steady-state premium and the deviation is $d_t - \bar{d} = -\chi_{os} \zeta_{os} \text{tot}_{HR,t}$, with

$$\zeta_{os} \equiv \mu^{bk} \frac{\phi(z)}{\sigma_\omega} + (1 - \mu^{bk}) \frac{\Phi(z - \sigma_\omega)}{\bar{\omega}}, \qquad z \equiv \Phi^{-1}\big(F(\bar{\omega})\big), \tag{D.51}$$

where ϕ and Φ are the standard normal density and distribution. The parameter ζ_{os} is the response of the expected loss to a one-percent rise in R's leverage. The premium on R's currency therefore acquires the term (27).

Transfer and deadweight. Of the response ζ_{os}, the part that reaches R's borrowers is $d(F - G/\bar{\omega})/d \log \bar{\omega} = G(\bar{\omega})/\bar{\omega}$, and the bankruptcy cost absorbs $\mu^{bk}\big(\bar{\omega} F'(\bar{\omega}) - G(\bar{\omega})/\bar{\omega}\big)$. The share transferred is therefore

$$\tau_T = \frac{\Phi(z - \sigma_\omega)/\bar{\omega}}{\zeta_{os}}, \tag{D.52}$$

and, in the external positions, R receives $\tau_T(-\kappa_{os} \bar{B}_R \text{tot}_{HR,t})$ while $(1 - \tau_T)(-\kappa_{os} \bar{B}_R \text{tot}_{HR,t})$ is subtracted from R's output.

Calibration. Given the quarterly default rate $F(\bar{\omega})$, three targets pin down σ_ω and μ^{bk} and hence τ_T: the loss given default ℓ and the response $\zeta_{os} = \kappa_{os}/\chi_{os}^0$. With an annual default rate of 3

percent (Bernanke et al., 1999), $\ell = 0.616$ (Hamilton et al., 2007, Exhibit 6) and $\kappa_{os} = 0.025$, the two conditions $\ell = 1 - (1 - \mu^{bk})G/(\bar{\omega}F)$ and (D.51) give, at $\chi^0_{os} = 0.52$, $\sigma_\omega = 0.27$, $\mu^{bk} = 0.58$ and $\tau_T = 0.143$. Fixing instead $\mu^{bk} = 0.12$, the value in Bernanke et al. (1999), and solving (D.51) for σ_ω gives $\tau_T = 0.153$; the baseline uses the midpoint, $\tau_T = 0.148$ (`calib_os_1013_8.py`).

E Proofs

This appendix proves the propositions and the lemma of Sections 2 and 3, gives the full proof of the invariance result, and collects the supporting arguments.

E.1 Propositions of Section 2

Proposition 2.1. By (25), neglecting the interest on revalued reserves, of order $\beta^{-1} - 1$, $\text{val}_{fx,t} \equiv 0$ for all $\dot{e}_{HF,t}$, $\dot{e}_{HR,t}$ requires $\theta^e_F = \theta^d_F$ and $\theta^e_R = \theta^d_R$, hence $\theta^e_H = \theta^d_H$. By (23), $\theta^e - \theta^d = (1 - \varsigma)(\theta^a - \theta^d)$, which is zero if and only if $\theta^a = \theta^d$ or $\varsigma = 1$. The sign statement follows directly. Neither θ^u nor any object depending on it appears in (F.45) or (25). □

Proposition 2.3. (i) Differentiate (26) and (27); no element of θ^d, θ^u, or ς appears in either. (ii) R's borrowers owe $\chi_{os}\bar{B}_R$ dollars against R-currency income. The issuer holds an effective R-currency position $\theta^e_R\bar{s}$ against R-currency liabilities $\theta^d_R\bar{s}$, which is (25). The financier, as counterparty to the issuer's hedge (23), holds $(\theta^a_R - \theta^e_R)\bar{s}$. Holders own $\theta^d_R\bar{s}$ of R's currency through the liability. Summing and using (26), the θ^a_R, θ^e_R, and θ^d_R terms cancel. □

Proposition 2.4. Substitute $\mathbb{E}_t\dot{e}_{Hk,t+1} = \psi_{Hk,t} - (i_{k,t} - i_{H,t})$ into (3) for $j = R$, which gives $g_{R,t} = \theta^d_F\mathbb{E}_t\dot{e}_{HF,t+1} - (1 - \theta^d_R)\mathbb{E}_t\dot{e}_{HR,t+1}$, and use (14) with $H_{R,t}$ from (16). With $d\varpi_{R,t} = d\psi_{HR,t}$, the coefficient on the shock is $(1 - \theta^d_R)\eta(B_R - 1)$, positive if and only if $B_R > 1$. □

Proposition 2.5. (i) Setting (F.53) to zero for all pairs gives $\bar{h}_k - \theta^e_k = \bar{h}_j - \theta^e_j$ for all j, k; since both vectors sum to one, the common difference is zero. (ii) Substitute $\theta^e_k = \bar{h}_k = \lambda_k\bar{s}_k/\bar{s}$ into (F.54)–(28): $\bar{h}_k dS_t = \lambda_k\bar{s}_k\tilde{s}_t$. (iii) With $\theta^e_{k,t}A_{sc,t} = S_{k,t}$, $Z_{k,t} = 0$ for every k and t. □

Proposition 2.5. (i) is (22). (ii) With zero inflation and constant exchange rates, the Euler equation (11) gives $\bar{R}_j = \beta^{-1}$ for every j, and the parity condition in levels then requires a zero steady-state premium, $\psi_b(\bar{b}_{jk} + \bar{x}^{sc}_{jk}) = 0$, given $\bar{\varpi}_R = 0$ and $\bar{E}_{sc} = 0$. By Sections A.3 and A.8, the steady-state positions are $\overline{\text{NFA}}_j = \lambda_j\bar{s}_j - \theta^e_j\bar{s} = \bar{s}(\bar{h}_j - \theta^e_j)$, so that $\bar{b}_{jk} = \overline{\text{NFA}}_j - \overline{\text{NFA}}_k = -\bar{x}^{sc}_{jk}$. A country whose weight in the effective position exceeds its residents' share of holdings owes the difference to the rest of the world. (iii) The first part is (21). For the second, take $s_{k,t} > 0$ and $s_{m,t} = 0$ for $m \neq k$: by (F.54)–(28), $x^{sc}_{jk,t} = \lambda_k\bar{s}_k s_{k,t}\big[1 - (\theta^e_k - \theta^e_j)\big]$, which is non-negative because $\theta^e_k - \theta^e_j \leq 1$. □

Proposition 2.5. By inspection: θ^u appears only in (4), which enters the U-price block and the CPI. □

Proposition 2.6. See Appendix E.3. The argument has three steps. First, by (F.62), θ^d and θ^u enter the household and price blocks only through the common terms G_t and $\mathcal{U}_t$, except for the factor $1-\theta^d_R$ in R's hedging demand (16), which is held fixed in (i)–(ii). Second, under (b) and (c) the issuer's balance sheet reaches countries only through terms that are common to all of them—its payout and the revaluation of the liability—and the payments on the bonds it holds depend on each country's own prices, not on θ^d or θ^u; original sin depends on θ^a_R alone. Third, subtracting country H's equations from country j's eliminates the common terms, because by (a) they enter every country's equations with the same coefficients, and leaves an autonomous system in exchange rates, premia, external positions, and cross-country differences that contains neither θ^u nor any element of θ^d other than θ^d_R. □

Proposition 2.7. The identity follows from the definition of x_R. With $\theta^e = \bar{h}$, $1-\theta^d_R-(1-\bar{h}_R) = x_R$; a zero open position, $x_R = 0$, is the condition of Proposition 2.1 for the issuer's equity to be insulated from R's currency. □

Proposition 2.8. (i) Holders' return is (3) and their hedging demand is (16); with $\theta^a = (1,0,0)$ and no hedge, $\theta^e_F = \theta^e_R = 0$ in (25); the reallocation of risk is Proposition 2.3(ii). (ii) follows from (4) and (F.6). (iii) follows from (22), (26), (F.54)–(28) with $\theta^e = \theta^a$, and (25) with $\theta^d = (1,0,0)$. Steady-state flows of seigniorage and of interest on the bonds the issuer holds have zero present value for each country (Section C.3). □

E.2 Welfare Non-Separability

Proposition C.1 states that the welfare effect of the unit of account depends on denomination and conversely. Along the restricted H–R edge, $\theta^d(x_d) = (1-x_d, 0, x_d)$ and $\theta^u(x_u) = (1-x_u, 0, x_u)$, the two margins interact through four channels. First, holders' purchasing power over U-priced goods (F.6) depends on the difference $x_d - x_u$, so the welfare value of moving x_u depends on x_d. Second, x_d governs holders' expected return (3) and therefore their holdings $s_{j,t}$, which drive the liability's footprint in (F.54)–(28) and the path of $e_{HR,t}$; x_u governs how that path passes into U-priced goods through $\Delta u_{j,t}$ and the price gap $z_{j,t}$, which enters the loss directly. Third, under alignment ($\theta^e = \theta^d$) x_d also sets the footprint. Fourth, the insurance motive scales R's holdings by $1-x_d$, so the flows that drive the footprint, and with it the path of the premium, depend on x_d, while x_u prices those paths. Because welfare (C.1) depends, to second order, on products of objects that both margins move through different equations, the mixed partial $\partial^2\mathcal{W}/\partial x_d \partial x_u$ is generically non-zero. The level component (C.5) is linear in θ^a and independent of θ^d and θ^u, so non-separability is a property of the stabilization component. The argument identifies the channels; their quantitative size at the baseline calibration is reported after Proposition C.1.

