”On the Predictability of the DJIA and S&P500 Indices”

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Abstract

We obtained from Standard and Poor’s Corporation, the complete 126-year history of the Dow Jones Industrial Average (DJIA) daily closing prices. We are applying rolling window averaging and adaptive learning methodologies, coupled with robust estimation methods, to examine which are the best forecasting models over a broad range of economic and financial conditions during the life of the index, based on daily and monthly stock index prices and daily, monthly, and semi-annual stock returns. Why is an AR(1) model a reasonable benchmark of stock prices? Why do we have it? What should be our forecasting benchmarks? Let us briefly re-visit the history of stock price research and efficient markets. Do we find forecasting improvements from the Hendry-Castle-Doornik-Clements approach using robust forecasting methodologies and saturation variables in the prices of the index? Given that the DJIA fell over 15% during the first half of 2022, is this one of the worst six-month periods ever? What has happened to the Dow, historically, during such periods in the past with regards to six-month, one-year, and three-year-ahead stock returns? Is capitalism dead or doomed? We report statistically significant forecasting improvement from saturation and robust forecasting techniques during the 1896 -June 2022 period. We report forecasted stock returns for the next 6 months and three years that are bullish. In the King’s English, June 30, 2022 was another excellent common stock buying opportunity and capitalism is not dead.

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I Introduction

Intelligent people have always been interested in modeling and forecasting stock prices and returns. In the United States, people have talked about the Dow Jones Industrial Average, commonly referred to as "the Dow", and its creator Charles Dow, the first editor of The Wall Street Journal, since the creation of the Dow in 1896. In this brief introduction to stock price forecasting, we introduce the reader to the edited stock price volume of Professor Paul Cootner (1964) and tests of stock market efficiency. The Cootner volume contained reprints of economists, statisticians, and operations research specialists from 1900 to 1963. The journals represented in the volume ranged from the *Journal of Finance*, the *Journal of Business*, the *Journal of the Royal Statistical Society*, *Operations Research*, *Econometrica*, *Kyklos*, the *Food Research Institute Studies*, to the *Industrial Management Review*. The Cootner volume included well known economists, such as Paul Samuelson and Holbrook Working and younger economists such as Clive Granger and Eugene Fama that would, like Samuelson, win the Nobel Prize in Economics. The volume included a practitioner, Alfred Cowles, founder of the Cowles Foundation, and Maurice Kendall, the eminent statistician, and M. F. M. Osborne, an operations research specialist. The weak form of the efficient markets hypothesis (EMH), put forth in Roberts (1959), held that price changes are random, and that the current price reflected no known information of the stock price history and technical analysis was futile. The Cootner volume papers supported the randomness of stock price changes and the EMH, as originally reported in Bachelier (1900), Cowles (1960), once transactions costs are admitted, and Granger and Morgenstern (1963). Granger and Morgenstern used spectral analysis to report the randomness of the S&P500 stock market index, 1875 -1952. Cootner (1962) tested 45 New York Stock Exchange, NYSE, regarding weekly price changes during the 1956 -1960 period. One-third of the stocks had statistically significant excessive mean-square successive difference tests, evidence of excessive reversals, and kurtosis. Non-randomness of stock prices was reported in Cootner (1962) and Moore (1962).

Professor Maurice Kendall, in his *Journal of the Royal Statistical Society* (1953) study of prices, studied 19 series of weekly indexes of industrial share prices during the 1928-1938 period and reported that there was little serial correlation within or between the share price series. "Unless individual stocks behave differently from the average of similar stocks, there is no hope of being able to predict movements on the exchange for a week ahead without extraneous information" (p. 85). Professor Kendall [1] also studied wheat and cotton prices and reported statistically significant first order and second-order serial correlations (also lags 4 and 6), but the coefficients reversed signs in the lags, and stated that the systematic elements were quite small compared to the random components. Professor Kendall noted that breaks were present in the

[1] The page numbers cited in this introduction are the page numbers of the articles reprinted in the Cottner (1964) volume.

cotton series because of the American Civil War. "Special events" can be present in the data. Finally, Professor Kendall concluded that more positive results must await further explorations of the series (index and individual stocks) and available statistical techniques for analyzing them (p.99).

Professor M.F.M. Osborne, in his *Operations Research* (1962) study of Brownian motion in stock prices, reported evidence of several that non-random movements in stock prices. Lower-priced stocks more volatile than higher-priced stock (p. 265). Stock price dispersions increased with stock volume. Daily stock volumes tended to more log normally distributed, rather than normally distributed, and stock volumes were higher on Tuesdays and Wednesdays (p. 279). Professor Osborne observed positive and negative skewness at higher concentrations than one would be expected of normal distributions (p. 293).

Professor Sidney Alexander, in his *Industrial Management Review* (1961) study, found that serial correlations of Professor Kendall's first-differenced stock indexes exhibited positive second-order serial correlation during the 1928 -1938 period. Furthermore, the variance in the correlations increased inversely proportionally to the number of observations. Professor Alexander reported that sixteen week intervals suggested further study (p. 204), and concluded that there are trends in stock market data, 1897 -1959 (p. 215). Professor Alexander developed a daily filter rule test of buying the Dow Jones Industrial Average (DJIA) and S&P from 1897 – 1959, A stock market upward movement of x % is likely to move up more than x % before it falls than by x %. The volatility of the trading strategy was successful particularly in the 1897 – 1914 and 1929 -1959 time periods, before commissions. Over time, speculative markets data appear to follow a random walk, but once a move is created, the trend can persist. A stock market upward movement of x % is likely to move up more than x % before it falls than by x %. Trends exist and persist (p. 218).

Professor Paul Cootner, in his *Industrial Management Review* (1962) study, specifically discussed the AR(1) model of prices of 45 stocks and random walk theory of successive-difference test of stock prices. Professor Cootner tested the variance structure of successive price changes and the variance of the price changes themselves, and reported excessive reversals, implying trends in the data (p. 241). Lo and MacKinlay (1988) studied four-week variance test ratio and reported results for the equal-weighted CRSP NYSE-AMEX that were statistically different from 1.0 for the September 6. 1962 – December 26, 1985 period, and equal-weighted variance test ratios that exceeded 1.0 during the 1962 -1985 period for two, four, eight, and 16-week periods. The variance test ratios were additional evidence of continued Dow non-randomness. Lo's (1991) reported long-term memory in stock prices for the 1962 – December 1987 period using a modified R/S statistic of the Mandelbrot (1971) range / standard deviation (R/S) test statistic. There is a long history of studying stock price index and returns data that suggest non-randomness, but the need for more appropriate statistical time series applications. Stock market movements are more complicated than random walks and more sophisticated testing is necessary (pp. 250-251).

Many of the contributors to the Cootner volume suggested that more sophisticated forecasting models needed to be developed and tested. The weak and semistrong form of the EMH, the latter holding that all publicly available information was reflected in the stock price [2].

During the 1983-88 period, studies on nonrandom evidence of stock market anomalies, including technical analysis inefficiencies, as well as monthly stock seasonality, small stock effects, and low price-earnings multiples, were reported in Dimson (1988). Haugen (1999) reported that 12-month price movements were useful as an input to a 13-factor stock selection model. The Lo and MacKinlay (1999) volume and Lo, Mamaysky, and Wang (2000) reported great progress and evidence in the non-random walk of stock market prices. In our analysis, we hope to contribute to the growing evidence that technical analysis, and price momentum analysis can produce statistically significant returns (results).

This paper tends to examine the predictability of the complete history on two of the major stock market indices, the Dow Jones Industrial Average (DJIA) and the Standard and Poor's 500 (S&P500). We examine the differences in the predictability of the stock market under a broad range of economic and financial conditions. Our main focus is about the predictability of both returns and prices and consider the use of (lags of) causal variables as possible predictors – the index of Leading Economic Indicators (LEI) the realized volatility of returns. See Guerard (2022, Chapters 7 and 10) for a time series transfer function analysis of the 1959 -2020 period of the LEI leading US real GDP and the unemployment rate and Xiao, Chen, and Guerard (2022) for an enhanced transfer function modeling of the US LEI leading the US unemployment rate, 1990 - 2019. We present that rolling window averaging (RWA) in forecast generation performs extremely well and it takes away some of the a priori uncertainty as to which part of the time series to use in computing a forecast. We find that prices and returns are, at least locally in time, predictable, that the use of the causal variables is improving forecasting performance and that a set of models of limited complexity has very robust performance under many different time periods. Our approach is suitable for mass production of forecasts across many time series, since it has low computational complexity and can run fast if implemented in parallel.

The rest of our paper is organized as follows. The next section briefly discusses relevant literature to motivate the paper. Section III describes both the data and evaluation of performance employed and the the empirical methodology we carefully implement across different sub-periods for both the two indices. Section IV discusses the empirical results from RWA analysis. Finally, Section V reaches to a conclusion.

[2]The Seminal Fama (1965) paper on the behavior of stock prices, Fama and Blume stock buying and selling filter rule test (1966) and the Fama, Fisher, Jensen, and Roll (1969) stock split test reported strong evidence of market efficiency. See Brealey (1969), Chapter two, and Lorie. Dodd, and Kimpton (1985), chapter 4, for readable treaties on market efficiency.

II Brief literature review

Forecasting price fluctuations in financial and commodity markets is a major challenge for both investors and businesses. Even more, several financial variables display features that seem to require time series models for descriptive and forecasting purposes. Montgomery, Zarnowitz, Tsay, and Tiao (MZTT, 1998) have measured forecasting performance in financial indexes on building vector models, while Hendry and Doornik (2014, 2015) outline the theory of model selection and the performance of several algorithms for forecasting.

The use of linear and non linear time series presenting forecasting performance for financial markets, goes back to Berndt et al. (1974), Hamilton (1989), Franses et al. (2004). They use theoretical and empirical issues, including predictive density, interval and point evaluation and model selection, loss functions, data-mining, and aggregation to argue the analyze and support the forecasting performance of financial and economic series. Furthermore, Clements et al. (2003) argue that the predominance of non-linear models in economics and finance is not inconsistent with the use of linear models by the applied practitioner. The seminal paper of Leybourne et al. (1996) summarized the perspective that randomized unit root models have a natural interpretation in the context of modelling financial time series and showed that both bond yields and stock market indices display evidence of randomized unit root behavior. Hamilton (1990) introduces – the then state of the art—an algorithm for obtaining maximum likelihood estimates of parameters for processes subject to discrete shifts in autoregressive parameters, influencing financial time series. The work of Perron (1989) is another work which have shown that allowing the possibility of occasional discrete shifts in trend can make a major difference for hypotheses about the persistence of innovations in key economic and financial time series.

Forecast models aim to predict the future trend of stock markets, using all the acquired knowledge from the past and the dynamic of trading strategies. The S&P500 is characterized as a benchmark index that all investors must taking into consideration. Lu and Wu (2011) stated the importance of forecast in decision making process, while Enke, Grauer, and Mehdiyev, (2011), highlighted the opportunities and challenges of market movement using these Indexes. Krauss, Do and Huck, (2017); Leigh, Purvis, and Ragusa, (2002); Leung, Daouk, and Chen, (2000) mentioned the significance of S&P500 index for developing and structuring trading strategies. The importance of such Indices is more significant considering their role in forecasting models to approximate future shifts Guerard (2013). Bodie, Kane, and Marcus, (2009) stated that the past price patterns have the tendency to repeat, which contribute to the predictability of future price shifts. Chen (2012) explored the linkages within stock returns and trading volume, and to what degree stock markets fluctuation played crucial role. Exploring the bull and bear stock markets on the S&P500 for 35 years until 2008, he stated significant relationship of asymmetry on contemporaneous relationship. That dynamic linkage indicate that stock returns could predict trading volume in both bull and bear markets but not the opposite. Nyberg (2013) predicts bull and bear markets for the S&P500 using a sample from 1957 to 2010. Through a dynamic probit model, he was able to forecast market sentiment, with results superior from its static model. Niaki and Hoseinzade (2013) used in their analysis a variety of 27 financial and economic factors for forecasting purposes, to reveal the direction of the daily S&P500 and beat the buy-and-hold strategy. Dhaoui, A., and Bacha, S. (2017), explored the dynamic relationship between trading volume and investors sentiments on S&P500. The sentiment indicators were the overconfidence and the net optimism-pessimism. They stated the asymmetric response of the stock market liquidity to both sentiment indicators in the long run had insignificant predictive power.

Lohrmann and Luukka (2019), separated the S&P500 open-to-close returns on daily base into four classes to acquire the highest level of its information. They ended up that the S&P500 revealed the highest importance of information from all markets about the forecast of their returns. Ku et al. (2020) detected that through an artificial neural network the different growth pattern of S&P500, driven to better prediction performance, considering the long-term development of the market, by using different combinations of fractal conditions. Kyrtsou et al. (2019), implemented Granger causality test on asymmetric partial transfer entropy to explore the relationship of VIX, S&P500 and its volume and conclude that the increasing volume affects the S&P500 returns.

III Data and Methodology

III.I Data and evaluation of performance

The authors received and gratefully acknowledge the Dow and S&P data used in this analysis from the Standards & Poor's global index group, for research-only purposes. We use Standard & Poor's Corporation, and DJIA to evaluate predictability from 1896-2021.In the first part of our analysis we use the daily closing prices of the DJIA for the period 1896-2022, in the full sample and subperiods of 5 and 10-years. The daily closing prices are analyzed via the OxMetrics package with an $AR(1)$ and saturation variables. In the second part we use the monthly returns of the DJIA from 1980 to 2022. As exogenous variables we use the first lag of the monthly realized volatility (VOL) and the monthly growth rate of Conference Board's leading economic indicator (LEI). We perform the analysis for three in-sample percentages defining the initial rolling window, 60%, 70% and 80% of the observations, with the rest 40%, 30% and 20% of observations left for evaluation. We repeat the analysis for the full sample, and every 5- and 10-year period. We consider sequences of 3, 5 and 7 rolling windows (with lengths chosen by an automatic sample split). We evaluate our forecasting performance using MSE and MAE and the Mincer-Zarnowitz (MZ) regression, with robust standard errors. Some of our analysis is repeated with the series of the S&P500 index.

