

Trade, Inequality, and the Endogenous Sorting of Heterogeneous Workers*

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Abstract

This paper presents a new framework to quantitatively investigate the effect of international trade on between-educational-type inequality and labor reallocation for a large number of countries. I embed workers' occupational choice problem into a multi-country, multi-industry, and multi-factor trade model. International trade and worker's comparative advantage affect workers' labor supply decision together, and, as a consequence, gains from trade differ across workers. I quantify the model for 32 countries, 5 educational types, 4 industries, and 5 occupations to examine the distributional effect of trade liberalization between 2000 and 2007, using the microdata from household surveys of each country. I find that (1) between-educational-type inequality increases in high-income countries and low-income countries with a manufacturing comparative advantage such as China; (2) occupation-level labor reallocation is an important channel by which trade shocks are disseminated across different workers; and (3) trade significantly contributes to macro patterns of industry- and occupation-level employment shifts.

Keywords: trade, worker heterogeneity, inequality, occupational choice, labor reallocation

JEL Codes: F16, F66, J24, C68, D33

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1 Introduction

This paper presents a new framework to quantitatively investigate the effect of international trade on domestic labor markets for a large number of countries. Two labor market outcomes are of main interest in this paper. First, workers with different levels of educational attainment gain or lose from trade by a different amount: *between-educational-type inequality*. Second, changes in trade environment make workers reallocate across different industries and occupations: *labor reallocation*. Although traditional trade theory predicts that trade simply increases inequality in high-income countries and decreases inequality in low-income countries (i.e., the Stolper-Samuelson theorem (1941)), this prediction is at odds with empirical evidence which paints much more complicated pictures – see [Goldberg and Pavcnik \(2003; 2004; 2007\)](#). This paper provides a multi-country, multi-industry, and multi-factor general equilibrium trade model with heterogeneous workers in order to better quantify the effect of changes in trade environment between 2000 and 2007 on those two sets of labor market outcomes for 32 countries around the world.

In this model, two distinct comparative advantage structures characterize the international trade environment and domestic labor markets, respectively. First, trade is driven by comparative advantage across countries based on productivity differences and relative factor endowments. Second, the effect of trade is disseminated differentially across workers within a country based on comparative advantage across workers. I characterize the worker-level comparative advantage by assuming that workers draw idiosyncratic industry- and occupation-specific productivities conditional on their exogenously endowed educational type. Then they endogenously sort into industry and occupation in order to maximize their incomes, as in the [Roy \(1951\)](#) model. International trade impacts this sorting mechanism and, as a consequence, gains from trade are different across workers based on their comparative advantage.

I also add another important ingredient to the model: workers not only choose an industry but also an occupation. Workers engage in different occupational tasks within the same industry based on their comparative advantage. As a result, they are affected by industry-level trade shocks differently depending on what they actually do within an industry. Occupation is another important margin through which trade shocks are disseminated across workers, because workers

with different skill levels show significantly different patterns of occupation-level labor allocation as shown in Figure A1. This model also quantitatively shows that occupation-level labor reallocation is much more tightly related to workers' education level, while industry-level labor reallocation patterns are similar across different worker types. Thus, ignoring the occupational dimension will significantly underestimate the differential effect of trade on workers with different characteristics.

I use this model to quantify the distributional effects of changes in the trade environment between 2000 and 2007 across 5 worker types defined by educational attainment, 4 industries, and 5 occupation categories in 32 countries and the rest of the world. This time period is particularly interesting, because international trade became an increasingly significant factor after China joined the World Trade Organization (WTO) in 2001. I use international microdata gleaned from household surveys for each country to quantify workers' differential responses to trade shocks in different countries. To take the model to the data, I estimate the key parameter, the *labor supply elasticity*, for four different countries and five educational types. This parameter is directly related to the degree of worker heterogeneity. I allow it to be country- and educational-type-specific rather than pre-commit to a specific assumption on the degree of worker heterogeneity.¹ The model in this paper also conveniently nests existing trade models in a tractable way using different values of this key parameter.