E.3 Invariance at the Neutral Position

This section proves Proposition 2.6. Write the log-linear system of Appendix A as $\mathcal{S}(\mathbf{x};\Theta)$, where $\mathbf{x}$ stacks all endogenous variables.

Step 1: where θ^d and θ^u enter. Denomination appears in the system in holders' returns (3), in the issuer's FX valuation (25), and, for a hedged issuer, in the effective position (23). The unit of account appears only in the local value of the U (4), which enters the U-price block (7)–(F.14) and the CPI (D.29)–(D.31). By (F.62), holders' returns and the local value of the U are each the sum of a term common to all countries (G_t or $\mathcal{U}_t$) and a country-specific term that depends only on exchange rates.

Step 2: the issuer's balance sheet reaches countries symmetrically. Under (b), $\theta^e = \theta^a = \lambda$ is independent of θ^d. The footprint term $(\theta^e_k - \theta^e_j)(dS_t + E_{sc,t})$ in (F.54)–(28) is zero, and so are the convenience-yield transfers $n^{cy}_{j,t}$, since with $\theta^a_j = \lambda_j$ every $\zeta_{y,j}$ is the same. Under (c), the capacity term $-\chi_E E_{sc,t}$ is zero. In the external positions (18), the issuer's payout enters every country with the same weight $\lambda_j = 1/3$, and so does the revaluation of the liability, $\lambda_j \bar{s}_j \Delta x^H_t$, which is the only place where θ^d enters them. The payments on the bonds the issuer holds, $(\theta^a_j \bar{s}/\beta) r_{j,t}$, differ across countries, but they depend only on country j's own interest rate, exchange rate, and bond price, with the common weight $\lambda_j \bar{s}/\beta$, and not on θ^d, θ^u, or the issuer's equity. The original-sin coefficient $\kappa_{os}(\theta^a_R)$ depends on backing only. Hence $E_{sc,t}$, and with it $\text{val}_{fx,t}$, reaches the rest of the system only through terms that are common to all countries, which proves (iii) and removes the second route through which θ^d enters. The duration of reserves enters through the bond prices in $r_{j,t}$, which have the same coefficients in every country and do not involve θ^d or θ^u, so that Steps 3–4 apply at any duration, which proves (iv).

Step 3: the difference system. For every country variable define $\tilde{x}_{j,t} \equiv x_{j,t} - x_{H,t}$. Under (a), the Phillips curves (6) and (7), the gap (F.14), marginal cost (10), the Taylor rule (F.67), the Euler equation (11), the closed forms (D.39)–(D.41), the CPI (D.29)–(D.31), goods-market clearing (32), and the bond prices have the same coefficients in every country. Subtracting country H's equations from country j's therefore eliminates G_t and $\mathcal{U}_t$, which enter them with those common coefficients, and leaves equations in $\tilde{x}_{j,t}$ and exchange rates. The terms that are specific to R or that differ across countries are the risk-off disturbance in (29), R's hedging demand $H_{R,t} = \eta B_R(1-\theta^d_R)\varpi_{R,t}$ (16) (and $H_{H,t} = H_{F,t} = -\eta\theta^d_R\varpi_{R,t}$), and original sin $-\kappa_{os}\text{tot}_{HR,t}$, a function of exchange rates and relative prices. The first and the last do not involve θ^d or θ^u; the hedging demand involves θ^d only through θ^d_R, which is held fixed. By Step 2, the premia reduce to $\psi_{jk,t} = \psi_b b_{jk,t} + \psi_b \lambda \bar{s}(\widetilde{dS}_{k,t} - \widetilde{dS}_{j,t})$ plus the R-specific terms; net exports (F.66) are functions of consumption, of the relative prices (F.64), and of the terms of trade; and the differences in external positions (18) accumulate net exports and the country-specific returns on reserves, in which neither θ^d nor θ^u appears. Together with the parity conditions (F.59)–(F.61), these equations form a closed linear system $\tilde{\mathcal{S}}$ in $\tilde{\mathbf{x}} \equiv (\tilde{x}_{F,t}, \tilde{x}_{R,t}, e_{HF,t}, e_{HR,t}, b_{HF,t}, b_{HR,t}, \psi_{HF,t}, \psi_{HR,t})$ that contains neither θ^u nor any element of θ^d other than θ^d_R.

Step 4: uniqueness. In the variables $(\tilde{\mathbf{x}}, \mathbf{x}_H, E_{sc})$, the full system is block recursive: $\tilde{\mathcal{S}}$ does not depend on $\mathbf{x}_H$ or E_{sc}. Its generalized eigenvalues are therefore a subset of those of the full system, and the bounded solution of the full system, which is unique under the Blanchard–Kahn conditions of Appendix B, restricts to the unique bounded solution of $\tilde{\mathcal{S}}$. That solution does not depend on θ^u or, given θ^d_R, on θ^d, which is (i). Writing $x_{j,t} = x_{H,t} + \tilde{x}_{j,t}$, such a change moves $x_{H,t}$ but not $\tilde{x}_{j,t}$, which is (ii).

Welfare. By (i)–(ii), the paths that enter welfare (C.1) change by common amounts; the insurance term depends on θ^d_R, and the level component (C.5) on θ^a alone. □

E.4 Intermediation Capacity and Separability

With $\chi_E = 0$, issuer equity enters the bilateral premia only through the footprint term $(\theta^e_k - \theta^e_j)E_{sc,t}$ of (F.54)–(28), which is zero for any pair of currencies with equal weight in the effective position. With $\chi_E > 0$, equity enters every premium directly. Since $E_{sc,t}$ depends on backing and denomination through (25), and exchange rates enter the U-price block through $\Delta u_{j,t}$, which depends on θ^u, all three clauses then interact through intermediation capacity, and the balance-sheet losses of Proposition 2.3 reach every currency market rather than the issuer alone.

E.5 Proof of Lemma 2.2

Let R's borrowers owe the share χ of their gross external debt $\bar{B}_R$ in dollars and $1-\chi$ in R's currency, and hold assets $\bar{A}_R$ valued in R's output. Their net worth in units of R's output is $\mathcal{N}_t = \bar{A}_R - (1-\chi)\bar{B}_R - \chi\bar{B}_R/\mathrm{Tot}_{HR,t}$, since the dollar debt is a claim on H's output (equation (F.50)). To second order in $\mathrm{tot}_{HR,t}$,

$$\mathcal{N}_t \approx \bar{A}_R - \bar{B}_R + \chi\bar{B}_R\big(\mathrm{tot}_{HR,t} - \tfrac{1}{2}\mathrm{tot}^2_{HR,t}\big),$$

so that, with $\mathbb{E}\,\mathrm{tot}_{HR,t} = 0$ at the steady state (assumption (b): the steady-state premium is zero, so the two currencies have the same expected cost to first order), $\mathbb{E}\mathcal{N}_t = \bar{A}_R - \bar{B}_R - \frac{1}{2}\chi\bar{B}_R\mathrm{Var}(\mathrm{tot}_{HR})$ and $\mathrm{Var}(\mathcal{N}_t) = \chi^2\bar{B}^2_R\mathrm{Var}(\mathrm{tot}_{HR})$. The expected value is decreasing and the variance increasing in χ, so any objective that is increasing in the mean and strictly decreasing in the variance of net worth is strictly decreasing in χ whenever $\mathrm{Var}(\mathrm{tot}_{HR}) > 0$. Borrowers therefore choose the smallest feasible dollar share. By assumption (c), R-currency debt can be placed with investors other than the issuer up to $(1-\chi^0_{os})\bar{B}_R$ and with the issuer up to its holdings $\theta^a_R\bar{s}$, so feasibility requires $(1-\chi)\bar{B}_R \le (1-\chi^0_{os})\bar{B}_R + \theta^a_R\bar{s}$, that is $\chi \ge \chi^0_{os} - \theta^a_R\bar{s}/\bar{B}_R$, together with $\chi \ge 0$. The optimum is $\chi = \max\{\chi^0_{os} - \theta^a_R\bar{s}/\bar{B}_R, 0\}$, which is (26); at the calibration of the main text the first term is positive in every regime. The default transfers of Section 2.5 do not change the argument: their expected value is zero to first order, because the coupon prices the steady-state default rate. □

E.6 Proof of Propositions 2.9 and 3.1

The issuer owes S at par and holds reserves with effective position θ^e and equity K. In a disaster, by Proposition 2.1, the value of its assets falls relative to its liabilities by $x_R JS$. If holders do not run, the shortfall is $\Delta_s = x_R JS - K$ when positive. If they run, the issuer must also sell its R-currency reserves, $\theta^e_R S$ to first order, at the discount ϕ^{fs}, and the shortfall is $\Delta_r = (x_R J + \phi^{fs}\theta^e_R)S - K$. The members cover a shortfall $\Delta > 0$ if and only if $\vartheta\Delta \le \bar{c}$, that is $\Delta \le \bar{G} - K$.

(i) Suppose holders expect the shortfall to be covered. Then no holder gains by redeeming early, nobody runs, and the shortfall is Δ_s; the expectation is correct if and only if $\Delta_s \le \bar{G} - K$ (or $\Delta_s \le 0$), that is $x_R JS \le \bar{G}$, or $S \le \bar{S}_1$. If $S > \bar{S}_1$ the members do not cover Δ_s, the last holders to redeem bear it, and every holder strictly prefers to redeem early, so no equilibrium without a run exists.