III.II Methodology

We carefully scrutinize whether is worthwhile to continue meddling with the subject of stock market predictability and then, we examine the impact of historically large drawdowns and the future performance of the market. We analyze the predictability of prices themselves: while the random walk hypothesis may or may not (locally or globally) hold, it is worthwhile to examine the behavior of some data-compatible models in generating accurate forecasts. Moreover, we tend to analyze the predictability of returns that may have characteristics (such as conditional variance, threshold effects and possibly other types of non-linearity, etc.) that are make them more amenable to successful forecasting. Insist on the use of models that are data-compatible and of low complexity (conceptual and computational). We suggest that the "trick" in establishing good forecasting benchmarks is conceptual clarity (why the forecast works) and computational simplicity (run regression!); the only cherry topping is that we use robust methods of estimation. We re-introduce and propose rolling window averaging as a method to overcome the problem of choosing the sample length to use when computing a forecast; this approach not only liberates one from this uncertainty but it appears to offer solid performance gains against the a priori choice of a rolling window. We highlight the need to examine several sub-periods of evaluation, to uncover the local predictability of our time series.

We apply the Hendry-Clements-Castle-Doornik approach on modeling structural breaks via several shift and saturation variables, as additional components to the standard AR(1) model. Consider the augmented, first order autoregression:

$$X_{t+1} = \phi_0 + \phi_1 X_t + \boldsymbol{\gamma}^\top \boldsymbol{Z}_{t+1} + \epsilon_{t+1} \tag{1}$$

where $\boldsymbol{Z}_{t+1}$ is a vector of a linear trend and structural breaks are modeled with impulse indicator saturation (IIS) variables (a dummy variable; 1, for each observation, and 0 for all other observations), differenced impulse indicator saturation (DIIS) variables (differences in the IIS variables, and trend saturation (TIS) variables. The model fits an overall trend and variables representing deviations that trend are detected. All these are designed to examine the sensitivity of the baseline model to structural shifts and unusual residuals and to aid in modeling the latter in the best possible fashion (our use of robust regression methods in the second part of the presentation is motivated by the need to address such outliers and structural change, e.g., the 2008 crisis or the Covid-19 crisis.)

We next briefly explain the idea behind rolling window averaging. Consider a sequence of rolling windows of increasing length $R_1 < R_2 < \cdots < R_M \leq n$ and let $\mathcal{F}_{t|t-j}$, for $j = R_1, R_2, \ldots, R_M$, denote the information set that covers a period of R_m, $m = 1, 2, \ldots, M$ observations from period t backwards. The sequence of rolling windows and their associated information sets are overlapping (not of major concern).

Given any of these rolling windows, and its associated information set, we can compute the corresponding forecast, say $\mu_{t|t-j}$, for the next value of the time series we work with X_{t+1}, i.e. we have that the forecast is given by $\mu_{t|t-j} \stackrel{\text{def}}{=} \underset{\mu}{\operatorname{argmin}} \mathsf{E}\left[X_{t+1} - \mu|\mathcal{F}_{t|t-j}\right]^2$ thus being the minimum variance or optimal forecast, for that information set.

The expanded information sets as $m \to M$, provide us with a set of forecasts for the same future value of the time series. It only makes sense to combine them with some form of corresponding weights, say $w_{t|t-j}$. We can do this if we cast the conditional expectation optimization problem in the following fashion:

$$\min_{\left\{\mu_{t|t-j}, w_{t|t-j}\right\}_{j=R_1}^{R_M}} \sum_{j=R_1}^{R_M} w_{t|t-j}^2 \mathsf{E}\left[X_{t+1} - \mu_{t|t-j}|\mathcal{F}_{t|t-j}\right]^2 \tag{2}$$

which is to be minimized subject to the summability condition that the weights sum-up to one. We ignore the overlapping nature of the rolling windows, concentrating on the variance properties of the objective function and not on any auto-correlation that will be present because of the overlapping information. Setting up the Lagrangian and taking derivatives we find the first order conditions:

$$\begin{aligned} 2w_{t|t-j}^2 \mathsf{E}\left[X_{t+1} - \mu_{t|t-j}|\mathcal{F}_{t|t-j}\right] = 0 \\ 2w_{t|t-j} \mathsf{E}\left[X_{t+1} - \mu_{t|t-j}|\mathcal{F}_{t|t-j}\right]^2 - 2\lambda = 0 \end{aligned} \tag{3}$$

and solving we obtain the the individual forecasts and their associated weights:

$$\begin{aligned} \mu_{t|t-j} &= \mathsf{E}\left[X_{t+1}|\mathcal{F}_{t|t-j}\right] \\ w_{t|t-j} &= \sigma_{t|t-j}^{-2} / \textstyle\sum_{j=R_1}^{R_M} \sigma_{t|t-j}^{-2} \end{aligned} \tag{4}$$

We have that $\sigma_{t|t-j}^2 \stackrel{\text{def}}{=} \mathsf{E}\left[X_{t+1} - \mu_{t|t-j}|\mathcal{F}_{t|t-j}\right]^2$ is the conditional variance. The weights are the inverses of the conditional variances and assign a higher contribution to the rolling window segment with smallest variance.

$$\mu_{t+1|t}^{RWA,ow} \stackrel{\text{def}}{=} \sum_{j=R_1}^{R_M} w_{t|t-j} \mu_{t|t-j} \tag{5}$$

The equal weights forecast is given by:

$$\mu_{t+1|t}^{RWA,ew} \stackrel{\text{def}}{=} M^{-1} \sum_{j=R_1}^{R_M} \mu_{t|t-j} \tag{6}$$

The length-based weights forecast is given by:

$$\mu_{t+1|t}^{RWA,lw} \stackrel{\text{def}}{=} \left(\sum_{m=1}^{M} R_m\right)^{-1} \sum_{j=R_1}^{R_M} R_j \mu_{t|t-j} \tag{7}$$

The simpler weights circumvent the problem of computing the appropriate model for the conditional variance.

The table below provides a list of the names and abbreviations of the models that we used in the context of rolling window averaging. Note that models with threshold non-linearity are not applicable to prices but only to returns.

Model	Explanation
Mean	The sample mean
AR(1)	1st order autoregression
Drift	Constant plus dummy for lagged negative returns
Drift Rob	Drift, RLS
Drift-AR(1)	Drift w/ autoregression
Drift-TAR(1)	Drift w/ threshold autoregression
Drift-AR(1) Rob	Drift w/ autoregression, RLS
Drift-TAR(1) Rob	Drift w/ threshold autoregression, RLS
Drift-AR(1)-X Rob	Drift w/ autoregression & X-variable, RLS
Drift-TAR(1)-X Rob	Drift w/ threshold autoregression & X-variable, RLS
Model Avg	Equally weighted forecast average
Model WAvg	RMSE-weighted forecast average

Models estimated; RLS is robust least squares.

IV Discussion of the results

The discussion of our results is for the analysis of price levels and of returns. The first 4 tables contain the analysis of models for price levels with and without accounting for trend saturation and related variables and also with and without accounting for the presence of large residuals (outliers). The rest of the tables are for the rolling window averaging analysis of both price levels and returns, for the Dow Jones and the S&P500 indices.

The estimation of an AR(1) model with least squares (LS) using OxMetrics during the (mainly) 5-year subperiods from 1896 – 2021 produced very large residual sum of squares (RSS) errors. These errors increase drastically over time, see Table 1. If one applies the automatic time series modeling (AutoMetrics) procedures of OxMetrics in each of these periods, first with estimated large residuals (using a standard 5% rejection level), the RSS fall by 32.8 percent, on average, in the sub-periods. If one applies OxMetrics in each of these periods, and estimated IIS, DIIS, and TIS variables, using a small, 1% rejection level, the RSS fall by 64.0 percent, on average, in the sub-periods. If one applies OxMetrics in each of these periods, estimating (only) TIS variables, using a small, 1% rejection level, the RSS fall by 72.2 percent, on average, in the sub-periods. It appears that trend indicator saturation variables are most important in modeling the DJIA, during the 1896 – 2021 period. Why? The plot of the data shows great exponential growth in stock prices! The OxMetrics results reported in Table 1 clearly demonstrate the need for modeling structural changes and outliers.

In Table 2, the AR(1) model shows that the first-order autoregressive parameter, theta 1, is approximately 1.0, as one would expect in a random walk series. However, the residuals are not normally distributed. Furthermore, the trend term is statistically significant in many subperiods, particularly during the 1985 -2021 time period. Many, if not most of our readers, were not born at the beginning of the statistically significant trend variables. In Table 3, we report the estimated AR(1) parameter with the estimated AutoMetrics models with the identified and estimated large residuals. Although the AR(1) term is still roughly 1.0, the RSS fall significantly than to the AR(1) LS benchmark, previously discussed. In Table 4, the AutoMetrics estimated AR(1) parameter falls in several periods to 0.90 or less with the IIS, DIIS, and TIS variables, and the RSS fall substantially, particularly during 1975-2021 period. The trend term is often very highly statistically significant throughout much of the 1896 -2021 sub-periods in Table 4.

The reader cannot fully appreciate the model complexity and residual reductions until the reader sees the model plots of scaled residuals for AutoMetrics estimates of LS (Figures 1, 5, and 9) for the 1896 -1899 period, containing the Spanish-American War); the 1905-1909 period, containing the "Panic of 1907"; the 1985-1989 period, containing Black Monday, October 19, 1987. LS estimations are almost all periods, are plagued with huge outliers. The AutoMetrics estimates of the AR(1) model with large residuals (Figures 2, 6, and 10) substantially dampen outliers, but do not produce normally distributed residuals in all periods (only in 5 sub-periods). The AutoMetrics estimates of the AR(1) model with IIS, DIIS, and TIS Saturation variables (Figures 3, 7, and 11) substantially dampen outliers, and produce normally distributed residuals in 11 sub-periods. The AutoMetrics estimates of the AR(1) model with TIS Saturation variables (Figures 4, 8, and 12) substantially dampen outliers, and produce normally distributed residuals in 9 sub-periods. We included Figures 13 and 14, of LS and full IIS, DIIS, and TIS Saturation variable residuals of the 1980 -1989 period to show the dominance of Black Monday in time series model estimation! Finally, we show Figure 15, the graph of the DJIA, since its inception, with the red dot for Black Monday, to illustrate its non-consequential impact of the entire history of the Dow. It was quite consequential for those of us living, and working, on Wall Street at that point in time.

We now turn to the discussion of the results from the application of rolling window averaging in forecasting price levels and returns. These results appear in Tables 5 through 11, and they collectively tell the same result: stock prices and returns' predictability is there, scattered in swaths of time and is thus a local and not a global phenomenon. The evaluation in different periods of different lengths shows that even when we evaluate in one period and might not find predictability when we evaluate on a shorter period then predictability arises. Our results are consistent throughout and show very considerable performance enhancements compared to the naive benchmark. Furthermore, we can easily see that including explanatory variables (realized volatility and the leading economic indicator) is, at times, a very productive approach to forecasting. Our results are statistically significant by the application of the MZ regression.

On the technical side, we note that the use of RWA takes away the need to consider which window to use in a forecasting evaluation, and provides considerable simplification to forecasting performance assessment. The use of sample length-based weights of equation (7) provides additional performance enhancements over the use of equal weights of equation (6). Our results on the application of RWA appear to be mildly insensitive to the number of windows that are being used; the main factor that impacts performance is the length of the evaluation sample. If the particular results of this study are to be any guide, we would suggest that a relatively smaller number of windows (3-4) is sufficient to achieve the performance enhancements we are after. Furthermore, the use of simpler models, with or without a drift-adjustment for negative returns, appear to be the top performers when the evaluation sample is larger. The use of models with our two explanatory variables of realized volatility and the leading indicator, appear to be top performers when a larger sample is used in estimation (and a smaller for evaluation). MAE improvements are consistently and considerably higher than MSE improvements, a result that appears in all periods of our analysis.[1]

In Tables 12 and 13 we just point out the strong market reversals that occur when the market exhibits very large drawdowns. In Table 12 we consider the worst periods with the semester drawdowns larger than the one from the fist semester of 2022. Then, we present the returns of the Dow Jones after 6 months, 12 months and 36 months and the corresponding average returns over all these exceptionally "bad"periods: the result is, no surprise here, that the sun will rise again tomorrow! Our six-months analysis was inspired by rereading Malkiel (1963), written referencing the first half-year returns of 1962 and Graham (1974), writing on the current and future state of common stock prices. There are many times when you must declare your opinion on the world in which you live and work. This is our modest attempt to do what greater thinkers have done before us. If history is any guide then we should not be but mildly alarmed of what we have seen the first semester of the year. And to keep-up with our understanding of predictability, in Table 13 we present the real-time forecast for the second semester of 2022 and its distribution, from some of the top performing models that were used in the analysis of our semester returns. Well corresponding with the historical averages of Table 12, we can see (last column of Table 13) that the distribution of the predicted returns ranges from a mild drop of -1.8% (10th percentile) over the second semester of 2022, with a mean increase of 5.7% (median is 4%) to a hefty rebound of 18% (90th percentile).

[1]All tables present results using the MAE criterion, results with the MSE criterion also available on request from the authors.

V Conclusions

We report statistically significant Impulse Indicator Saturation (IIS), Differenced Impulse Indicator Saturation (DIIS), and Trend Saturation (TIS) variables with the Hendry et. al OxMetrics application, when forecasting the price level of the Dow Jones Industrial Average. Monthly prices and returns are predictable, by standard forecast metrics, at least locally in time and with varying degrees of success. Our results are significant based on the MZ methodology. We present a large set of results on the efficacy of rolling window averaging, RWA, where forecasts of monthly returns are generated by averaging over different information periods. This alleviates the problem of choosing the rolling window of estimation and forecasting and improves performance in real-time applications. We illustrate that the use of realized volatility and the leading indicator can lead to performance enhancements, but this result is period-dependent. We perform an additional analysis using semi-annual returns, identify the periods in time that were as of the last 6 months or worse and forecast higher 6-monh, 1-year, and 3-year DJIA returns; we then couple our historical analysis with a real-time forecast for the second semester of 2022. The sun will come out tomorrow, as happy days should be here again!