Armed with the parameter estimates, I separately introduce two types of trade shocks to perform counterfactuals. I first measure trade shocks by changes in bilateral trade costs, which are calibrated to match changes in bilateral trade flows in the data. The calibration result shows that trade costs have decreased primarily in the manufacturing industry between 2000 and 2007. I also separately look at the effect of changes in trade costs with China, since China has been on the rise in the global market during the time of interest. The second trade shock of main interest is the change in China's factor-neutral productivity as estimated in Hsieh and Ossa (2016).²

¹Workers are homogeneous in most trade models, including the Ricardian and the Heckscher-Ohlin trade models. The specific factors model is the other extreme case, where workers are extremely heterogeneous and thus constrained to a certain industry.

²Many empirical papers, such as Autor et al. (2013), connect the import competition from China in high-income countries to the increase of productivity in China, which eventually improves

The result from counterfactual experiments show that changes in the trade environment between 2000 and 2007 have raised between-educational-type inequality in most high-income countries and in low-income countries with a comparative advantage in the manufacturing industry such as China, India, and Indonesia. For example, combining two trade shocks, U.S. workers with advanced degrees have had a 1.98% increase in welfare, whereas welfare of high school dropouts has increased by 1.213%.³ For China, this discrepancy is predicted to be even larger: e.g., 2.62% increase and 1.45% increase, respectively. When between-educational-type inequality is measured by the skill premium, changes in trade environment explain, 11.42% and 17.07% of the actual changes in the skill premium in the U.S. and in China, respectively.⁴ In contrast, in some Latin American countries such as Brazil and Argentina, between-educational-type inequality has decreased due to trade, which is consistent with recent empirical evidence in those countries.

This paper also quantifies trade-induced labor reallocation across industries and occupations. The result shows that the occupation-level labor reallocation is very important.⁵ In order to generate differential response to trade shocks across different worker types, it is important to look at not only industry-level labor reallocation but also occupation-level labor reallocation. Moreover, the result shows that trade induces a significant contraction of manufacturing employment as well as a job polarization in high-income countries.⁶ In contrast, the model shows that liberalized trade between 2000 and 2007 generates a contraction of agricultural employment in low-income countries such as China and an expansion of agricultural employment in Latin American countries.

The motivation for this paper stems from many previous empirical studies that

China's export supply capability mainly through their cost advantage.

³In line with the international trade literature, welfare gains from trade are measured by changes in real income caused by changes in the trade environment assuming consumers have a homothetic preference.

⁴I define the skill premium by the wage premium of college graduates over non-college graduates.

⁵This is consistent with results in the macro literature; e.g., [Kambourov and Manovskii \(2008\)](#) and [Groes et al. \(2015\)](#). A recent paper by [Traiberman \(2016\)](#) also emphasizes the occupational dimension to explain the labor market effect of import competition in Denmark.

⁶The polarization across skill levels of occupation is both theoretically and empirically well-studied in the labor economics literature – see [Baumol \(1967\)](#), [Acemoglu \(1999\)](#) and [Autor et al. \(2003\)](#) on models of the skill-biased technical change, as well as [Autor et al. \(2008\)](#) and [Goos and Manning \(2007\)](#) for empirical evidence. A recent paper by [Harrigan et al. \(2016\)](#) studies the effect of trade on polarization.

document the relationship between trade and inequality: e.g., Autor et al. (2013; 2015) and Ebenstein et al. (2014) for developed countries, Goldberg and Pavcnik (2003; 2005) and Topalova (2007) for developing countries. I provide a structural model that complements empirical findings in those papers. This paper is not the first to use a quantitative general equilibrium framework to examine trade-induced inequality in a large number of countries. Burstein and Vogel (2016) focus on the reallocation of factors across heterogeneous firms within a sector, and Parro (2013) focuses on capital-skill complementarity. Unlike these papers, I focus on workers' heterogeneous productivities and endogenous sorting as the key channel through which trade impacts inequality. In addition, this paper uncovers the effect of trade on both industry- and occupation-level labor reallocation within many countries.

Most importantly, this paper contributes to the fast-growing literature on the Roy-like assignment model with worker heterogeneity, by Lagakos and Waugh (2013), Hsieh et al. (2013), and Burstein et al. (2015). I embed worker-level comparative advantage into the gravity structure of standard trade models based on country-level comparative advantage.⁷ This paper is distinct from previous works in three important ways. First, workers have heterogeneous productivities across both industries and occupations. I show quantitatively that considering both dimensions is important to quantify the distributional effect of trade. Second, this model provides a full picture of the interplay between country-level comparative advantage and within-country worker-level comparative advantage. Lastly, this paper quantifies a high-dimensional model of trade, inequality, and worker heterogeneity with rich microdata from household surveys across a large number of countries instead of focusing on the outcome of a single country.