(ii) Suppose holders expect the shortfall not to be covered. Since redemptions are served at par in sequence, each holder strictly prefers to redeem early, all run, and the shortfall is Δ_r; the expectation is correct if and only if $\Delta_r > \bar{G} - K$, that is $(x_R J + \phi^{fs}\theta^e_R)S > \bar{G}$, or $S > \bar{S}_0$.

(iii) follows from (i) and (ii), since $\bar{S}_0 < \bar{S}_1$ when $\phi^{fs}\theta^e_R > 0$. (iv) With $x_R = 0$, $\Delta_s = -K \le 0$ for every S, so by the argument of (i) a safe equilibrium exists at every scale. A run equilibrium may still exist when $\phi^{fs}\theta^e_R S > \bar{G}$, a liquidity run on peripheral reserves that does not depend on the open position; it is absent at the dollar corner, where $\theta^e_R = 0$. By Proposition 2.7, $x_R = 0$ at a neutral position means $\theta^d_R = \bar{h}_R$, the consistent design. □

Proposition 3.1. With capital $K = \mathcal{K}S$ held in the denomination bundle, a disaster leaves K unchanged, and the shortfalls in the proof above become $\Delta_s = (x_R J - \mathcal{K})S$ and $\Delta_r = (x_R J + \phi^{fs}\theta^e_R - \mathcal{K})S$. The members cover a shortfall if and only if it does not exceed $\bar{g} = \bar{c}/\vartheta$. (i) A safe equilibrium exists if and only if $\Delta_s \le \bar{g}$: at every S when $\mathcal{K} \ge x_R J$, and for $S \le \bar{g}/(x_R J - \mathcal{K})$ otherwise. (ii) A run equilibrium exists if and only if $\Delta_r > \bar{g}$: at no S when $\mathcal{K} \ge x_R J + \phi^{fs}\theta^e_R$, and for $S > \bar{g}/(x_R J + \phi^{fs}\theta^e_R - \mathcal{K})$ otherwise. □

F The Model in Full

This appendix states the model of Section 2 in full, with the derivations that the main text summarizes. Propositions are stated and numbered in the main text; their proofs are in Appendix E.

The model studies a multilateral digital monetary liability whose contract is chosen by a governance authority and implemented by an institutional issuer that holds reserve assets against it.

Three economies, indexed by $j \in \{H, F, R\}$, issue distinct currencies and share the common digital liability. Home (H) and Foreign (F) are advanced economies with deep financial markets; the Rest of the World (R) is a peripheral economy whose currency carries a time-varying risk premium and whose external debt is partly denominated in dollars. Each economy has a representative household,

a continuum of monopolistically competitive firms, a government that issues bonds, and a central bank that follows an inertial Taylor rule. A competitive international financier intermediates cross-border positions.

F.1 The Contract: Three Clauses

The digital liability is defined by a contract with three clauses. Each is a vector of non-negative weights on the three currencies that sum to one,

$$\Theta = (\theta^d, \theta^u, \theta^a), \qquad \theta^x = (\theta^x_H, \theta^x_F, \theta^x_R), \quad \textstyle\sum_j \theta^x_j = 1, \quad x \in \{d, u, a\}, \tag{F.1}$$

and each is a bundle of *currencies*, not of price levels.

- *Denomination* θ^d defines what one unit of the liability *is*: a fixed amount of each currency, with value shares θ^d at the steady state. Redeeming one unit delivers those amounts.
- *Unit of account* θ^u defines a separate accounting unit, denoted U, which is also a fixed amount of each currency, with value shares θ^u. Producers who price in the liability quote sticky prices in U.
- *Backing* θ^a defines the currency composition of the government bonds the issuer holds as reserves.

The government of each country, the households that hold the liability, and the issuer all take Θ as given. It is chosen by a governance authority (Section 3).

Two reference points organize the analysis. The *dollar corner*, $\theta^d = \theta^u = \theta^a = (1, 0, 0)$, describes existing dollar stablecoins: the liability is a dollar, prices are quoted in dollars, and reserves are dollar bonds. *Bundling* a clause means moving its weights from the dollar corner to a basket of currencies, with the other two clauses held fixed.

The value of one unit. One unit of the liability contains c_k units of currency k, chosen so that $c_k \bar{E}_{Hk} = \theta^d_k \bar{X}^H$, where $E_{jk,t}$ is the price of one unit of currency k in units of currency j and $\bar{X}^H$ the steady-state value of one unit in H's currency. Its value in H's currency is $X^H_t = \sum_k c_k E_{Hk,t}$, with $E_{HH,t} \equiv 1$. Log-linearizing, and dividing by $E_{Hj,t}$ to express the value in country j's currency,

$$d_{j,t} = \sum_k \theta^d_k \, e_{Hk,t} - e_{Hj,t}, \qquad e_{HH,t} \equiv 0, \tag{F.2}$$

where lower-case letters denote log deviations from the steady state. A unit acquired at t earns, in j's currency, the expected appreciation

$$g_{j,t} \equiv \mathbb{E}_t \, \Delta d_{j,t+1} = \mathbb{E}_t \Big[\theta^d_F \dot{e}_{HF,t+1} + \theta^d_R \dot{e}_{HR,t+1} \Big] - \mathbb{E}_t \dot{e}_{Hj,t+1}, \tag{F.3}$$

where $\dot{e}_{jk,t} \equiv e_{jk,t} - e_{jk,t-1}$. A fall in $e_{HR,t}$ is a depreciation of R's currency. Equation (3) is the liability's return relative to money in country j; it enters household portfolio choice in Section 2.3.

The value of one U. The same steps applied to the unit of account give the value of one U in country j's currency,

$$u_{j,t} = \sum_k \theta^u_k \, e_{Hk,t} - e_{Hj,t}, \tag{F.4}$$

and the exchange rate between one unit of the liability and one U, which is the same in every country,

$$d_{j,t} - u_{j,t} = \sum_k \big(\theta^d_k - \theta^u_k\big) e_{Hk,t}. \tag{F.5}$$

When $\theta^u = \theta^d$ the two coincide and prices are quoted in units of the liability itself. The log number of U-priced goods that one unit of the liability buys in country j is

$$d_{j,t} - \big(p^U_{j,t} + u_{j,t}\big) = \sum_k \big(\theta^d_k - \theta^u_k\big) e_{Hk,t} - p^U_{j,t}, \tag{F.6}$$

where $p^U_{j,t}$ is the index of prices quoted in U (Section 2.2). When $\theta^u = \theta^d$ the first term vanishes, and the liability's purchasing power over U-priced goods is insulated from exchange-rate movements in every country at once. The unit of account must be a currency bundle for quoted prices to be sticky: a price quoted in a basket of price levels would be an indexed price and would move automatically with those price levels.

Aggregate holdings and adoption. Country-level holdings aggregate into

$$s_t = \sum_j \lambda_j s_{j,t}, \qquad \textstyle\sum_j \lambda_j = 1, \tag{F.7}$$

where λ_j is country j's population share ($\lambda_j = 1/3$ in the benchmark). Steady-state holdings are tied to adoption ω_j,

$$\bar{s}_j = \kappa_{S,j} \omega_j, \qquad \bar{s} = \sum_j \lambda_j \bar{s}_j, \tag{F.8}$$

with $\kappa_{S,j} = 1$ in the benchmark (Section D.2). Two further objects are used throughout: the change in the total value of holdings, in units of steady-state output,

$$dS_t = \sum_j \lambda_j \bar{s}_j \, s_{j,t}, \tag{F.9}$$

and the steady-state share of holdings belonging to residents of country j,

$$\bar{h}_j \equiv \frac{\lambda_j \bar{s}_j}{\bar{s}}, \qquad \textstyle\sum_j \bar{h}_j = 1, \tag{F.10}$$

which equals $1/3$ for every j under symmetric adoption.

F.2 Firms and Price Setting

In each country a continuum of monopolistically competitive firms produces differentiated varieties with labor alone, faces CES demand with elasticity $\mu > 1$, and sets prices subject to Calvo rigidity, a fraction $1-\alpha$ resetting each period (Sections D.1 and D.8). Domestic-goods inflation follows the New Keynesian Phillips curve

$$\pi_{jd,t} = \beta\,\mathbb{E}_t\pi_{jd,t+1} + \kappa\, mc_{j,t}, \qquad \kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}, \tag{F.11}$$

where real marginal cost $mc_{j,t} = w_{j,t} - p^d_{j,t}$ is determined by the household block, equation (10) below.

Prices Quoted in the Unit of Account A share $\Upsilon_j \in [0,1)$ of country j's producers quote their prices to domestic customers in the unit of account U of Section F.1 rather than in local currency. These firms face the same technology, wages, and Calvo friction as all others, but the price they keep fixed is a number of U. The share is tied to adoption,

$$\Upsilon_j = \upsilon\,\omega_j, \tag{F.12}$$

so that pricing in U spreads with the use of the liability; $\upsilon = 0$ recovers producer-currency pricing throughout. The specification follows the evidence that prices set in a vehicle currency are sticky in that currency (Gopinath et al., 2020; Boz et al., 2022); here the vehicle unit is a design choice of the contract.