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Appendix

	AR1 Model				RSS Reductions		
Period	LS	Large Residuals	IIS, DIIS TIS Sat.	TIS Sat.	Large Residuals	IIS, DIIS TIS Sat.	TIS Sat.
1896-1899	4.7E+02	3.3E+02	1.4E+02	**1.5E+02**	0.303	0.709	0.688
1900-1904	6.4E+02	4.6E+02	2.9E+02	**2.6E+02**	0.279	0.552	0.599
1905-1909	9.2E+02	7.5E+02	**4.8E+02**	**5.3E+02**	0.194	0.479	0.430
1910-1914	9.8E+03	4.3E+02	**2.8E+02**	3.0E+02	0.956	0.971	0.970
1915-1919	1.5E+03	1.1E+03	**7.1E+02**	**7.2E+02**	0.239	0.526	0.519
1920-1924	9.8E+02	7.9E+02	**5.6E+02**	6.2E+02	0.193	0.427	0.365
1925-1929	1.5E+04	6.6E+03	1.6E+03	6.6E+02	0.544	0.888	0.955
1930-1934	9.6E+03	7.3E+03	2.4E+03	6.0E+02	0.246	0.746	0.938
1935-1939	4.5E+03	3.3E+03	**1.9E+03**	1.5E+03	0.268	0.572	0.672
1940-1944	1.3E+03	7.8E+02	6.1E+02	**2.1E+02**	0.389	0.522	0.837
1945-1949	2.6E+03	1.7E+03	1.1E+03	8.9E+02	0.324	0.574	0.656
1950-1954	3.3E+03	2.6E+03	1.9E+03	1.9E+03	0.217	0.422	0.443
1955-1959	1.6E+04	1.3E+04	**5.3E+03**	1.9E+03	0.175	0.660	0.881
1960-1964	2.5E+04	**1.9E+04**	**1.4E+04**	1.1E+04	0.213	0.449	0.556
1965-1969	3.6E+04	**3.3E+04**	3.2E+04	**6.4E+03**	0.073	0.116	0.822
1970-1974	8.2E+04	**6.8E+04**	**2.7E+04**	6.2E+03	0.167	0.668	0.924
1975-1979	6.3E+04	**5.8E+04**	**3.9E+04**	**4.2E+04**	0.079	0.381	0.332
1980-1984	1.2E+05	**1.0E+05**	**8.2E+04**	**8.4E+04**	0.141	0.302	0.290
1985-1989	9.2E+05	4.5E+05	1.9E+05	**1.7E+05**	0.508	0.798	0.812
1990-1994	7.3E+05	5.8E+05	**4.4E+05**	6.2E+05	0.204	0.396	0.147
1995-1999	8.0E+06	5.1E+06	8.9E+05	1.3E+06	0.365	0.889	0.834
2000-2004	1.7E+07	1.2E+07	6.8E+06	9.4E+06	0.287	0.612	0.458
2005-2009	2.4E+07	1.3E+07	6.3E+06	3.7E+06	0.450	0.734	0.842
2010-2014	1.7E+07	1.2E+07	7.9E+06	4.7E+06	0.263	0.520	0.718
2015-2019	4.3E+07	2.8E+07	**1.5E+07**	4.4E+06	0.347	0.644	0.899
2017-2021	1.2E+08	6.8E+07	**2.5E+07**	3.3E+07	0.452	0.798	0.737
Average	9.8E+06	5.9E+06	2.6E+06	2.4E+06	0.328	0.640	0.722

Table 1: DIJA analysis with indicator and saturation variables only, daily prices

Period	$\widehat{\phi_1}(t)$	$\widehat{\phi_0}(t)$	$\widehat{\tau}(t)$	$\widehat{\sigma}$	$\bar{R}^2$	$RESET$
1896-1899	0.989 (226)	0.362 (2.51)	0.000 (2.35)	0.663	0.995	0.686
1900-1904	0.997 (414)	0.155 (0.36)	0.000 (0.00)	0.657	0.993	2.574
1905-1909	0.998 (560)	0.195 (1.29)	0.000 (0.26)	0.785	0.995	0.731
1910-1914	0.995 (252)	0.362 (1.13)	0.000 (-1.19)	0.777	0.982	4.211
1915-1919	0.995 (226)	0.412 (2.34)	0.000 (0.71)	1.006	0.994	1.005
1920-1924	0.995 (460)	0.297 (1.66)	0.000 (3.44)	0.810	0.995	1.266
1925-1929	0.995 (360)	0.362 (2.51)	0.000 (1.34)	3.137	0.998	0.348
1930-1934	0.998 (671)	-.047 (-.14)	0.000 (0.60)	2.547	0.998	1.159
1935-1939	0.996 (515)	0.601 (2.11)	0.000 (-.66)	1.727	0.994	0.017
1940-1944	0.997 (580)	0.312 (1.46)	0.000 (2.74)	0.924	0.996	0.280
1945-1949	0.994 (345)	1.157 (2.25)	0.000 (0.11)	1.352	0.988	0.150
1950-1954	1.002 (418)	-.397 (-.79)	0.000 (0.11)	1.579	0.999	5.567
1955-1959	0.999 (455)	0.823 (0.92)	0.000 (1.32)	3.525	0.997	2.147
1960-1964	0.989 (226)	2.230 (1.58))	0.000 (2.43)	4.440	0.997	1.625
1965-1969	0.995 (420)	4.761 (2.78)	-.001 (-1.19)	5.415	0.989	1.009
1970-1974	0.999 (444)	1.323 (0.67)	-.001 (-1.70)	8.051	0.994	2.174
1975-1979	0.991 (373)	8.687 (3.65)	-.001 (-2.21)	7.087	0.991	0.589
1980-1984	0.995 (345)	4.535 (1.90)	0.002 (1.45)	9.679	0.996	1.865
1985-1989	0.992 (284)	11.778 (2.39)	0.008 (1.90)	26.960	0.996	0.503
1990-1994	0.981 (188)	49.835 (3.70)	0.021 (3.59)	23.602	0.997	1.188
1995-1999	0.985 (239)	59.816 (3.72)	0.062 (3.55)	67.686	0.999	0.185
2000-2004	0.993 (368)	61.777 (2.17)	0.000 (0.18)	97.640	0.988	6.126
2005-2009	0.997 (590)	33.719 (1.62)	-.004 (-.77)	114.033	0.995	2.708
2010-2014	0.989 (259)	105.420 (2.87)	0.056 (3.00)	103.673	0.988	3.128
2015-2019	0.986 (216)	227.69 (1.89)	0.152 (3.26)	185.000	0.988	1.506
2017-2021	0.988 (220)	246.566 (2.68)	0.136 (2.54)	315.162	0.995	3.061

Table 2: DIJA analysis with AR(1) model, including a trend, daily prices

Period	$\widehat{\phi}_1(t)$	$\widehat{\sigma}$	RSS	Normalized Residuals	$RESET$
1896-1899	1.001 (3166)	0.557	328.6	13.24	1.258
1900-1904	0.999 (4200)	0.561	462.4	15.30	0.051
1905-1909	1.000 (4589)	0.709	745.2	10.32	1.325
1910-1914	0.999 (5510)	0.560	428.8	742.90	2.908
1915-1919	1.001 (3854)	0.883	1146.3	14.09	1.367
1920-1924	0.998 (500)	0.733	792.1	13.41	0.163
1925-1929	0.999 (998)	2.106	6448.5	278.90	0.213
1930-1934	0.999 (2384)	2.283	7257.9	28.40	2.891
1935-1939	0.999 (3782)	1.491	3268.3	38.73	2.339
1940-1944	0.999 (6840)	0.727	782.9	54.48	0.834
1945-1949	0.999 (5990)	1.120	1742.2	40.14	0.960
1950-1954	1.001 (7024)	1.407	2605.9	8.58	5.636
1955-1959	1.000 (2307)	3.216	12875.2	12.84	0.041
1960-1964	1.000 (6310)	3.958	19444.9	1.75	0.045
1965-1969	0.999 (5946)	5.230	33366.3	2.41	0.902
1970-1974	1.003 (478)	7.042	67995.5	3.37	2.077
1975-1979	0.992 (388)	6.822	58224.0	3.45	1.106
1980-1984	0.999 (4025)	9.013	101453.0	4.77	1.828
1985-1989	1.001 (3786)	19.024	450594.4	96.77	1.182
1990-1994	0.985 (207)	21.239	581928.4	8.93	0.452
1995-1999	0.984 (286)	54.520	3091757.0	197.62	0.412
2000-2004	0.995 (450)	83.270	12404706.6	88.25	4.342
2005-2009	1.000 (5524)	85.523	13034004.4	269.78	0.590
2010-2014	0.990 (296)	89.813	12196312.2	76.83	3.237
2015-2019	0.991 (260)	152.114	28414418.6	36.70	3.410
2017-2021	0.990 (254)	235.117	68271080.4	84.63	0.222

Table 3: DIJA analysis with AR(1) model and large residuals, daily prices

Period	$\widehat{\phi_1}(t)$	$\widehat{\phi_0}(t)$	$\widehat{\tau}(t)$	$\widehat{\sigma}$	$\bar{R}^2$	$RESET$
1896-1899	0.906 (84.1)	0.000 (0.00)	0.006 (8.36)	0.397	0.999	2.013
1900-1904	0.984 (277)	0.000 (0.00)	0.000 (0.00)	0.453	0.998	3.510
1905-1909	0.883 (86.7)	0.000 (0.00)	0.008 (11.50)	0.587	0.998	2.335
1910-1914	0.965 (156)	0.000 (0.00)	0.001 (5.19)	0.467	0.998	1.073
1915-1919	0.998 (664)	0.000 (0.00)	0.000 (0.00)	0.726	0.998	1.293
1920-1924	0.962 (178)	0.000 (0.00)	0.003 (7.24)	0.641	0.996	1.343
1925-1929	0.908 (117)	0.000 (0.00)	0.018 (13.40)	1.143	0.999	2.320
1930-1934	0.913 (95.5)	0.576 (9.14)	0.000 (0.00)	1.417	0.997	0.646
1935-1939	0.939 (118)	0.000 (0.00)	0.006 (7.70)	1.099	0.998	0.434
1940-1944	1.000 (5609)	0.000 (0.00)	0.000 (0.00)	0.657	0.998	2.935
1945-1949	1.001 (1311)	0.000 (0.00)	0.000 (0.00)	0.925	0.999	7.176
1950-1954	0.976 (187)	0.000 (0.00)	0.000 (0.00)	1.257	0.999	4.641
1955-1959	0.815 (65.8)	0.000 (0.00)	0.010 (15.00)	2.233	0.997	0.215
1960-1964	0.948 (137)	17.697 (2.26)	0.000 (0.00)	3.393	0.999	2.357
1965-1969	1.000 (726)	0.000 (0.00)	0.000 (0.00)	4.477	0.998	9.070
1970-1974	0.998 (740)	0.000 (0.00)	0.000 (0.00)	5.140	0.999	2.085
1975-1979	0.884 (67.7)	0.000 (0.00)	0.104 (12.40)	5.795	0.994	0.940
1980-1984	0.999 (2993)	0.000 (0.00)	0.000 (0.00)	8.276	0.999	8.282
1985-1989	0.885 (83.2)	0.000 (0.00)	.252 (11.00)	12.908	0.992	3.062
1990-1994	0.943 (119)	214.715 (7.14)	0.000 (0.00)	18.948	0.999	2.064
1995-1999	0.899 (115)	0.000 (0.00)	0.000 (0.00)	25.702	0.999	8.314
2000-2004	0.962 (183)	381.321 (7.07)	0.000 (0.00)	64.379	0.995	0.202
2005-2009	0.987 (321)	0.000 (0.00)	0.000 (0.00)	52.915	0.999	9.805
2010-2014	0.979 (197)	0.000 (0.00)	0.248 (4.26)	74.927	0.999	3.045
2015-2019	1.000 (1943)	0.000 (0.00)	0.000 (0.00)	116.694	0.999	8.216
2017-2021	0.997 (485)	0.000 (0.00)	0.000 (0.00)	153.323	0.999	20.420

Table 4: DIJA analysis with AR(1) model, indicator and saturation variables, daily prices

Model	1900-10	1910-30	1930-40	1940-50	1950-60
Rec Mean	1.000	≥ 1	1.000	1.000	1.000
Roll Mean	1.000	≥ 1	1.000	1.000	1.000
Rec AR(1)	1.016	≥ 1	1.004	1.010	0.914*
Roll AR(1)	1.022	≥ 1	0.977*	1.007	0.952*
Rec AR(1) Rob	0.997	≥ 1	0.995	0.994	0.830*
Roll AR(1) Rob	0.999	≥ 1	0.977*	1.001	0.889*
Rec AR(1)-V Rob	0.995	≥ 1	0.994	0.989	0.836*
Roll AR(1)-V Rob	1.007	≥ 1	1.085	1.003	0.887*
Model Avg	1.004	≥ 1	0.999	1.000	0.913*
Model WAvg	1.004	≥ 1	0.999	1.000	0.913*

Table 5: DJIA with in-sample/out-of-sample split is 80%/20%, 3 rolling windows, RWA with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly prices

Model	1990-95	1995-00	2000-05	2005-10	2010-15	2015-22
Rec Mean	1.000	1.000	1.000	1.000	1.000	1.000
Roll Mean	1.000	1.000	1.000	1.000	1.000	1.000
Rec AR(1)	1.025	0.970*	1.031	1.047	0.968*	0.986
Roll AR(1)	1.016	1.034	1.031	1.125	1.015	1.032
Rec AR(1) Rob	1.018	0.905*	1.025	0.964*	0.926*	0.885*
Roll AR(1) Rob	1.019	0.966*	1.026	1.050	1.032	0.945*
Rec AR(1)-V Rob	1.011	0.926*	1.032	1.005	0.935*	0.850*
Roll AR(1)-V Rob	1.023	0.970*	0.996	1.085	1.039	0.913*
Rec AR(1)-L Rob	0.812*	1.007	1.017	1.073	0.877*	1.226
Roll AR(1)-L Rob	0.699*	1.093	1.034	1.084	0.912*	1.115
Model Avg	0.960*	0.979*	0.992	1.035	0.876*	0.901*
Model WAvg	0.960*	0.979*	0.992	1.035	0.876*	0.901*