This paper also contributes to the literature by providing a quantitative strategy to experiment with a wide range of trade liberalization episodes regarding changes in trade costs or partner countries' productivity instead of restricted trade episodes such as moving to autarky. Moreover, I estimate the key parameter – the labor supply elasticity, which is directly related to the degree of worker heterogeneity – in a more general setup accounting for heterogeneous wage distributions between

⁷Galle et al. (2015) follow a similar approach with heterogeneity defined only across industries not across occupations. Later in this paper, I discuss a limit case of my model to match a case without occupation-level worker heterogeneity or labor reallocation.

worker types and countries.

Generalizing the quantitative Ricardian model of [Eaton and Kortum \(2002\)](#) with heterogeneous workers, I provide a quantitative framework for theoretical foundations of workers' comparative advantage and trade studied by [Ohnsorge and Trefler \(2007\)](#) and [Costinot and Vogel \(2010\)](#). This model also differs from the search and matching model in an open economy ([Helpman et al. \(2016\)](#)) or from the model with transitional dynamics of industry-level reallocation ([Artuç et al. \(2010\)](#), [Dix-Carneiro \(2014\)](#), and [Caliendo et al. \(2015\)](#).) With the gravity structure that is in line with the welfare analysis of [Arkolakis et al. \(2012\)](#), the model remains quantitatively tractable by applying the technique of 'hat' algebra used by [Dekle et al. \(2008\)](#). The algorithm to solve the model is based on [Alvarez and Lucas \(2007\)](#) and [Caliendo and Parro \(2015\)](#), but with multiple production factors–occupations.

The structure of this paper is as follows. In Section 2, I develop a general equilibrium trade model with endogenous sorting of heterogeneous workers, and derive welfare and distributional effects of trade. Section 3 discusses the quantitative strategy, including the estimation of parameters and the calibration of trade shocks. In Section 4, I present counterfactual results to discuss the effect of trade on labor market outcomes. Section 5 presents sensitivity analyses, and Section 6 concludes.

2 Model

In this section, I construct a general equilibrium trade model that describes interrelation between international trade and workers' occupational choice problem within a country. Two comparative advantage structures characterize the model: one, across countries and the other, across workers within each country. Workers choose an industry and an occupation to work in based on their heterogeneous productivities as in [Roy \(1951\)](#). The parametrization of worker heterogeneity is closely related to [Hsieh et al. \(2013\)](#) and [Burstein et al. \(2015\)](#).

2.1 Environment

Consider an economy with N countries indexed by $i \in \{1, \dots, N\}$. Each country has J industries indexed by $j \in \{1, \dots, J\}$ and a continuum of products $e^j \in [0, 1]$

within each industry j . The trade environment of each industry follows [Eaton and Kortum \(2002\)](#) (EK, hereafter).⁸

Preferences Individuals have common nested CES preferences over J industries and within-industry product varieties:

$$U_i = \left(\sum_j (C_i^j)^{\frac{\eta_1-1}{\eta_1}} \right)^{\frac{\eta_1}{\eta_1-1}}$$

where $C_i^j = \left(\int_0^1 C_i(e^j)^{\frac{\eta_2-1}{\eta_2}} de^j \right)^{\frac{\eta_2}{\eta_2-1}}$ is a CES aggregate consumption bundle, and $\eta_1, \eta_2 > 0$ are elasticities of substitution across industries and across product varieties, respectively.

Workers Workers inelastically supply one unit of time and earn labor income. Workers are exogenously classified by their types $\tau \in \{1, \dots, T\}$ *ex ante*, which are mutually exclusive and exhaustive groups empirically defined by observable worker characteristics, including educational attainment, age, or gender. The total number of type τ workers in country i is exogenously given by $L_{i,\tau}$. Each worker solves an occupational choice problem by simultaneously choosing the industry and occupational affiliation generating the highest labor income, as in the [Roy \(1951\)](#) model. There are O occupations indexed by $o \in \{1, \dots, O\}$.