A U-pricing firm that resets at t chooses $P^{U*}_{j,t}$ in U. Its revenue in local currency is $P^{U*}_{j,t}U_{j,t+k}$, where $U_{j,t}$ is the local value of one U, so the problem is (D.1) with nominal marginal cost converted into U. Following the steps that led to (6), with the log of nominal marginal cost in U equal to $mc_{j,t} + p^d_{j,t} - u_{j,t}$, the reset price is

$$p^{U*}_{j,t} = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k\,\mathbb{E}_t\big[mc_{j,t+k} + p^d_{j,t+k} - u_{j,t+k}\big],$$

and, with the U-price index $p^U_{j,t} = \alpha p^U_{j,t-1} + (1-\alpha)p^{U*}_{j,t}$,

$$\pi^U_{j,t} = \beta\,\mathbb{E}_t\pi^U_{j,t+1} + \kappa\big(mc_{j,t} + z_{j,t}\big), \tag{F.13}$$

where the price gap

$$z_{j,t} \equiv p^d_{j,t} - u_{j,t} - p^U_{j,t}, \qquad z_{j,t} = z_{j,t-1} + \pi_{jd,t} - \Delta u_{j,t} - \pi^U_{j,t}, \tag{F.14}$$

measures the local-currency price of local-priced goods, converted into U, relative to the U price. The gap is a new predetermined state variable. It anchors U prices: when the U appreciates against

local currency, $z_{j,t}$ falls, U prices are high relative to costs, and resetting firms lower them. No central bank targets $\pi^U_{j,t}$; firms' costs do the anchoring.

F.3 Households and Liquidity Demand

The representative household in country $j \in \{H, F, R\}$ chooses consumption $C_{j,t}$, labor supply $N_{j,t}$, and holdings of conventional money $M_{j,t}$ and the digital liability $S_{j,t}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{j,t}^{1-\sigma} - 1}{1-\sigma} - \frac{\varphi_l}{2} \left(C_{j,t} - L_{j,t} \right)^2 - \frac{\varphi_n}{1+\nu} N_{j,t}^{1+\nu} + \chi_L \log L_{j,t} \right], \tag{F.15}$$

where $L_{j,t}$ denotes composite liquidity services, normalized to one at the steady state. Liquid assets are worth κ_L of a quarter's consumption at the steady state, real money $\kappa_L(1-\omega_j)\tilde{m}_{j,t}$ and the liability $\kappa_L \omega_j \tilde{s}_{j,t}$. The parameter $\varphi_l > 0$ governs the welfare cost of deviations between consumption and liquidity services, and $\chi_L = \kappa_L(1-\beta)$ the direct marginal value of liquidity. The expectation includes a state that the first-order dynamics do not display, a currency disaster in the periphery, which gives the liability its value as insurance (Section 2.3).

The marginal utilities of consumption and liquidity services are

$$U_{C,j,t} = C_{j,t}^{-\sigma} - \varphi_l \left(C_{j,t} - L_{j,t} \right), \tag{F.16}$$

$$U_{L,j,t} = \frac{\chi_L}{L_{j,t}} + \varphi_l \left(C_{j,t} - L_{j,t} \right). \tag{F.17}$$

Adoption $\omega_j \in [0,1]$ is a characteristic of the monetary environment that sets the weight of the digital liability in the liquidity technology; given it, the household chooses the scale and composition of its liquid assets.

Liquidity Aggregator Conventional money and the digital liability provide liquidity services through a CES aggregator,

$$L_{j,t} = \left[(1-\omega_j) \tilde{m}_{j,t}^{\rho_L} + \omega_j \tilde{s}_{j,t}^{\rho_L} \right]^{1/\rho_L}, \qquad \rho_L < 1, \tag{F.18}$$

where

$$\tilde{m}_{j,t} \equiv \frac{m^R_{j,t}}{\bar{m}_j(\omega_j)}, \qquad \tilde{s}_{j,t} \equiv \frac{s^R_{j,t}}{\bar{s}_j(\omega_j)}, \qquad \eta_L \equiv \frac{1}{1-\rho_L}. \tag{F.19}$$

with $m^R_{j,t}$ and $s^R_{j,t}$ real holdings normalized by their adoption-specific steady-state values, so that $\bar{L}_j = 1$ for every ω_j (Section D.2), and η_L the elasticity of substitution. To first order,

$$l_{j,t} = (1-\omega_j) mb_{j,t} + \omega_j s_{j,t}, \tag{F.20}$$

where $mb_{j,t}$ is the log deviation of real conventional-money holdings.

Steady State, Marginal Cost, and Consumption Steady-state holdings of the liability are tied to adoption, $\bar{s}_j = \kappa_L \omega_j$, and money balances to $\bar{m}_j = \kappa_L(1-\omega_j)$, with $\kappa_L = 1/3$ of a quarter's consumption; both liquid assets have user cost $1-\beta$ at the steady state, and $\chi_L = \kappa_L(1-\beta)$ makes $\bar{C}_j = \bar{L}_j = 1$ the model's own steady state. Labor supply, linear production, and an employment subsidy $\tau = 1/\mu$ that makes the steady-state allocation of labor efficient give real marginal cost, the real wage in units of domestic goods (Sections D.2–D.3),

$$mc_{j,t} = \nu y_{j,t} + (\sigma + \varphi_l) c_{j,t} - \varphi_l l_{j,t} + \mathrm{rp}_{j,t}, \tag{F.21}$$

where $\mathrm{rp}_{j,t} \equiv p_{j,t} - p^d_{j,t}$ is the price of consumption relative to domestic goods (Section 2.7). The liability affects marginal cost through the liquidity services households demand. The Euler equation for the domestic safe asset, $U_{C,j,t} = \beta R_{j,t} \mathbb{E}_t[U_{C,j,t+1}/\Pi_{j,t+1}]$, with $\Pi_{j,t+1}$ gross CPI inflation, gives

$$(\sigma + \varphi_l) c_{j,t} - \varphi_l l_{j,t} = \mathbb{E}_t\big[(\sigma + \varphi_l) c_{j,t+1} - \varphi_l l_{j,t+1}\big] - \big(i_{j,t} - \mathbb{E}_t \pi_{j,t+1}\big). \tag{F.22}$$

Portfolio Choice and Asset-Specific User Costs For conventional money, which is non-interest-bearing, the period user cost relative to the safe nominal asset is

$$\mathcal{U}^m_{j,t} \equiv 1 - \frac{1}{R_{j,t}}. \tag{F.23}$$

The digital liability is likewise non-interest-bearing contractually, but its value in local currency is not fixed. One unit is a fixed bundle of currencies (Section F.1), whose price in country j's currency, $X_{j,t}$, moves with the exchange rates of the currencies in the bundle, $\Delta \log X_{j,t} = \Delta d_{j,t}$ by (2). A unit acquired at t and held until $t+1$ therefore earns the gross nominal return $X_{j,t+1}/X_{j,t}$, against one for money and $R_{j,t}$ for the safe asset. The corresponding user cost is

$$\mathcal{U}^{sc}_{j,t} \equiv 1 - \frac{\mathbb{E}_t\big[X_{j,t+1}/X_{j,t}\big]}{R_{j,t}}. \tag{F.24}$$

Both user costs follow from the household's nominal budget constraint (Section D.5), and the first-order condition for the liability also contains the covariance between the household's marginal utility and the liability's return, $h_{j,t} \equiv \beta \operatorname{Cov}_t\big(\lambda_{j,t+1}/\lambda_{j,t}, X_{j,t+1}/X_{j,t}\big)$. Covariances generated by the model's own fluctuations are second order and drop out of the log-linear system. The covariance generated by a currency disaster does not, because the disaster is a large event whose probability moves at first order; it is derived in Section 2.3. CPI inflation erodes money and the liability alike and drops out of their relative cost, so what distinguishes the liability from money is its *expected* nominal appreciation in j's currency, $g_{j,t} \equiv \mathbb{E}_t \Delta d_{j,t+1}$ of (3).

The household's first-order conditions for conventional money and the digital liability are

$$\frac{U_{L,j,t}}{U_{C,j,t}}\frac{\partial L_{j,t}}{\partial m^R_{j,t}} = \mathcal{U}^m_{j,t}, \tag{F.25}$$

$$\frac{U_{L,j,t}}{U_{C,j,t}}\frac{\partial L_{j,t}}{\partial s^R_{j,t}} + h_{j,t} = \mathcal{U}^{sc}_{j,t}. \tag{F.26}$$

For the liability, the marginal value includes its value as insurance, $h_{j,t}$, which is positive when it pays off in the states in which the household's marginal utility is high.

Log-linearizing the two conditions around the steady state, at which both user costs equal $1-\beta$, and subtracting them eliminates the common shadow price of liquidity (Section D.5). With the elasticity of substitution $\eta_L = 1/(1-\rho_L)$ of (F.19), define the composite semi-elasticity

$$\eta \equiv \eta_L \frac{\beta}{1-\beta}, \tag{F.27}$$

equations (D.26)–(D.27) give the portfolio-composition condition

$$mb_{j,t} - s_{j,t} = -\eta\, g_{j,t} - H_{j,t}, \qquad H_{j,t} \equiv \frac{\eta}{\beta}\, h_{j,t}, \tag{F.28}$$

where $H_{j,t}$ is the hedging demand derived in Section 2.3. Households substitute from money into the digital liability when it is expected to appreciate in their own currency (the transactions motive, governed by η) and when it insures them against a disaster in their own currency (the insurance motive). Both are inherited from preferences, technology, and the distribution of risk rather than imposed as a reduced-form asset demand.

Transactions and Insurance: Why Households Hold the Liability Households hold the digital liability for two reasons, which the composition condition (14) separates. The first is the transactions (medium-of-exchange) motive: the liability provides liquidity services and is held more when it is expected to appreciate against local money. The second is the insurance (store-of-value) motive: the liability is a claim on a bundle that includes foreign currencies, so holding it hedges part of the household's exposure to its own currency, and it is held more when that currency is under pressure.

The two motives respond to different wedges (Calvo and Végh, 1992). In peripheral economies foreign-currency deposits serve as a store of value (Feige, 2003), deposit dollarization responds to interest-rate differentials (Basso et al., 2011), and financial dollarization can be read as a hedging choice: in the minimum-variance portfolio of Ize and Levy Yeyati (2003), dollarization rises with the volatility of inflation relative to that of the real exchange rate.