Table 6: DJIA with in-sample/out-of-sample split is 80%/20%, 3 rolling windows, RWA with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly prices

Model	Full	Full w/ LEI	1990-00	2000-10	2010-22
Rec Mean	1.000	1.000	1.000	1.000	1.000
Roll Mean	1.000	1.000	1.000	1.000	1.000
Rec AR(1)	0.980	0.973*	0.957*	0.992	0.977*
Roll AR(1)	0.981	0.974*	0.975*	1.016	0.992
Rec AR(1) Rob	0.975*	0.966*	0.976*	0.976*	0.939*
Roll AR(1) Rob	0.978*	0.968*	0.973*	1.004	0.947*
Rec AR(1)-V Rob	0.974*	0.963*	0.982	1.035	0.896*
Roll AR(1)-V Rob	0.981	0.966*	0.977*	1.036	0.896*
Rec AR(1)-L Rob	N.A.	0.963*	0.981	0.982	1.124
Roll AR(1)-L Rob	N.A.	0.970*	0.984	1.025	1.099
Model Avg	0.975*	0.965*	0.934*	0.992	0.943*
Model WAvg	0.975*	0.965*	0.934*	0.992	0.943*

Table 7: DJIA with in-sample/out-of-sample split is 80%/20%, 3 rolling windows, RWA with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly prices

Model	Full	1990-1999	2000-2009	2010-2022
Rec AR(1)	1.004	0.983	0.979*	1.025
Roll AR(1)	1.012	0.989	0.972*	1.043
Rec Drift	1.001	1.024	0.991	1.007
Roll Drift	1.003	1.004	0.995	1.008
Rec Drift Rob	0.998	1.019	0.987	0.989
Roll Drift Rob	1.000	1.008	0.991	0.989
Rec Drift-AR(1)	1.006	0.978*	0.991	1.029
Roll Drift-AR(1)	1.016	0.965*	0.974*	1.048
Roll Drift-TAR(1)	1.016	1.028	0.972*	1.051
Rec Drift-AR(1) Rob	0.997	0.967*	1.004	1.010
Roll Drift-AR(1) Rob	1.002	0.951*	0.986	1.026
Rec Drift-TAR(1) Rob	0.994	1.007	1.045	0.964*
Roll Drift-TAR(1) Rob	0.996	0.957	1.003	0.958*
Rec Drift-AR(1)-V Rob	0.985	1.071	1.067	0.931*
Roll Drift-AR(1)-V Rob	0.987	0.968	0.995	0.930*
Rec Drift-TAR(1)-V Rob	0.981	1.171	1.023	0.932*
Roll Drift-TAR(1)-V Rob	0.975*	1.000	0.937*	0.994
Rec Drift-AR(1)-L Rob	1.003	0.979*	1.003	1.167
Roll Drift-AR(1)-L Rob	1.019	0.984	0.988	1.051
Rec Drift-TAR(1)-L Rob	0.999	1.024	1.034	1.060
Roll Drift-TAR(1)-L Rob	1.005	0.994	1.000	1.035

Table 8: DJIA with in-sample/out-of-sample split is 80%/20%, 3 rolling windows, RWA with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly returns

Model	1990-1994	1995-1999	2000-2004	2005-2009	2015-2022
Rec AR(1)	0.955*	0.951*	1.022	0.941*	1.023
Roll AR(1)	0.927*	0.966*	1.008	0.969*	1.033
Rec Drift	1.046	1.008	1.007	0.933*	1.002
Roll Drift	1.046	1.017	1.055	0.928*	0.991
Rec Drift Rob	1.031	1.011	1.007	0.931*	0.964*
Roll Drift Rob	1.042	1.032	1.059	0.928*	0.954*
Rec Drift-AR(1)	0.979*	0.948*	1.233	0.958*	1.031
Roll Drift-AR(1)	0.958*	1.001	1.063	0.986	1.072
Rec Drift-TAR(1)	0.987	0.987	1.291	0.976*	1.009
Roll Drift-TAR(1)	0.978*	1.073	1.108	0.988	1.041
Rec Drift-AR(1) Rob	0.975*	0.916*	1.102	0.939*	1.041
Roll Drift-AR(1) Rob	0.965*	0.917*	0.919*	0.977*	1.072
Rec Drift-TAR(1) Rob	0.994	0.945*7	1.162	0.961*	0.978*
Roll Drift-TAR(1) Rob	0.982	1.046	0.956*	0.984	1.034
Rec Drift-AR(1)-V Rob	0.981	0.941*	1.115	1.269	0.935*
Roll Drift-AR(1)-V Rob	0.986	1.041	0.963*	1.130	0.955*
Rec Drift-TAR(1)-V Rob	0.985	0.964*	1.017	0.931*	0.920*
Roll Drift-TAR(1)-V Rob	0.974*	1.023	1.167	1.235	1.000
Rec Drift-AR(1)-L Rob	1.006	0.972*	1.116	0.937*	1.035
Roll Drift-AR(1)-L Rob	1.003	1.041	0.970*	0.987	1.059
Rec Drift-TAR(1)-L Rob	1.086	0.990	1.196	0.952*	0.991
Roll Drift-TAR(1)-L Rob	0.996	1.170	0.975*	1.032	1.087

Table 9: DJIA with in-sample/out-of-sample split is 80%/20%, 3 rolling windows, with length-based weights; a * denotes significance based on the MZ-regression - entries are relative MAE, monthly returns

Model	Full	1990-1999	2000-2009	2010-2022
Rec AR(1)	1.003	1.000	0.956*	1.024
Roll AR(1)	1.005	0.985	0.910*	1.042
Rec Drift	1.001	1.014	0.988	1.011
Roll Drift	0.997	0.994	0.992	1.013
Rec Drift Rob	0.989	1.014	0.987	0.985
Roll Drift Rob	0.980	0.991	0.991	0.979 *
Rec Drift-AR(1)	1.021	1.006	0.940*	1.030
Roll Drift-AR(1)	1.024	0.992	0.901*	1.054
Rec Drift-TAR(1)	1.022	0.990	0.945*	1.027
Roll Drift-TAR(1)	1.021	0.998	0.914*	1.051
Rec Drift-AR(1) Rob	1.005	0.995	0.944*	1.002
Roll Drift-AR(1) Rob	1.002	0.977*	0.895*	1.023
Rec Drift-TAR(1) Rob	1.008	1.038	0.948*	0.955*
Roll Drift-TAR(1) Rob	1.000	0.953*	0.883*	1.020
Rec Drift-AR(1)-V Rob	1.005	1.121	0.944*	0.913*
Roll Drift-AR(1)-V Rob	0.982	1.033	0.901 *	0.928*
Rec Drift-TAR(1)-V Rob	0.992	1.184	1.053	0.917*
Roll Drift-TAR(1)-V Rob	0.962	1.159	1.071	0.918*
Rec Drift-AR(1)-L Rob	1.018	1.010	0.954*	1.173
Roll Drift-AR(1)-L Rob	1.040	0.989	0.898*	1.060
Rec Drift-TAR(1)-L Rob	1.019	1.070	0.957*	1.063
Roll Drift-TAR(1)-L Rob	1.031	0.973*	0.907*	1.099

Table 10: S&P500 in-sample/out-of-sample split is 80%/20%, 3 rolling windows, RWA with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly returns

Model	1990-1994	1995-1999	2000-2004	2005-2009	2015-2022
Rec AR(1)	0.932*	0.996	0.981	0.898*	1.058
Roll AR(1)	0.958*	1.014	0.872*	0.924*	1.051
Rec Drift	1.033	1.004	0.957*	0.960*	1.012
Roll Drift	1.060	0.988	0.815*	0.990*	0.986
Rec Drift Rob	1.040	1.001	0.901*	0.930*	0.956*
Roll Drift Rob	1.067	0.987	0.745*	0.963*	0.919*
Rec Drift-AR(1)	0.938*	1.006	0.959*	0.864*	1.073
Roll Drift-AR(1)	0.969*	1.036	0.844*	0.900 *	1.082
Rec Drift-TAR(1)	0.915*	1.005	0.973*	0.903*	1.059
Roll Drift-TAR(1)	0.945*	1.082	1.044	0.912*	1.055
Rec Drift-AR(1) Rob	0.948 *	0.991	0.913*	0.842*	1.064
Roll Drift-AR(1) Rob	0.979*	1.033	0.777*	0.888*	1.069
Rec Drift-TAR(1) Rob	0.908*	1.016	0.924*	0.893*	1.033
Roll Drift-TAR(1) Rob	0.996	1.095	0.972*	0.911 *	1.036
Rec Drift-AR(1)-V Rob	0.921*	1.008	0.947*	0.894*	0.968*
Roll Drift-AR(1)-V Rob	0.936*	1.141	0.826*	0.816 *	0.956*
Rec Drift-TAR(1)-V Rob	0.941*	1.016	1.010	0.932*	0.916 *
Roll Drift-TAR(1)-V Rob	0.931*	1.145	0.932*	1.055	0.888*
Rec Drift-AR(1)-L Rob	0.981	1.017	0.979*	0.855*	1.096
Roll Drift-AR(1)-L Rob	0.969*	1.078	0.871*	0.845*	1.045
Rec Drift-TAR(1)-L Rob	1.002	1.022	0.975*	0.921*	1.051
Roll Drift-TAR(1)-L Rob	0.931*	1.141	0.878*	0.960*	1.121

Table 11: S&P500 in-sample/out-of-sample split is 80%/20%, 3 rolling windows, with length-based weights, a * denotes significance based on the MZ-regression - entries are relative MAE, monthly returns

Dates	Returns	6m ahead	1y ahead	3y ahead
6/30/1900	-16.87%	28.73%	41.89%	17.04%
12/31/1901	-17.17%	-0.39%	-0.42%	-23.71%
12/31/1903	-16.88%	0.29%	41.74%	77.17%
12/31/1907	-26.89%	23.56%	46.64%	38.18%
6/30/1910	-18.04%	0.22%	5.91%	8.24%
12/31/1914	-32.33%	28.36%	81.66%	75.65%
12/31/1917	-22.42%	11.16%	10.51%	22.02%
6/30/1920	-15.36%	-20.72%	-24.58%	8.78%
12/31/1920	-20.72%	-4.86%	12.72%	22.10%
12/31/1929	-25.56%	-8.91%	-33.77%	-82.76%
12/31/1930	-27.29%	-8.75%	-52.67%	-40.37%
12/31/1931	-48.13%	-45.01%	-23.07%	22.88%
6/30/1932	-45.01%	39.89%	129.08%	142.86%
12/31/1937	-28.63%	10.78%	28.06%	0.84%
6/30/1939	-15.59%	15.01%	-6.71%	-15.06%
6/30/1940	-18.88%	7.60%	1.04%	-2.03%
6/30/1962	-23.23%	16.18%	25.94%	55.74%
12/31/1974	-23.20%	42.64%	38.32%	48.69%
12/31/1987	-19.83%	10.46%	11.85%	48.58%
12/31/2008	-22.68%	-3.75%	18.82%	41.45%
6/30/2022	-15.31%			
	Average TR	7.12%	17.65%	23.31%

Table 12: Total returns of current 6m, next 6m, next 12m and next 36m for DJIA dates with semi-annual returns worse than 2022/01-2022/06

Model	MAE 2000/06-2022/06	Forecast 2022/07-2022/12	Distribution	
			Mean	5.71%
Roll Mean	99.749%	4.41%	10%	-1.78%
Rec AR(1)	99.834%	0.68%	25%	2.06%
Rec Drift	98.408%	3.98%	50%	3.98%
Roll Drift	97.673%	3.45%	75%	4.35%
Rec Drift Rob	98.345%	4.19%	90%	18.81%
Roll Drift Rob	97.333%	3.81%		
Roll Drift-TAR(1) Rob	83.227%	18.81%		
Rec Drift-AR(1)-V Rob	92.772%	-1.78%		
Roll Drift-AR(1)-V Rob	99.661%	4.28%		
Rec Drift-TAR(1)-V Rob	87.539%	-12.53%		
Roll Drift-TAR(1)-V Rob	95.770%	33.52%		

Table 13: Backtested MAE of best models, in-sample/out-of-sample split is 90%/10%, 3 rolling windows, with length-based weights, real-time forecast & distribution - 6-month DJIA returns

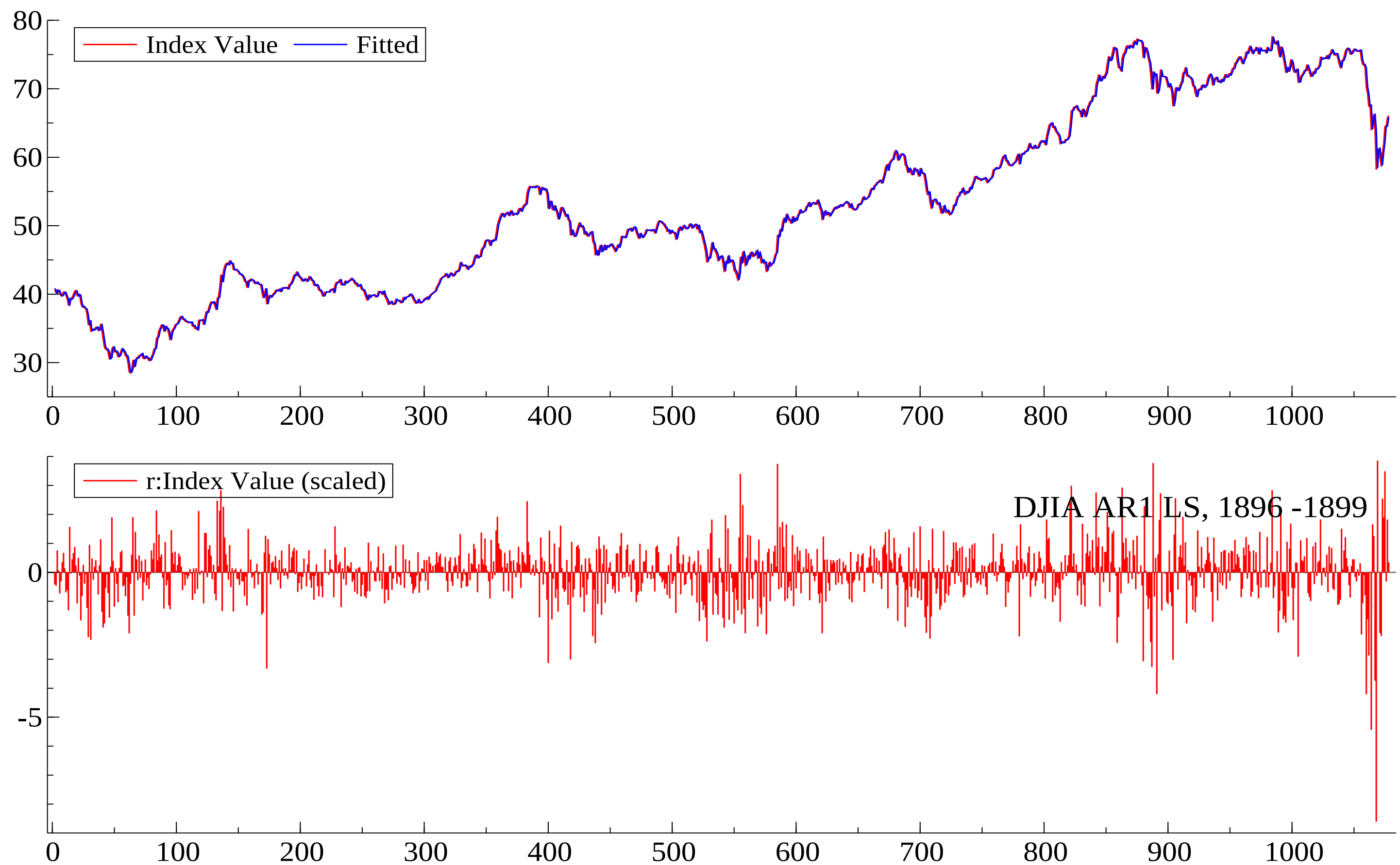

Figure 1: Model DJIA AR1 LS, from 1896 to 1899.

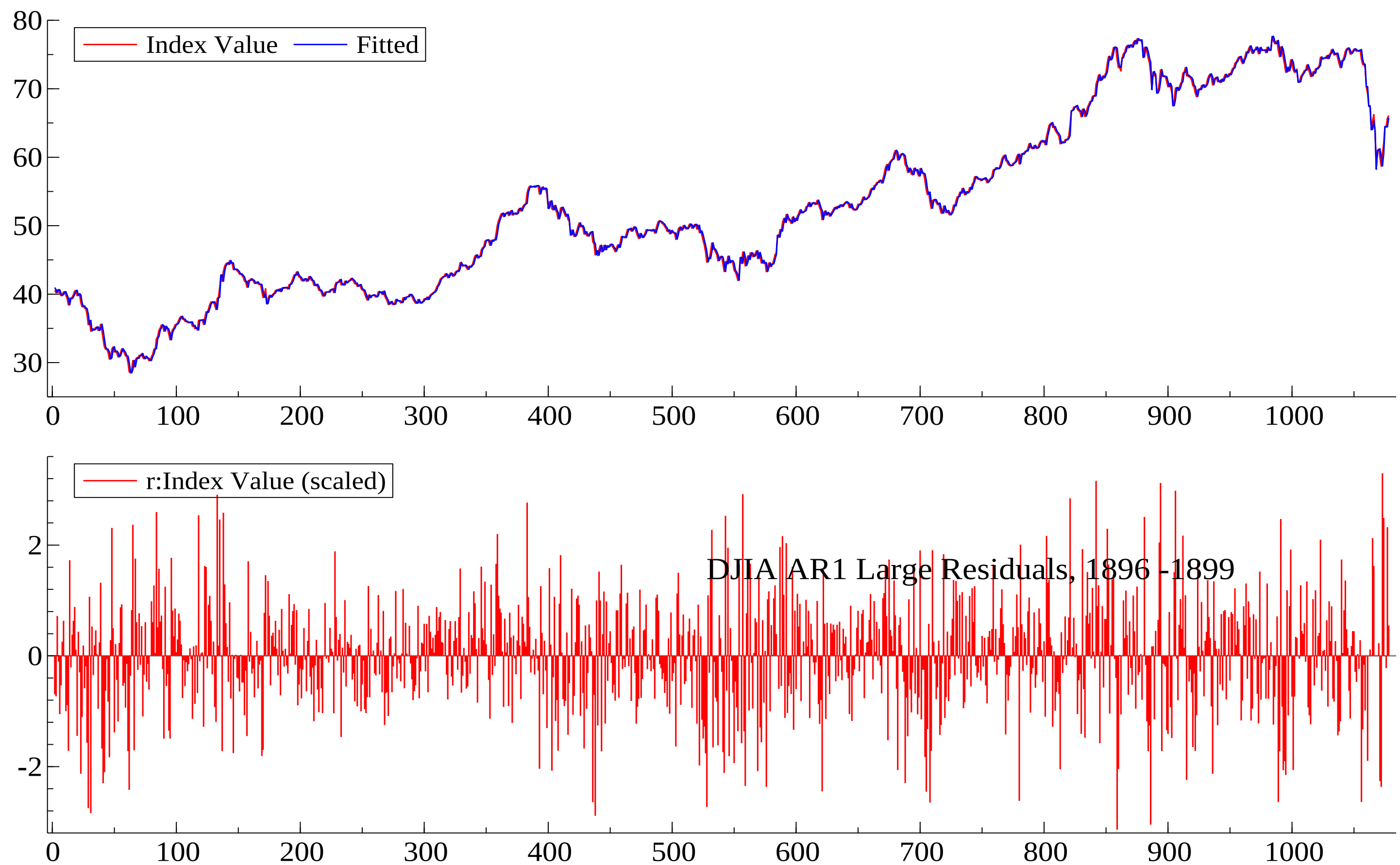

Figure 2: Model DJIA AR1 Large Residuals, from 1896 to 1899.

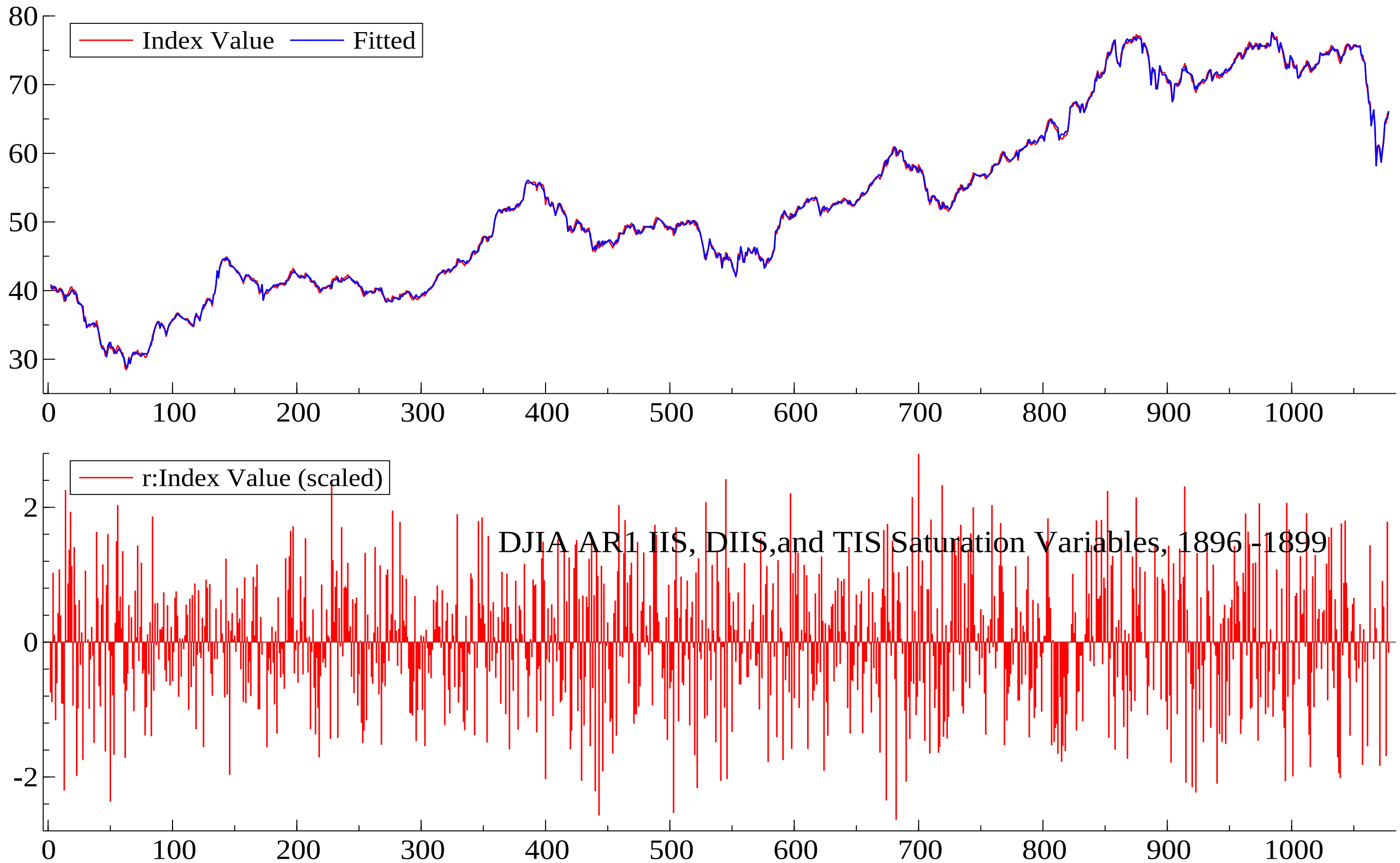

Figure 3: ModelDJIA AR1 IIS, DIIS, and TIS Saturation Variables, from 1896 to 1899.

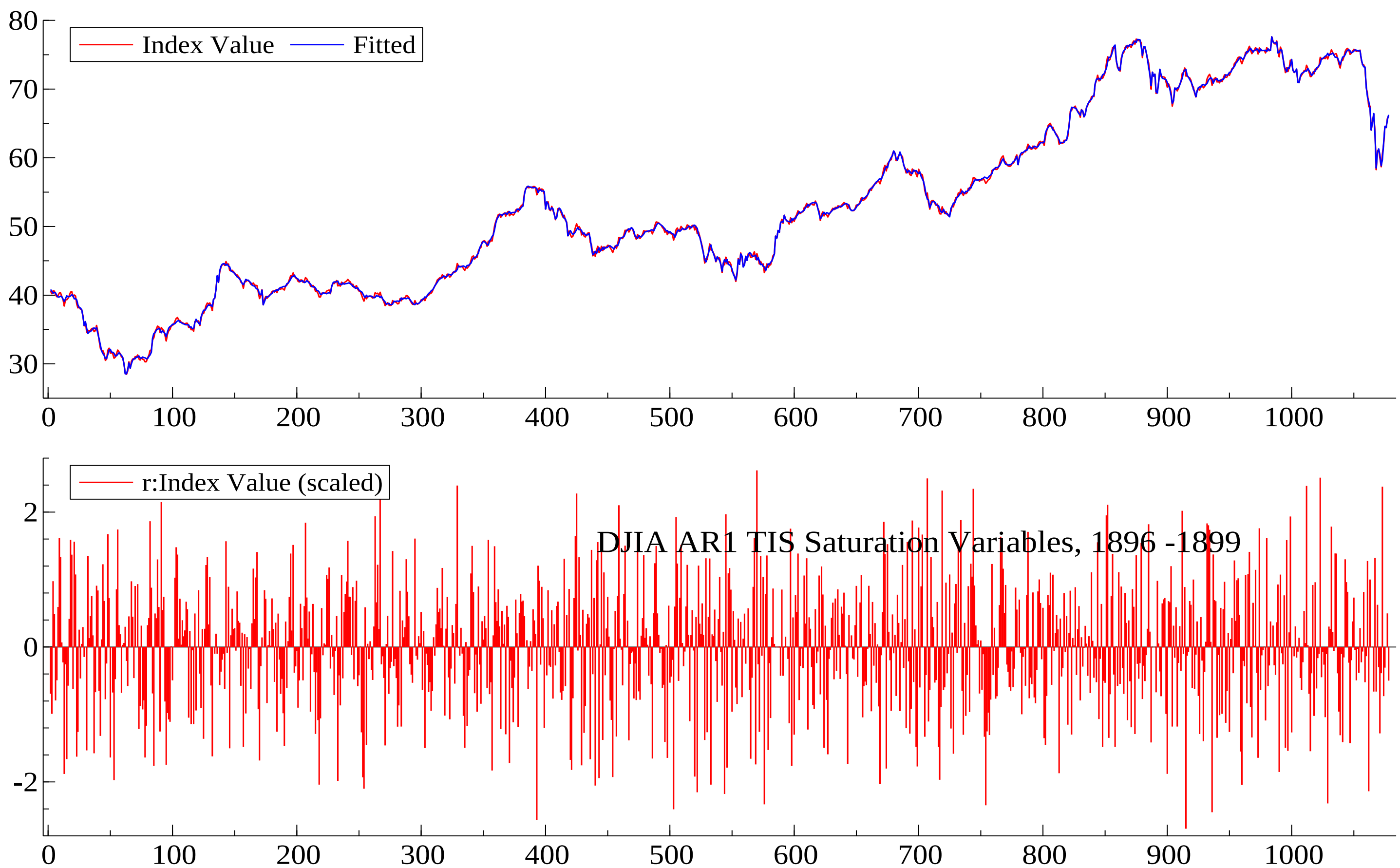

Figure 4: Model DJIA AR1 TIS Saturation Variables, from 1896 to 1899.