Currency disasters. The insurance motive comes from the risk of a currency disaster. In each period, country j's currency suffers a disaster with probability $p_{j,t}$: it loses a fraction J of its

value against the other two currencies, and the marginal utility of j's households rises by the factor $B_j > 1$, because the disaster destroys part of their real income and wealth. Following the literature on time-varying disaster risk (Barro, 2006; Gourio, 2012; Farhi and Gabaix, 2016), the probability of a disaster moves over time, and the dynamics of the model are those of the economy in which a disaster has not yet occurred. In the model only the periphery's currency is exposed, $p_{H,t} = p_{F,t} = 0$; a risk-off episode is a rise in $p_{R,t}$. The steady state is the non-stochastic one, $\bar{p}_R = 0$, and the financier prices the expected loss on R's currency,

$$\varpi_{R,t} \equiv J\, p_{R,t}, \tag{F.29}$$

which is the risk-off disturbance in R's premium (29). With a positive average probability of disaster, the disaster-state value derived below would also have a steady-state component.

Hedging demand. In a disaster in R's currency, one unit of the liability changes value, in j's currency, by $\pi_j J$: it rises by $(1-\theta_R^d)J$ in R's currency, $\pi_R = 1-\theta_R^d$, and falls by $\theta_R^d J$ in H's and F's, $\pi_H = \pi_F = -\theta_R^d$. R's marginal utility is multiplied by $B_R > 1$; households in H and F are not hit by the disaster, $B_H = B_F = 1$. The expected return $g_{j,t}$ in (12) is that of the no-disaster path and already carries the drift with which the financier compensates holders of R's currency for the expected loss. The disaster-state payoff, weighted by marginal utility in that state, is the remaining term, $h_{j,t} = \beta B_j \pi_j \varpi_{R,t}$ to first order, and substituting into (14),

$$H_{j,t} = \eta\, B_j\, \pi_j\, \varpi_{R,t}. \tag{F.30}$$

The expected loss is counted once, in the drift. Net of it, R's households expect the excess return $(B_R - 1)(1-\theta_R^d)\varpi_{R,t}$ on the liability; for H and F the drift and the disaster payoff cancel, and the liability is fairly priced. The hedging demand has three properties that a reduced-form store-of-value demand would not deliver. It is driven by risk—the probability of a disaster—and not by the endogenous components of the premium, such as the liability's own footprint. It is proportional to $1-\theta_R^d$, the share of foreign currencies in denomination: a liability that contains the holder's own currency insures the holder less, and one denominated entirely in it insures not at all. And its strength is governed by B_R, the jump in marginal utility in a disaster, which can be disciplined by the size of consumption declines in currency crises (Section 4.1).

The disaster-state value of the liability. Welfare is evaluated on the no-disaster path. Holding $\kappa_L \omega_j \tilde{s}_{j,t}$ of the liability adds to period utility on that path the disaster-state payoff, weighted by marginal utility in that state,

$$\beta\, B_j\, \pi_j\, \varpi_{R,t}\, \kappa_L \omega_j \tilde{s}_{j,t}, \tag{F.31}$$

which is zero at the steady state, where $\varpi_{R,t} = 0$, and enters the welfare criterion (Section C.1). It is the same object that generates the hedging demand: the household holds the liability up to the point at which its marginal value in the disaster state equals its user cost net of the drift. The

welfare measure counts the disaster state only through this term; the losses that a realized disaster inflicts on money, bonds, the liability and the issuer's reserves are not counted.

Whether a risk-off episode produces a flight *into* the liability or *out of* it depends on the balance of the two motives, and is characterized in Proposition 2.4 once exchange rates have been determined.

Liquidity Demand The composition condition (14) restricts only $mb_{j,t} - s_{j,t}$; the money condition, used on its own with the aggregator, pins down the scale of liquidity (Section D.8). Liquidity falls with the policy rate and rises with consumption, with the expected appreciation of the liability, and with hedging demand. The insurance motive acts exactly like an additional expected return $H_{j,t}/\eta$ on the liability: it shifts the composition of liquid assets toward the liability and, because the money condition does not contain it, also raises total liquidity. Through the liquidity gap the same terms reach marginal cost (10): a rate increase raises it, and an expected appreciation of the liability, or a rise in own-currency risk that makes it more valuable as insurance, lowers it.

Liquidity gap. The monetary component of household welfare is the liquidity gap,

$$\text{gap}_{j,t} \equiv c_{j,t} - l_{j,t}, \tag{F.32}$$

the deviation of consumption from the liquidity services available to the household. The contract reaches it through every channel above: denomination through holders' return and hedging demand, backing through the premia, and the unit of account through the prices of goods quoted in it.

F.4 International Asset Markets

Before introducing the digital liability, it is useful to fix how international bond markets clear and how the resulting risk premium is priced, since the digital liability will be shown to operate through exactly this channel rather than through a channel of its own.

External Balance Sheets Households trade goods with the other two countries and hold bonds only in their own currency. Cross-border positions in bonds are intermediated by an international financier, who takes the other side of each country's position in that country's currency (Gabaix and Maggiori, 2015). The digital liability adds two cross-border positions: residents' holdings of the liability, which are claims on the issuer whose value moves with the currencies of θ^d, and the issuer's holdings of each government's bonds. In units of H's goods, country j's net foreign assets are

$$\text{NFA}_{j,t} = S_{j,t} - \theta_j^a A_{sc,t} + F_{j,t}, \tag{F.33}$$

where $S_{j,t}$ is the value of its residents' holdings of the liability, $\theta_j^a A_{sc,t}$ the value of its bonds held by the issuer (Section 2.4), and $F_{j,t}$ its position with the financier, in its own currency. Each position earns its own return: the liability pays no interest and revalues with the bundle θ^d; country j pays the holding-period return on its bonds, including any capital gain and the movement of its

currency; and $F_{j,t}$ earns country j's policy rate. Let $b_{j,t}$ denote the deviation of $\text{NFA}_{j,t}$ from its steady state, in units of steady-state quarterly output. To first order,

$$b_{j,t} = \beta^{-1} b_{j,t-1} + nx_{j,t} + \lambda_j \bar{s}_j \big(\Delta x_t^H - \pi_{Hd,t}\big) - \frac{\theta_j^a \bar{s}}{\beta}\big(r_{j,t} - \pi_{Hd,t}\big) + \frac{\bar{F}_j}{\beta}\big(i_{j,t-1} + \dot{e}_{Hj,t} - \pi_{Hd,t}\big) - \left(\beta^{-1} - 1\right) dS_{j,t-1} + \alpha_j^T \,\text{div}_{sc,t} + n_{j,t}^{cy} + \text{os}_{j,t}, \tag{F.34}$$

with $\sum_j b_{j,t} + E_{sc,t} = 0$. Here $nx_{j,t}$ is net exports (Section 2.7); $\Delta x_t^H = \theta_F^d \dot{e}_{HF,t} + \theta_R^d \dot{e}_{HR,t}$ is the revaluation of the liability in H's currency, and $\pi_{Hd,t}$ deflates nominal positions into H's goods; $r_{j,t} = i_{j,t-1} + \dot{e}_{Hj,t} + \beta \delta_j \Delta q_{j,t}$ is the return on country j's bonds in H's currency, with $\dot{e}_{HH,t} \equiv 0$; $\bar{F}_j = (\theta_j^a - \theta_j^e)\bar{s}$ is the steady-state position with the financier, non-zero only for a hedged issuer; $dS_{j,t}$ is the deviation of $S_{j,t}$, on which no interest is paid; $\text{div}_{sc,t}$ is the issuer's payout and $n_{j,t}^{cy}$ the convenience-yield transfer; and $\text{os}_{j,t}$ is the default transfer on R's dollar debt, $-\kappa_{os} \bar{B}_R \text{tot}_{HR,t}$ for R and its opposite for H's creditors (Section 2.5). The issuer's income is paid by the countries whose bonds it holds, through the fourth term, and returned to all countries in population shares through its payout. Because the issuer's equity is the world's remaining asset, the external positions sum to minus that equity. The difference between two countries' positions, $b_{jk,t} \equiv b_{j,t} - b_{k,t}$, is what the financier must absorb in market jk. Because $b_{j,t}$ is predetermined at t, it is a state variable of the equilibrium system; what remains to be determined is how it is priced.

International Portfolio-Balance Frictions: Source of the Risk Premium International bond markets are segmented. Following the intermediary-based approach to imperfect asset substitutability (Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021; Camanho et al., 2022), a competitive but capacity-constrained financier intermediates the cross-border positions that countries' external balances leave outstanding.

In bilateral market jk, let $x_{jk,t}$ denote the net stock of k-currency claims the financier must absorb. For now the only source of $x_{jk,t}$ is the difference between the two countries' external positions, $x_{jk,t} = b_{jk,t}$; Section 2.6 shows that the digital liability adds a second, additive source, through the currencies households sell to acquire it and the reserves the issuer buys. The financier chooses $x_{jk,t}$ each period to maximize the expected excess return on the position net of a quadratic intermediation cost,

$$\max_{x_{jk,t}} \mathbb{E}_t \left[R_{k,t+1} \frac{E_{jk,t+1}}{E_{jk,t}} - R_{j,t} \right] x_{jk,t} - \frac{\psi_b}{2} x_{jk,t}^2, \tag{F.35}$$

where $E_{jk,t}$ is the price of currency k in units of currency j and $R_{j,t}$, $R_{k,t}$ are the gross returns in each currency; the cost parameter $\psi_b > 0$ is common across markets to preserve triangular consistency. The first-order condition is

$$\mathbb{E}_t \left[R_{k,t+1} \frac{E_{jk,t+1}}{E_{jk,t}} - R_{j,t} \right] = \psi_b \, x_{jk,t}. \tag{F.36}$$

Log-linearizing the expected excess return around the zero-inflation symmetric steady state gives $i_{k,t} + \mathbb{E}_t[e_{jk,t+1} - e_{jk,t}] - i_{j,t}$, so that, with $x_{jk,t} = b_{jk,t}$, rearranging in terms of the exchange-rate level $e_{jk,t}$ gives

$$e_{jk,t} = \mathbb{E}_t e_{jk,t+1} + i_{k,t} - i_{j,t} - \psi_b \, b_{jk,t}. \tag{F.37}$$

More generally, (F.36) is a pricing *rule*: whatever currency exposure enters $x_{jk,t}$, the financier prices it at the same rate ψ_b, since its cost does not distinguish the source of the position. Section 2.6 applies this rule to the exposure created by the digital liability, and Section 2.5 extends the financier's problem to the periphery's dollar debt.

F.5 Government Bonds and the Convenience Yield

The issuer must hold eligible bonds in the shares θ^a whatever their yield. This section derives the effect of that demand on bond yields, and the cross-border transfer it generates.

Bond supply. The government of country k has outstanding a stock of one-period bonds B^g_k, in units of steady-state quarterly output, which is held fixed; its budget is balanced by lump-sum taxes on domestic households. The bonds are held by the issuer, whose holdings are $\theta^a_k A_{sc,t}$ by the reserve rule (F.40) below, and by households of all three countries.

The demand for convenience. Government bonds provide collateral and settlement services that other assets do not (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang et al., 2021), which we model as a saving of resources for the households that hold them, in the spirit of the transaction-cost foundation of money demand (Feenstra, 1986). Holdings of foreign bonds are hedged at covered interest parity on policy rates. Households' demand for these services makes the convenience yield $\mathrm{CY}_{k,t} \equiv 1 - R^g_{k,t}/R_{k,t}$, the spread between the policy rate and the yield on country k's bonds, common to all holders and decreasing in the stock of bonds in private hands, $B^g_k - \theta^a_k A_{sc,t}$ (Section D.8). Because the services save resources rather than yield utility, it does not depend on households' marginal utility of consumption.

The yield effect of reserve demand. When the issuer buys a larger share of country k's bonds, households hold fewer, the marginal value of their services rises, and the bond yield falls relative to the policy rate. With $A_{sc,t} = \bar{s} + dS_t + E_{sc,t}$, (D.45) gives the steady-state increase $\Delta\overline{\mathrm{CY}}_k = b_{cy} \log[\bar{B}^p/(\bar{B}^p - \theta^a_k \bar{s})]$ and, around it,

$$\mathrm{cy}_{k,t} \equiv \mathrm{CY}_{k,t} - \overline{\mathrm{CY}}_k = b_{cy} \log \frac{\bar{B}^p - \theta^a_k \bar{s}}{\bar{B}^p - \theta^a_k(\bar{s} + dS_t + E_{sc,t})} \approx \zeta_{y,k}\, \theta^a_k \big(dS_t + E_{sc,t}\big), \qquad \zeta_{y,k} \equiv \frac{b_{cy}}{\bar{B}^p - \theta^a_k \bar{s}}, \tag{F.38}$$

where $\bar{B}^p$ is private holdings before the issuer's purchases, common across countries. The nonlinear model uses the exact expression. The convenience yield is attached to backing, θ^a, and not to the effective position θ^e: hedging changes the issuer's currency exposure but not which bonds it holds.

Who gains and who pays. A fall in the yield on country k's bonds lowers the interest its government pays on its whole debt, and the saving is rebated to domestic households. Holders of the bonds receive less interest. Because households' holdings satisfy (D.44), the envelope theorem implies that a change in the yield affects their net income from bonds, to first order, only through the change in interest on the quantity they hold: the resources they spend on collateral services when they hold fewer bonds are exactly offset by the interest they no longer forgo (Section A.5). The welfare effect of the convenience yield is therefore its pecuniary transfer. Payments between residents of the same country cancel nationally; what remains is the transfer on the part of the debt held abroad. Let φ^f denote the share of each government's debt held by non-residents, including the issuer, and let non-resident holders be distributed across countries in the population shares λ_j; the issuer's own share reaches them through its payout, which is distributed in the same shares. Country k then receives, from the rest of the world,

$$N_{k,t} = \varphi^f B^g_k \,\mathrm{CY}_{k,t-1} - \lambda_k \sum_m \varphi^f B^g_m \,\mathrm{CY}_{m,t-1}, \qquad \textstyle\sum_k N_{k,t} = 0, \tag{F.39}$$

with the one-period lag reflecting that bonds bought at $t-1$ pay at t. The part of this transfer due to the liability is, for a common debt stock $B^g_k = B^g$, equation (22). The transfer per unit of the liability held as reserves can exceed the issuer's own seigniorage rate $\beta^{-1} - 1$ because the yield effect applies to the whole stock of foreign-held debt. Each country pays in proportion to its population share and receives in proportion to its weight in backing; the transfer enters the external positions (18).

Exchange rates. The convenience yield is a spread within each currency and does not enter the parity conditions of Section 2.6. The issuer's purchases reach exchange rates through a separate channel, the currencies it buys, so the same purchase lowers the yield on the bonds bought, through θ^a, and supports the currencies of the issuer's effective position, through θ^e.

F.6 The Institutional Issuer

The issuer sells units of the liability at their value and invests the proceeds in reserves. It does not choose the architecture; it implements the contract and manages the resulting balance sheet.

Reserves and effective currency position. The issuer holds reserve bonds with value shares θ^a,

$$A_{sc,j,t} = \theta^a_j A_{sc,t}, \qquad j \in \{H, F, R\}, \tag{F.40}$$

and may hedge part of its currency exposure by forward contracts with the international financier. A share $\varsigma \in [0,1]$ of the exposure hedged into the denomination currencies gives an *effective* currency position

$$\theta^e \equiv (1-\varsigma)\,\theta^a + \varsigma\,\theta^d. \tag{F.41}$$

An unhedged issuer ($\varsigma = 0$) carries $\theta^e = \theta^a$; a fully hedged issuer ($\varsigma = 1$) carries $\theta^e = \theta^d$ whatever its reserves. The baseline sets $\varsigma = 0$. The reserve rule (F.40) holds at every date, so the issuer rebalances: when the value of its holdings of one country's bonds falls relative to the others, because that country's interest rate rises or its currency depreciates, it buys that country's bonds and sells the others to restore the shares θ^a. Section 4.9 relaxes the speed of rebalancing.

Balance sheet. Let L_t denote the value, in H's currency, of the liability outstanding and $E_{sc,t}$ the issuer's equity. At every date,

$$A_{sc,t} = L_t + E_{sc,t}. \tag{F.42}$$

New units are sold at their value and the proceeds are invested in reserves, so issuance raises both sides of (F.42) equally and leaves equity unchanged. Between $t-1$ and t, reserves earn their realized return R_t^A and the liability gains value as the currencies in θ^d move, so that, with dividends D_t,

$$E_{sc,t} = R_t^A A_{sc,t-1} - \frac{X_t^H}{X_{t-1}^H} L_{t-1} - D_t.$$

Reserve returns. Reserve assets are bonds paying a coupon of 1 at $t+1$, decaying geometrically at rate $\delta_j \in [0,1]$ thereafter. For the issuer's accounts they are valued at the net policy rate $I_{j,t} \equiv R_{j,t} - 1$; the convenience yield it forgoes is recorded in the transfer (F.39). Their price is $Q_{j,t} = 1/(1 + I_{j,t} - \delta_j)$, a one-period bill at $\delta_j = 0$ and a perpetuity at $\delta_j = 1$; with Macaulay duration $D_j \equiv 1/(1 - \beta\delta_j)$, $q_{j,t} = -D_j\, i_{j,t}$, and the log one-period return is $r_{j,t} = i_{j,t-1} + \beta\delta_j \Delta q_{j,t}$, the yield locked in at purchase plus a capital gain that is zero for a one-period bill (Section D.8). The baseline sets $\delta_j = 0$, so that $D_j = 1$ quarter, consistent with reserve-management practice and existing reserve requirements; Section 4.9 varies it up to the perpetuity. The realized return on the effective reserve position, in H's currency, is

$$r_t^A = \sum_j \theta_j^e \big(i_{j,t-1} + \beta\delta_j \Delta q_{j,t}\big) + \theta_F^e \dot{e}_{HF,t} + \theta_R^e \dot{e}_{HR,t}. \tag{F.43}$$

Equity. At the steady state the issuer earns $(\beta^{-1} - 1)\bar{s}$ per period: interest on its reserves, while the liability pays none. This seigniorage is paid out with dividends, so that steady-state equity is zero. Dividends in excess of seigniorage are a fraction ϕ_{div} of last period's equity. Log-linearizing the balance sheet, using (F.43) and $\Delta \log X_t^H = \theta_F^d \dot{e}_{HF,t} + \theta_R^d \dot{e}_{HR,t}$, gives

$$E_{sc,t} = \big(\beta^{-1} - \phi_{\text{div}}\big) E_{sc,t-1} + \pi_t^A + \text{val}_{q,t} + \text{val}_{fx,t}, \tag{F.44}$$

with three components:

$$\pi^A_t = \frac{\bar{s}}{\beta} \sum_j \theta^e_j \, i_{j,t-1} + \bar{s}\,(1-\beta^{-1})\,\pi_{Hd,t}, \tag{F.45}$$

$$\mathrm{val}_{q,t} = \bar{s} \sum_j \theta^a_j \, \delta_j \, \Delta q_{j,t}, \tag{F.46}$$

$$\mathrm{val}_{fx,t} = \bar{s}\Big[(\theta^e_F - \theta^d_F)\dot{e}_{HF,t} + (\theta^e_R - \theta^d_R)\dot{e}_{HR,t}\Big] + (\beta^{-1}-1)\,\bar{s}\big(\theta^e_F \dot{e}_{HF,t} + \theta^e_R \dot{e}_{HR,t}\big). \tag{F.47}$$

The first is the carry earned on yields fixed a period earlier, the second the capital gain on reserves of longer duration, and the third the revaluation of the open currency position: reserves in currencies other than those the issuer owes. Its last term, the interest earned on the revaluation of reserves, is of order $\beta^{-1}-1$ and is neglected in the statements below. Every return is booked once, when realized. The unit of account does not appear: the issuer owes currency amounts, not a price index, so it carries neither unit-of-account nor indexation risk.