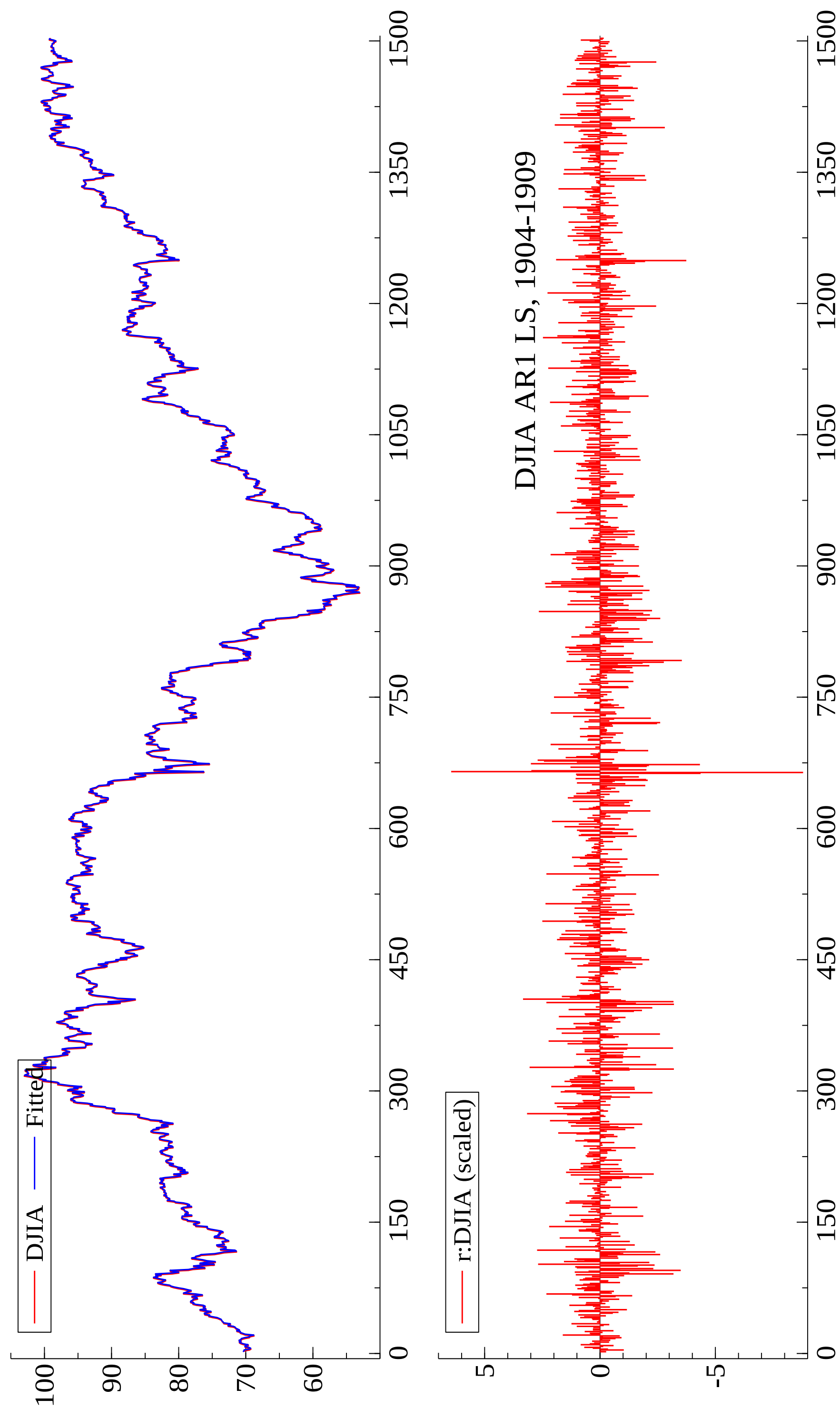

Figure 5: Model DJIA AR1 LS, from 1905 to 1909.

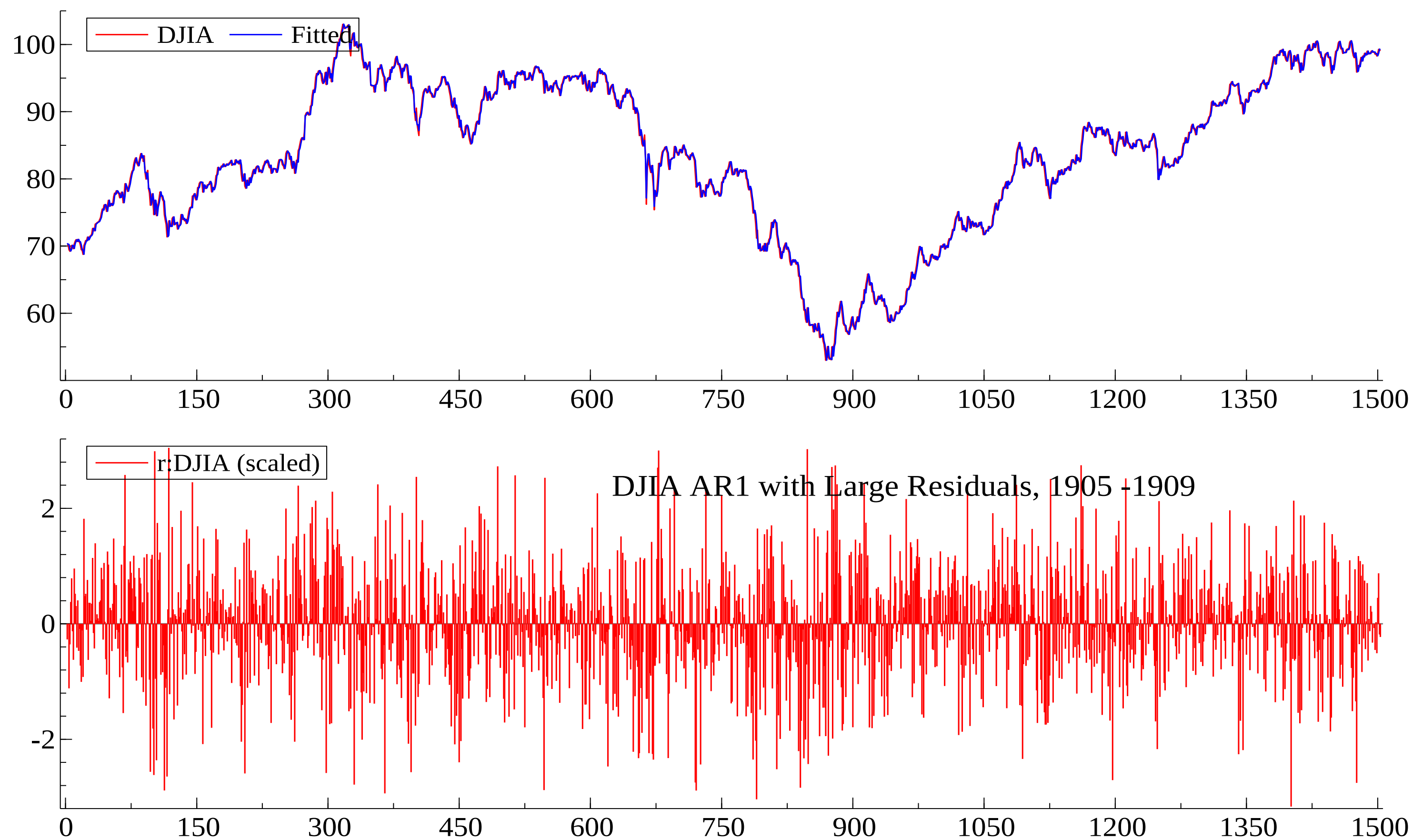

Figure 6: Model DJIA AR1 LS, from 1905 to 1909.

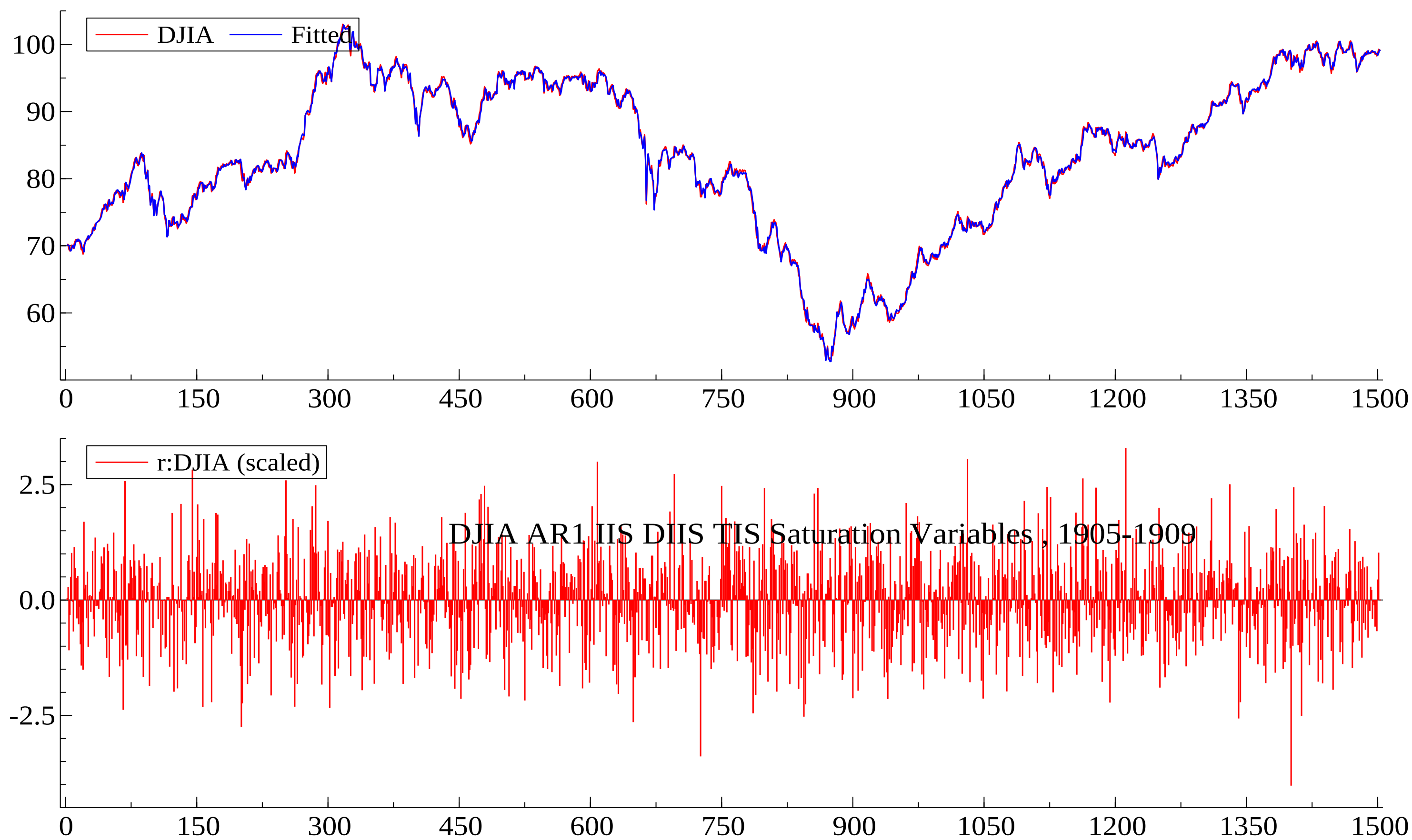

Figure 7: Model DJIA AR1 IIS DIIS TIS Saturation Variables, from 1905 to 1909.

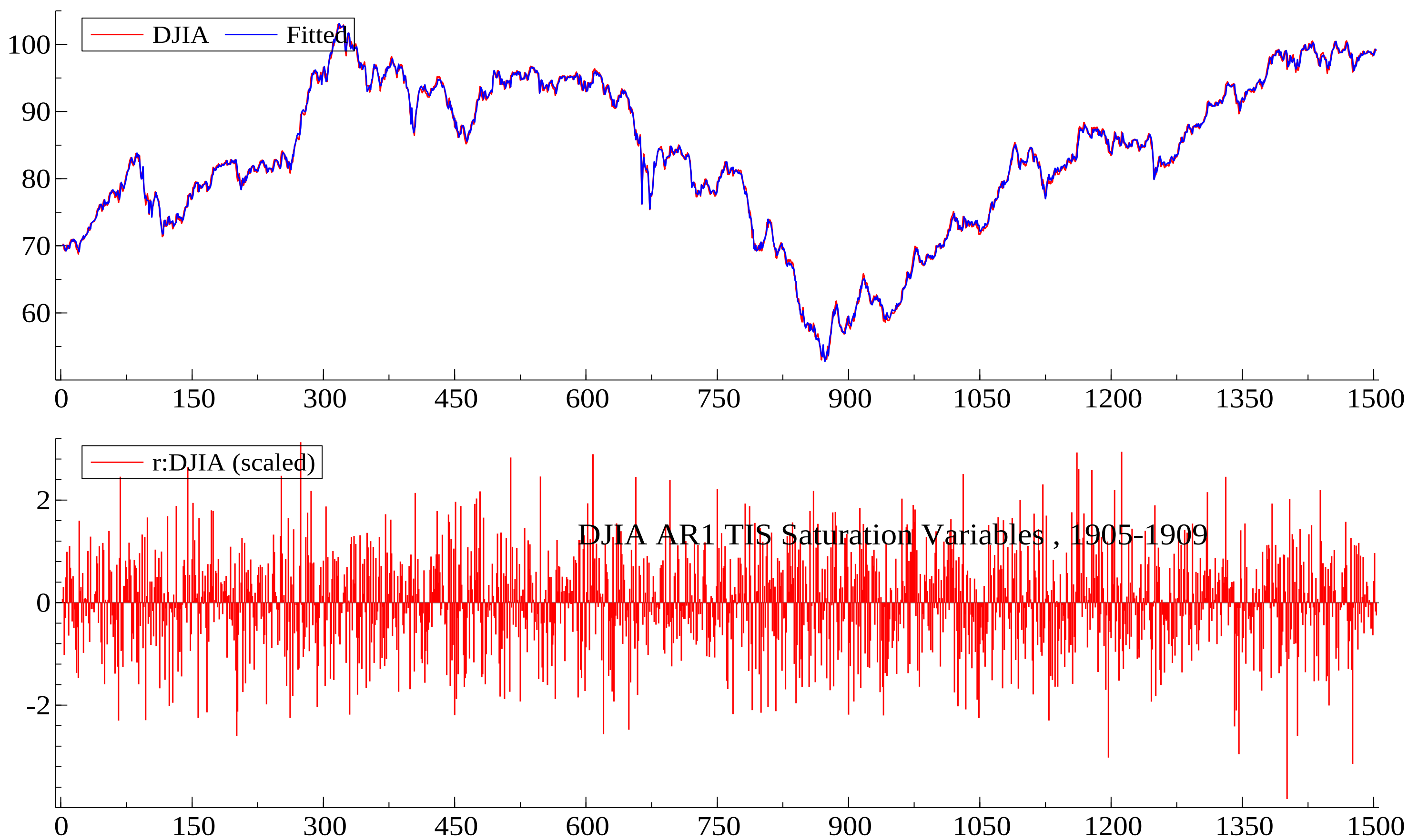

Figure 8: Model DJIA AR1 TIS Saturation Variables, from 1905 to 1909.