Duration risk is governed separately: $\mathrm{val}_{q,t}$ depends on θ^a and δ_j and is zero for one-period bills whatever the alignment. Whether it matters for anyone other than the issuer depends on the issuer's effective position.

Payout and transfers. Dividends beyond seigniorage are paid from the accumulated equity stock, $\mathrm{div}_{sc,t} = \phi_{\mathrm{div}} E_{sc,t-1}$; with $\phi_{\mathrm{div}} < 1$ part of each accrual is retained, which is the buffer mechanism of Section 4.5. Dividends are distributed as lump-sum transfers to member countries in fixed shares α^T_j, population shares λ_j in the baseline and backing weights θ^a_j in the alternative of Section 3.2,

$$\mathrm{div}_{j,t} = \alpha^T_j \, \mathrm{div}_{sc,t}, \tag{F.48}$$

and the seigniorage on the deviation of the liability outstanding, $(\beta^{-1}-1)dS_{t-1}$, is paid out with them. Because they are lump sum, no holder can raise its transfer by holding more of the liability, which therefore remains non-interest-bearing from the holder's point of view. The issuer's income is paid, in turn, by the countries whose bonds it holds, as the return on those bonds in their external positions (18), so the issuer neither creates nor destroys resources at the world level.

The consolidated accounting is in Section D.7.

F.7 Peripheral External Debt and Original Sin

Peripheral economies borrow abroad predominantly in foreign currency (Eichengreen and Hausmann, 1999). When they do, a depreciation raises the domestic-currency value of their external debt, weakens their balance sheets, and raises the premium investors require on their currency, so that the exchange rate feeds back on its own risk premium (Céspedes et al., 2004). The digital liability changes this exposure, because the issuer's reserves include R-currency bonds whenever $\theta^a_R > 0$: foreign demand for R-currency debt is exactly what original sin denies the periphery, and

what proposals to create an international market for emerging-market currency debt seek to supply (Eichengreen and Hausmann, 2005).

Debt and its currency composition. R has a steady-state stock of gross external debt $\bar{B}_R$, in units of steady-state output. Absent the liability, a share χ^0_{os} of it is denominated in H's currency. The issuer's steady-state holdings of R-currency bonds, $\theta^a_R \bar{s}$, are foreign lending to R in R's own currency. We assume that R's borrowers use this demand to refinance dollar debt one for one, so that the dollar share of R's external debt is

$$\chi_{os}(\theta^a_R) = \chi^0_{os} - \frac{\theta^a_R\, \bar{s}}{\bar{B}_R}, \qquad 0 \le \chi_{os}(\theta^a_R) \le \chi^0_{os}. \tag{F.49}$$

The currency composition of the debt is a slow-moving feature of R's balance sheet, set by the steady-state architecture; it is held fixed over the business cycle, as adoption is.

Balance-sheet burden. Debt and the capacity to repay it are measured in real terms: the dollar part of the debt is a claim on H's output, while the R-currency part and borrowers' assets are claims on R's output. What matters for repayment is then the relative price of the two outputs, the terms of trade $\text{Tot}_{HR,t} = E_{HR,t} P^d_{R,t} / P^d_{H,t}$ of Section D.6, not the nominal exchange rate. The value of the dollar debt in units of R's output is

$$\Gamma_{R,t} = \frac{\chi_{os}\, \bar{B}_R}{\text{Tot}_{HR,t}}, \qquad \hat{\Gamma}_{R,t} \equiv \log \frac{\Gamma_{R,t}}{\bar{\Gamma}_R} = -\text{tot}_{HR,t}, \tag{F.50}$$

so a real depreciation of R's currency, a fall in $\text{tot}_{HR,t}$, raises the burden one for one. On impact, with sticky prices, real and nominal depreciation nearly coincide; a nominal depreciation that only offsets higher inflation in R leaves the burden unchanged.

Default and the premium. R's external debt is owed by a continuum of borrowers whose assets are dispersed; a borrower defaults when its assets fall short of its debt, and its creditors then lose a fraction ℓ of their claim (Section D.8). A real depreciation raises the value of the dollar debt in R's output and pushes more borrowers below the default threshold. The debt is long-term with a coupon that prices the steady-state default rate, so defaults in excess of the steady state are a transfer from R's creditors to R's borrowers: default works as partial insurance for R, paid for by its creditors, and enters the external positions (18). The financier prices the expected loss on its R-currency positions. With ζ_{os} the response of that loss to a one-percent rise in R's leverage (D.51), which rises with the loss given default and falls with the dispersion of borrowers' assets, the premium on R's currency acquires the term

$$-\kappa_{os}\, \text{tot}_{HR,t}, \qquad \kappa_{os}(\theta^a_R) \equiv \zeta_{os}\, \chi_{os}(\theta^a_R), \tag{F.51}$$

which enters $\psi_{HR,t}$ in (29). Within the period, $\text{tot}_{HR,t}$ moves one for one with $e_{HR,t}$ at given inflation, so collecting $e_{HR,t}$ in the H–R parity condition (30), its coefficient becomes $1-\kappa_{os}$: for $0<\kappa_{os}<1$, the impact of a risk-off disturbance on R's currency is scaled by $1/(1-\kappa_{os})>1$, holding interest rates, expectations, and inflation fixed. A depreciation raises the burden, the burden raises the premium, and the premium deepens the depreciation. Because R households' hedging demand responds to the risk of a disaster and not to the premium itself (16), original sin deepens the depreciation without strengthening the insurance motive.

Backing and the allocation of peripheral currency risk. Equation (26) makes original sin a function of backing. The following result shows what backing does, and does not, change.

Proposition 2.3 is the sense in which backing is a substitute for, and not a remedy of, original sin. The architecture cannot reduce the total exposure to R's currency; it can only choose who bears it. What changes is how that exposure is priced: on R's borrowers it amplifies depreciation through κ_{os}; on the issuer it reduces equity and reaches premia only through the footprint and, if $\chi_E>0$, intermediation capacity; on holders it is spread across countries.

F.8 Currency-Market Clearing and the Financial Wedge

Section F.4 showed that the financier prices any currency exposure it must absorb at the rate ψ_b. This section derives the exposure the digital liability creates from the clearing of each currency's asset market, under four assumptions: (A1) households pay for the liability with assets in their own currency and convert redemption proceeds back into their own currency; (A2) the issuer keeps its effective position at θ^e; (A3) central banks set interest rates and supply money elastically at those rates, so that a fall in money demand in country k becomes an increase in the supply of currency-k assets to the market; (A4) the financier absorbs the residual relative imbalance between any two currencies.

The liability's footprint. Let $S_{k,t}$ be the value, in H's goods, of the liability held by residents of country k, so that $L_t=\sum_k S_{k,t}$. By (A1) and (A3), those residents have released $S_{k,t}$ of currency-k assets to the market; by (A2) the issuer has absorbed $\theta^e_k A_{sc,t}$ of them. The liability's net addition to the supply of currency-k assets is $Z_{k,t}=S_{k,t}-\theta^e_k A_{sc,t}$. A financier with a symmetric cost of holding each currency, choosing positions q_k with $\sum_k q_k=0$, requires an excess return on currency k over currency j of $\psi_b(q_k-q_j)$; with $q_k=Z_{k,t}$, it holds the relative imbalance

$$x^{sc}_{jk,t}=Z_{k,t}-Z_{j,t}. \tag{F.52}$$

A positive $x^{sc}_{jk,t}$ is an excess supply of currency-k assets relative to currency j, for which the financier requires a higher return on k. At the steady state,

$$\bar{x}^{sc}_{jk}=\bar{s}\Big[(\bar{h}_k-\theta^e_k)-(\bar{h}_j-\theta^e_j)\Big]. \tag{F.53}$$

Log-linearizing, with $dS_{k,t} \equiv S_{k,t} - \lambda_k \bar{s}_k = \lambda_k \bar{s}_k \big(s_{k,t} + \mathrm{rp}_{k,t} + \mathrm{tot}_{Hk,t}\big)$, the real holdings $s_{k,t}$ valued at the CPI of country k and converted into H's goods, and $dS_t = \sum_k dS_{k,t}$, the digital-liability component of the bilateral premium is

$$\psi^{sc}_{HF,t} = \psi_b \Big[dS_{F,t} - dS_{H,t} - (\theta^e_F - \theta^e_H)\big(dS_t + E_{sc,t}\big) \Big], \tag{F.54}$$

$$\psi^{sc}_{HR,t} = \psi_b \Big[dS_{R,t} - dS_{H,t} - (\theta^e_R - \theta^e_H)\big(dS_t + E_{sc,t}\big) \Big], \tag{F.55}$$

with $\psi^{sc}_{FR,t} = \psi^{sc}_{HR,t} - \psi^{sc}_{HF,t}$. The financier's total position in market jk is $x_{jk,t} = b_{jk,t} + x^{sc}_{jk,t}$, the difference between the two countries' external positions plus the footprint. At the steady state it is zero: the imbalance (F.53) is held by countries as external positions, $\bar{b}_{jk} = -\bar{x}^{sc}_{jk}$ (Proposition 2.5(ii)). Because the financier's steady-state positions are zero, their revaluation is of second order; the revaluation of countries' steady-state positions enters their external accounts (18). Equity enters (F.54)–(28) because retained earnings are invested in the effective position.

For a dollar-backed liability, an increase Δ in R residents' holdings gives $x^{sc}_{HR} = 2\Delta$: R residents sold R-currency assets and the issuer bought dollar assets. With $\theta^e = (0, 0, 1)$, $x^{sc}_{HR} = 0$: the issuer recycles the proceeds into R's own bonds.

Where denomination enters. Denomination appears in (F.54)–(28) only through θ^e. On redemption the issuer converts its effective position into the currencies of θ^d and holders convert those into their own currency; after both conversions the end-of-period stocks are the reverse of the original purchase, and the intermediate step washes out. Denomination moves currency markets when the issuer aligns its backing with denomination or hedges into it, so that $\theta^e = \theta^d$, and, outside the model, through convertibility risk when holders retain the settlement currencies.

The complete premium. The bilateral premia that enter the parity conditions are

$$\psi_{HF,t} = \psi_b b_{HF,t} + \psi^{sc}_{HF,t} - \chi_E E_{sc,t}, \tag{F.56}$$

$$\psi_{HR,t} = \psi_b b_{HR,t} + \psi^{sc}_{HR,t} - \chi_E E_{sc,t} - \kappa_{os}\, \mathrm{tot}_{HR,t} + \varpi_{R,t}, \tag{F.57}$$

with $\psi_{FR,t} = \psi_{HR,t} - \psi_{HF,t}$. The first term is the difference between countries' external positions (Section F.4), the second the liability's footprint, the fourth original sin (27). The coefficient $\chi_E \geq 0$ captures the extent to which institutional equity eases intermediation capacity, a primitive distinct from ψ_b in the spirit of net-worth-constrained intermediary asset pricing (Gabaix and Maggiori, 2015): replacing ψ_b by $\psi_b/(1 + \chi'_E E_{sc,t}/\bar{E})$ in the financier's problem and log-linearizing generates the additive term. The single exogenous disturbance is a risk-off shock to R's premium,

$$\varpi_{R,t} = \rho_\varpi \varpi_{R,t-1} + \varepsilon_{\varpi_R,t}, \qquad \varepsilon_{\varpi_R,t} \sim \mathrm{iid}(0, \sigma^2_\varpi). \tag{F.58}$$

With $e_{jk,t}$ the log price of currency k in units of currency j, the exchange rates satisfy

$$e_{HF,t} = \mathbb{E}_t e_{HF,t+1} + i_{F,t} - i_{H,t} - \psi_{HF,t}, \tag{F.59}$$

$$e_{HR,t} = \mathbb{E}_t e_{HR,t+1} + i_{R,t} - i_{H,t} - \psi_{HR,t}, \tag{F.60}$$

$$e_{FR,t} = e_{HR,t} - e_{HF,t}. \tag{F.61}$$

Flight into or out of the liability. The parity conditions determine the expected exchange-rate changes that enter holders' return (3): $\mathbb{E}_t \dot{e}_{Hk,t+1} = \psi_{Hk,t} - (i_{k,t} - i_{H,t})$ for $k \in \{F, R\}$. A currency under pressure is expected to recover, so a liability that does not contain it is expected to lose value against it.

The transactions motive pushes R households out of a liability that is expected to lose value as R's currency recovers; the insurance motive pulls them in. Both work through $1 - \theta_R^d$, so the contract decides how large a flight is but not its direction. A monetary tightening in R, which raises $i_{R,t}$ and reduces the expected recovery of R's currency, favors flight into the liability. Original sin, which raises $\psi_{HR,t}$ without raising the risk of a disaster, weakens it.

Neutrality, privilege, and orthogonality. The following results characterize what the liability does to currency and bond markets.

The yield privilege follows the bonds the issuer holds, θ^a; the balance-sheet privilege and the safe-haven appreciation follow its effective position, θ^e. Under alignment, $\theta^a = \theta^d$, both follow denomination, which is the configuration of the dollar system.

Invariance at the neutral position. The preceding results describe each clause separately. The next one shows that, once the issuer's position is neutral, denomination and the unit of account are cut off from exchange rates altogether. It rests on a property of the contract: both clauses move every country's variables by the same amount. By (3) and (4), with $\dot{e}_{HH,t} \equiv 0$,

$$g_{j,t} = G_t - \mathbb{E}_t \dot{e}_{Hj,t+1}, \qquad \Delta u_{j,t} = \mathcal{U}_t - \dot{e}_{Hj,t}, \tag{F.62}$$

where $G_t \equiv \theta_F^d \mathbb{E}_t \dot{e}_{HF,t+1} + \theta_R^d \mathbb{E}_t \dot{e}_{HR,t+1}$ and $\mathcal{U}_t \equiv \theta_F^u \dot{e}_{HF,t} + \theta_R^u \dot{e}_{HR,t}$ are common to all countries. Denomination enters holders' return only through G_t, and the unit of account enters prices only through $\mathcal{U}_t$.

Proposition 2.6 is the formal sense in which a neutral position separates the clauses. There, the unit of account and, except for the periphery's weight, denomination act only on the common component of the world economy's response to shocks. Countries still evaluate that common change differently, because their welfare losses are quadratic in paths that differ across countries, but no country can use these clauses to shift exchange-rate adjustment onto another, whatever the duration of reserves. Each condition is necessary: with an effective position that differs from population shares,

denomination reaches currency markets through the footprint and the issuer's transfers, and with $\chi_E > 0$ through the equity that $\text{val}_{fx,t}$ creates. Section 4.4 confirms the invariance numerically.

F.9 Market Clearing and Aggregation

The remaining equilibrium conditions are standard: the mapping from domestic to CPI inflation, goods-market clearing, the determination of net exports (which Section 2.4 took as given), and the monetary-policy rule that closes the nominal side of the model.

Prices and trade. CPI inflation in each country combines the inflation of local-priced domestic goods, of U-priced domestic goods converted into local currency, and of imports, with import share γ; the CPI of country R is

$$\pi_{R,t} = (1-\gamma)\big[(1-\Upsilon_R)\pi_{Rd,t} + \Upsilon_R(\pi^U_{R,t} + \Delta u_{R,t})\big] + \tfrac{\gamma}{2}\big(\pi_{Hd,t} - \dot{e}_{HR,t}\big) + \tfrac{\gamma}{2}\big(\pi_{Fd,t} - \dot{e}_{FR,t}\big), \quad \text{(F.63)}$$

and those of H and F are analogous (Section D.6). With the terms of trade $\text{tot}_{jk,t} \equiv e_{jk,t} + p^d_{k,t} - p^d_{j,t}$, the price of consumption relative to domestic goods is

$$\text{rp}_{j,t} \equiv p_{j,t} - p^d_{j,t} = \tfrac{\gamma}{2}\big(\text{tot}_{jk,t} + \text{tot}_{jl,t}\big) - (1-\gamma)\Upsilon_j z_{j,t}, \quad \text{(F.64)}$$

for $k, l \neq j$: imports are dearer when the terms of trade are high, and the domestic bundle is cheaper than local-priced goods when the U-priced goods in it are relatively cheap.

Goods-Market Clearing and Monetary Policy Each household spends a share $\gamma/2$ of its consumption expenditure on the goods of each trading partner, which are priced in the producer's currency, and a share $1-\gamma$ on the domestic bundle. Output is demand determined,

$$y_{j,t} = (1-\gamma)\big(c_{j,t} + \text{rp}_{j,t} + \Upsilon_j z_{j,t}\big) + \sum_{k \neq j} \tfrac{\gamma}{2}\big(c_{k,t} + \text{rp}_{k,t} + \text{tot}_{jk,t}\big), \quad \text{(F.65)}$$

and net exports, in units of domestic goods, are exports less imports,

$$nx_{j,t} = \sum_{k \neq j} \tfrac{\gamma}{2}\big(c_{k,t} + \text{rp}_{k,t} + \text{tot}_{jk,t}\big) - \gamma\big(c_{j,t} + \text{rp}_{j,t}\big), \qquad \sum_j nx_{j,t} = 0. \quad \text{(F.66)}$$

The transfers generated by the liability, (F.48) and (22), are cross-border payments; they enter the external positions (18) rather than the expenditure identity.

Each central bank follows an inertial Taylor rule in CPI inflation and output,

$$i_{j,t} = \rho_i\, i_{j,t-1} + (1-\rho_i)(\phi_\pi \pi_{j,t} + \phi_y\, y_{j,t})\,. \quad \text{(F.67)}$$

so that imported inflation, including the pass-through of a peripheral depreciation into $\pi_{R,t}$, feeds

directly into policy rates.

Together, these conditions close the general-equilibrium system. The full log-linear system implied by the nonlinear equilibrium conditions of Sections 2.1–2.7 is derived in Appendix A, and the Blanchard–Kahn conditions are verified in Appendix B.