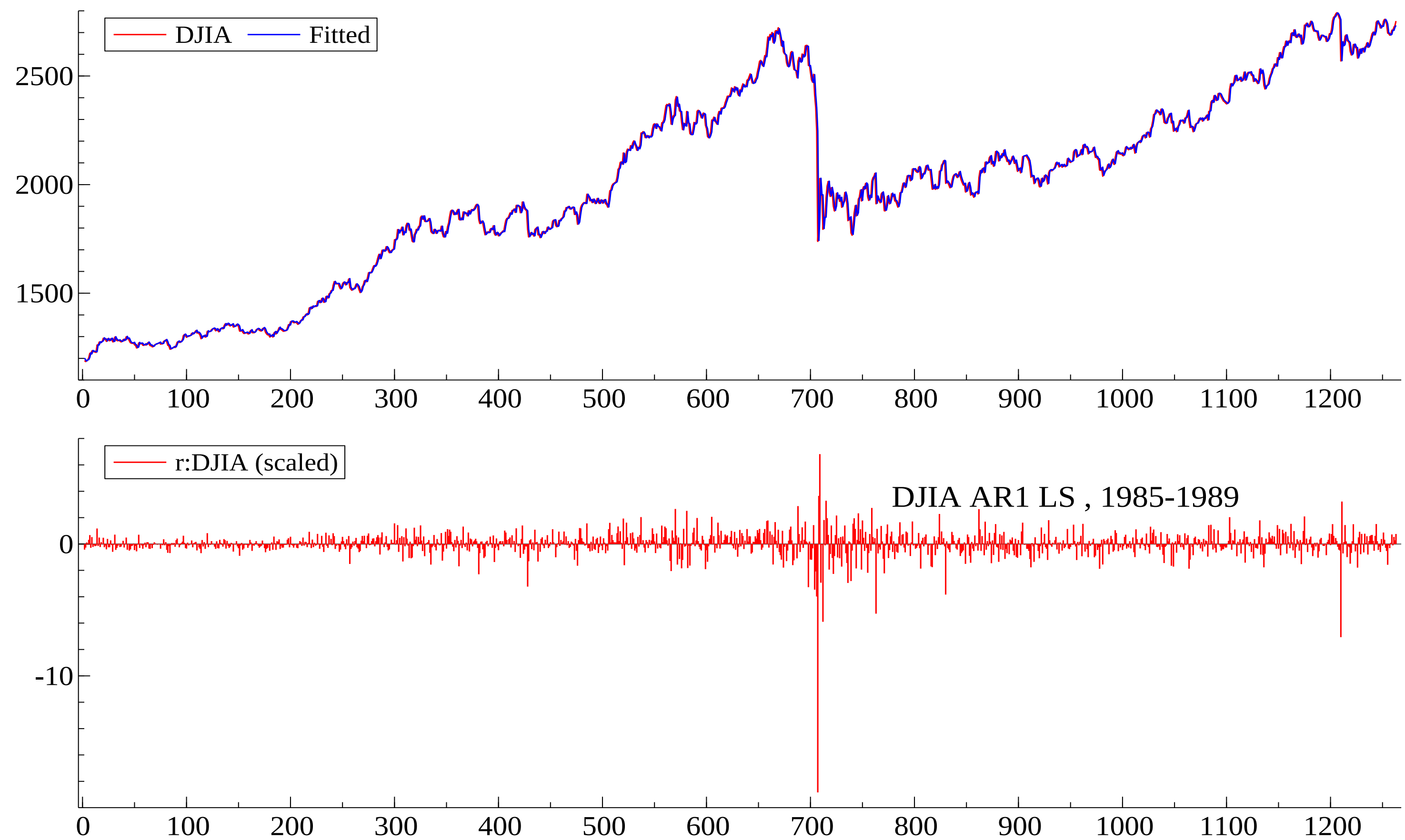

Figure 9: Model DJIA AR1 LS, from 1985 to 1989.

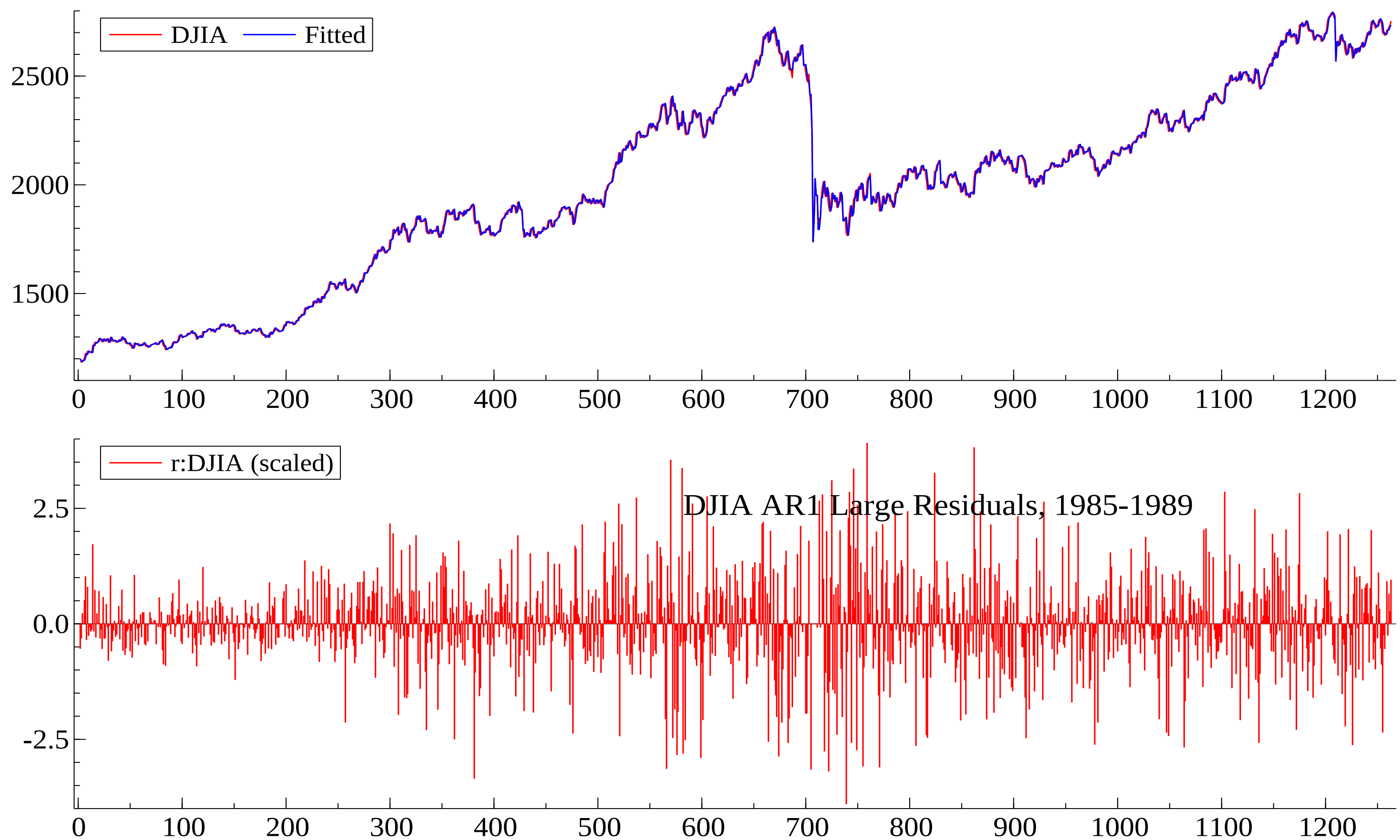

Figure 10: Model DJIA AR1 Large Residuals, from 1985 to 1989.

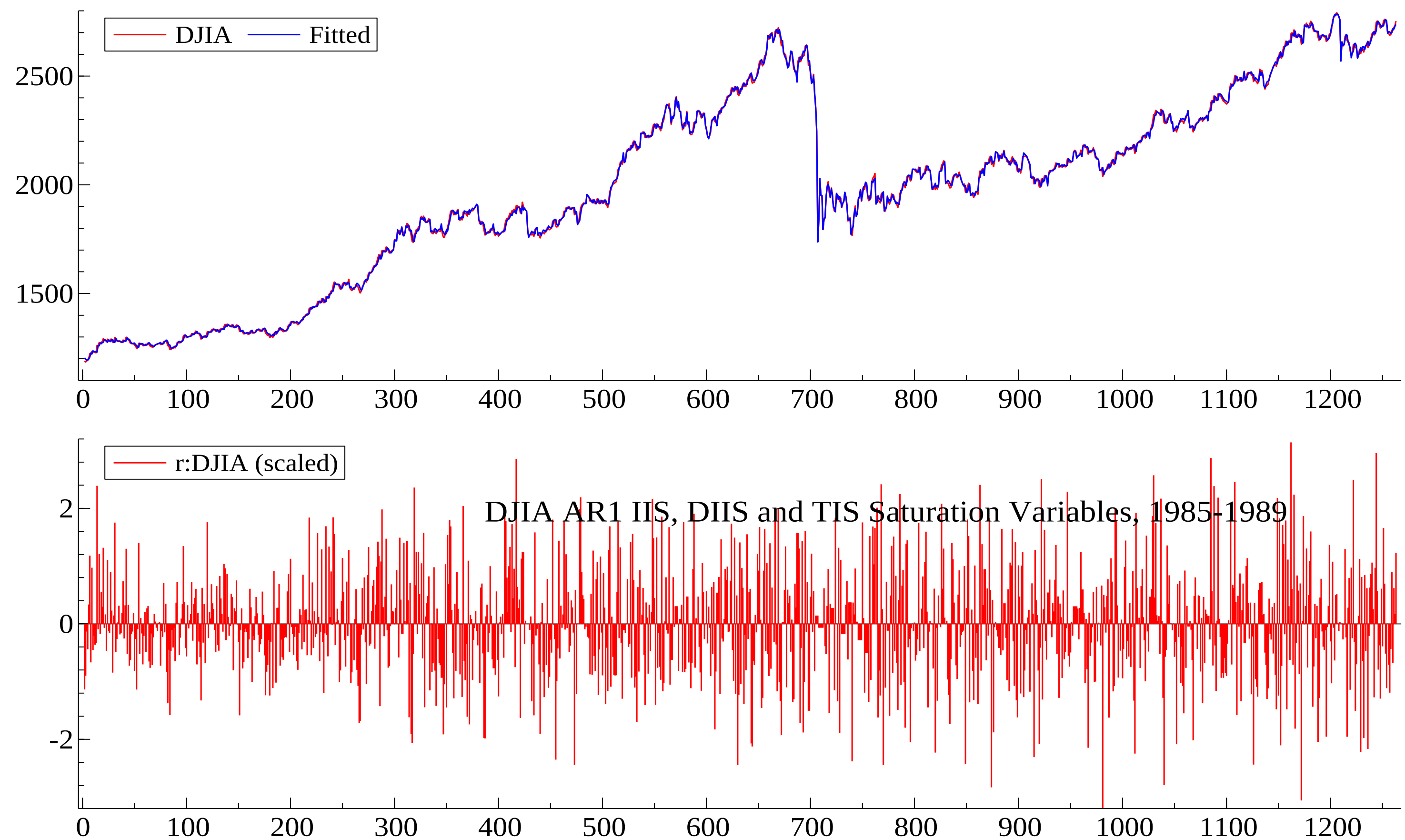

Figure 11: Model DJIA AR1 IIS DIIS and TIS Saturation Variables, from 1985 to 1989.

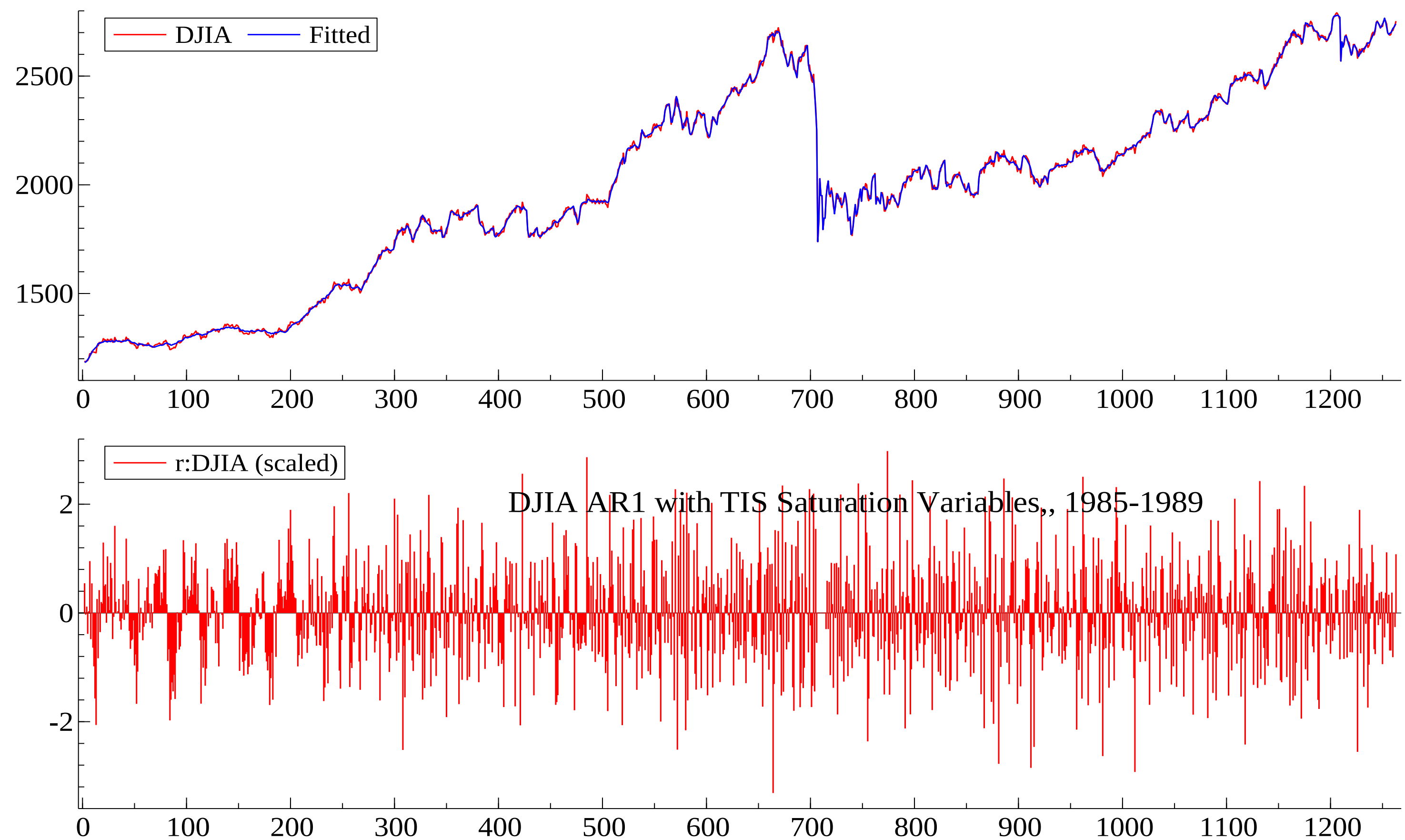

Figure 12: Model DJIA AR1 with TIS Saturation Variables, from 1985 to 1989.

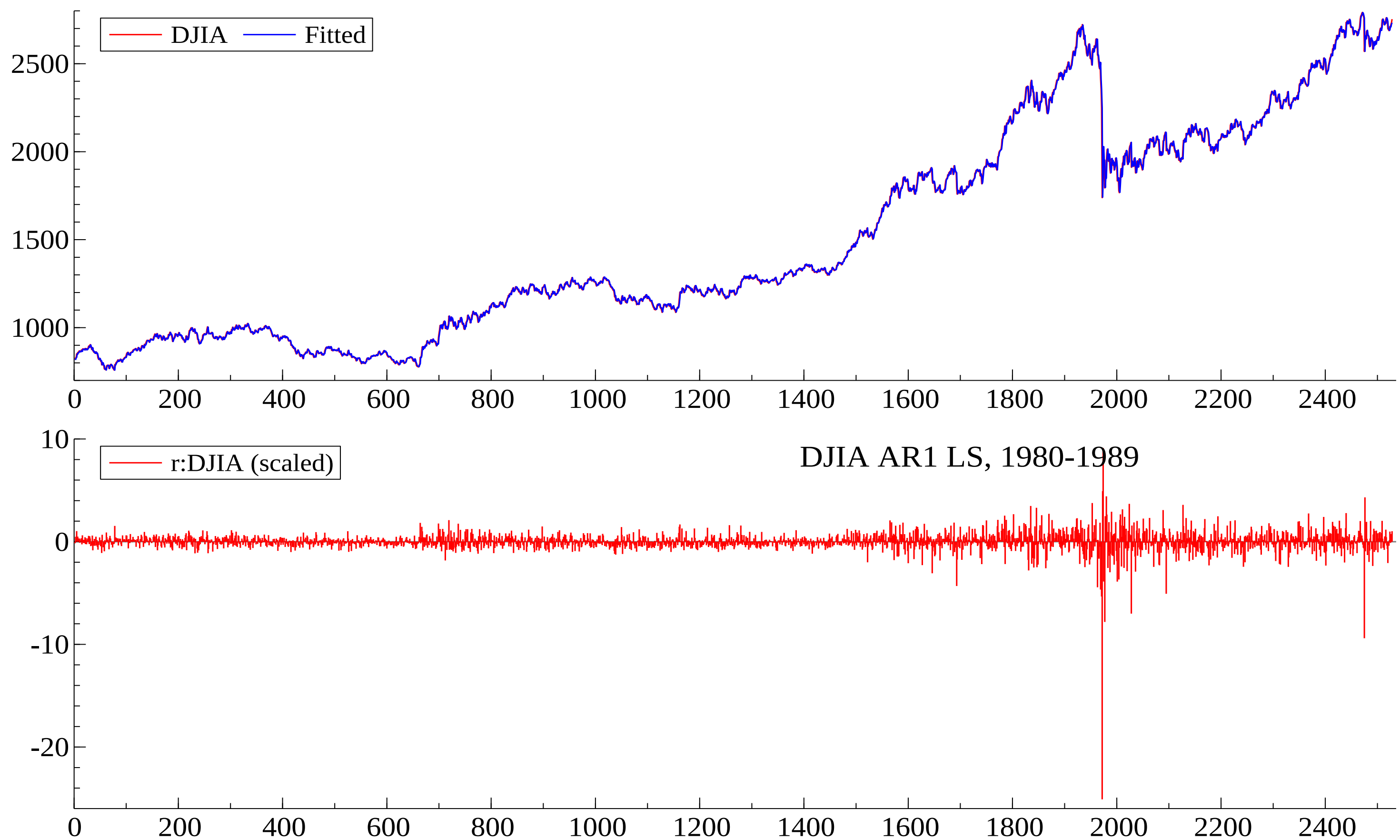

Figure 13: Model DJIA AR1 LS, from 1980 to 1989.

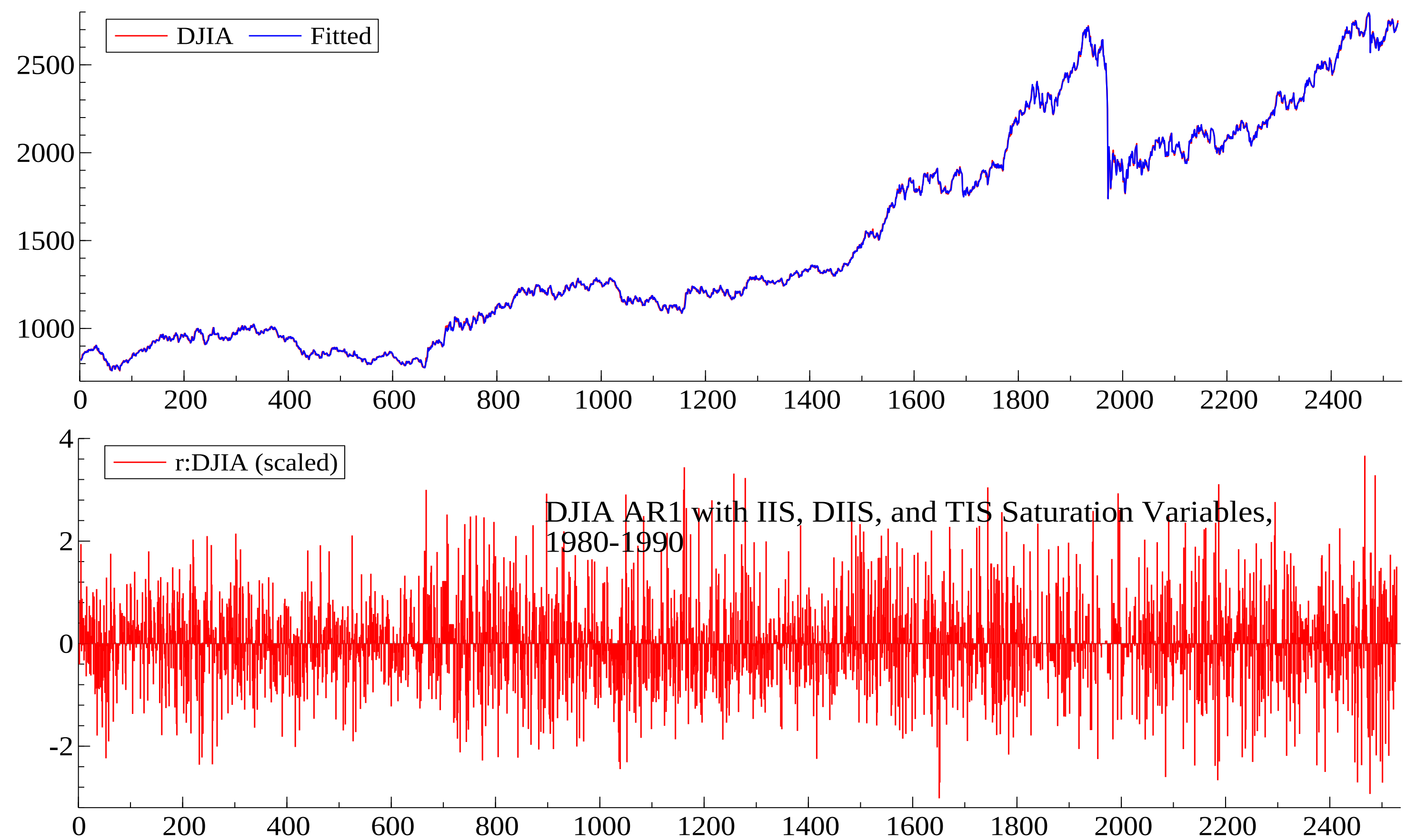

Figure 14: DJIA AR1 with IIS, DIIS, and TIS Saturation Variables from 1980 to 1989.

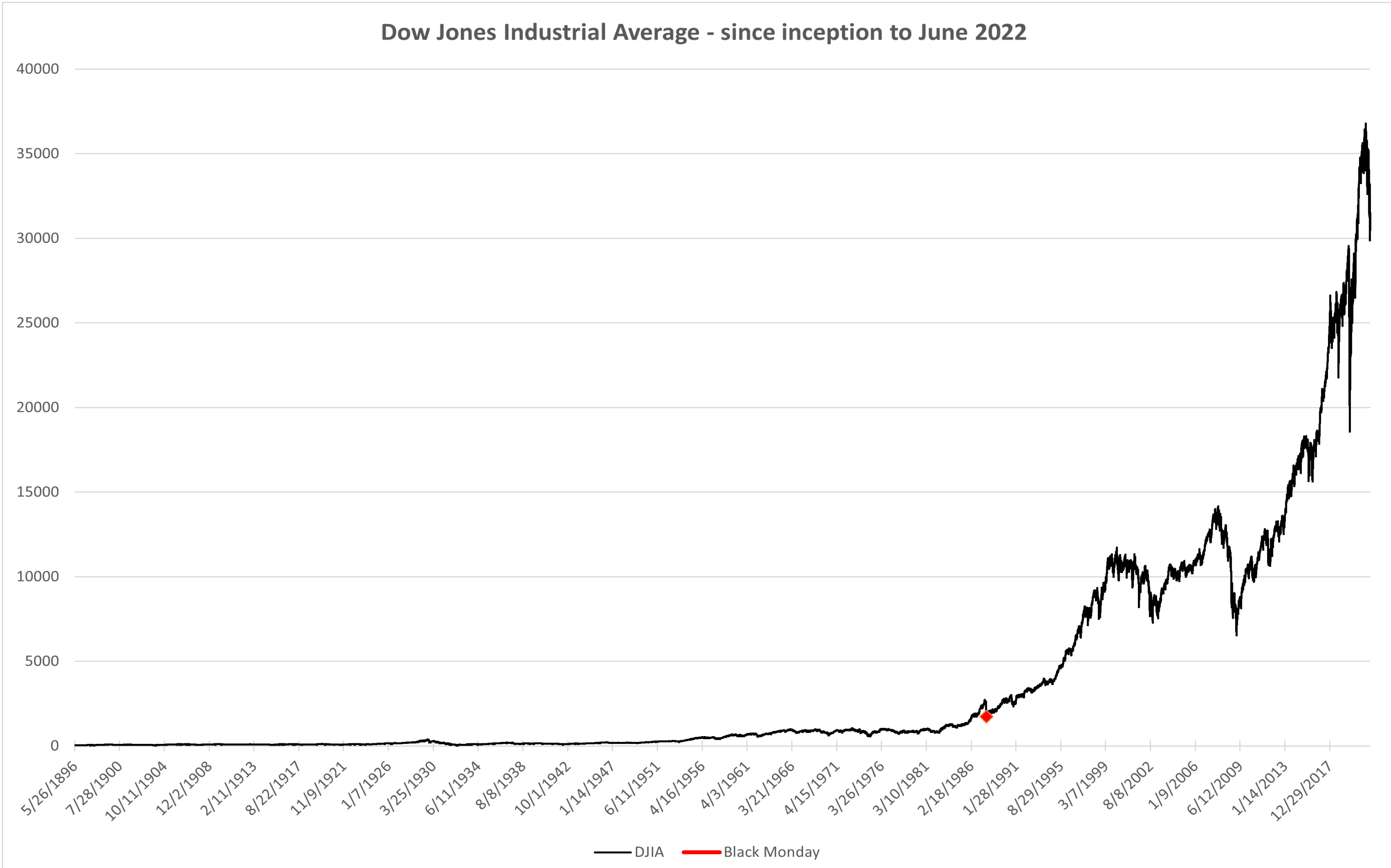

Figure 15: The whole history of the Dow Jones and the 1987 Black Monday

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