The labor market is perfectly competitive, so that workers earn their marginal revenue product. The workers' occupational choice problem depends on workers' productivity and the market value of labor in different industries and occupations. I assume that an individual worker ω of type τ has an idiosyncratic productivity $\epsilon_\omega^{j,o}$ for each pair of industry j and occupation o , where $\epsilon_\omega^{j,o}$ is randomly drawn from a Fréchet distribution:

$$F_{i,\tau}^{j,o}(\epsilon) = \exp(-T_{i,\tau}^{j,o} \epsilon^{-\theta_{i,\tau}}).$$

This idiosyncratic productivity is interpreted as efficiency units of labor that worker ω is able to provide to industry j with occupation o . For simplicity, it is assumed that there is no correlation between industry- and occupation-specific draws, but

⁸A Ricardo-Roy model combines the assignment-based Roy model and the Ricardian trade environment. [Costinot and Vogel \(2010\)](#) provide a theoretical foundation based on the notion of log-supermodularity. [Costinot and Vogel \(2015\)](#) provide an authoritative overview of both theory and empirics in this literature.

this assumption can be easily generalized to allow correlations.⁹ This parametrization is analogous to the quantitative Ricardian trade model pioneered by Eaton and Kortum (2002). The Fréchet distribution is a type II extreme value distribution, and thus the maximum of independently drawn Fréchet random variables again follows another Fréchet distribution. This feature lends great tractability to derive simple analytic solutions for equilibrium outcomes.

First, the shape parameter of this distribution $\theta_{i,\tau}$ governs the within-type dispersion of productivity, which can potentially differ across countries. As shown in Section 2.5, this parameter is related to the elasticity of labor supply at the industry and occupation level. Hence, I will call it the “labor supply elasticity” parameter. Worker types with higher $\theta_{i,\tau}$ have a more elastic labor supply at the industry and the occupation level. This is due to the fact that types with higher $\theta_{i,\tau}$ have fewer outliers in productivity, making them more likely to adjust to changes in per-unit wages by industry and occupation. Second, the scale parameter $T_{i,\tau}^{j,o}$ represents the level of workers’ productivities, which governs the absolute advantage of type τ workers in country i for (j, o) . The worker-level comparative advantage across types is determined by ratios of this parameter: for example, type τ workers have a comparative advantage in (j, o) compared to type τ' workers in (j', o') if

$$\frac{T_{i,\tau}^{j,o}}{T_{i,\tau}^{j',o'}} > \frac{T_{i,\tau'}^{j',o'}}{T_{i,\tau'}^{j,o}}.^{10}$$

Workers’ comparative advantage is two-fold. On the one hand, across-type worker comparative advantage is determined by the ratio of $T_{i,\tau}^{j,o}$ as described above. On the other hand, within-type worker comparative advantage is determined by $\theta_{i,\tau}$. Worker types with a smaller $\theta_{i,\tau}$ have stronger comparative advantage structure for industries and occupations within their types, as their productivities are more heterogeneous. Both across-type and within-type worker comparative advantages are important to determine the equilibrium labor allocation and reallocation in this model.

Production Workers engage in the production of final goods by choosing an

⁹If a correlation is allowed, the joint distribution function will be

$$F_{i,\tau}(\epsilon) = \exp[-\{\sum_{j,o} (T_{i,\tau}^{j,o} \epsilon^{-\theta_{i,\tau}})^{1/(1-\tilde{\rho})}\}^{1-\tilde{\rho}}]$$

where $\tilde{\rho}$ is a correlation parameter.

¹⁰This is a stochastic version of log-supermodularity as Costinot and Vogel (2015) point out.

industry and an occupation, where occupations are factors of production. Production of a product variety e^j follows a CES technology:

$$Y_i(e^j) = z_i(e^j) \left(\sum_o \mu_i^{j,o} (y_i^{j,o}(e^j))^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad (1)$$

where $z_i(e^j)$ is a country i 's factor-neutral productivity of producing e^j . The occupational labor input from all workers with occupation o is denoted by $y_i^{j,o}(e^j)$. The occupation-intensity parameter is given by $\mu_i^{j,o}$, and sums to one for each industry, and γ is the elasticity of substitution between occupations. In the quantitative analysis, I consider occupations as complementary production inputs, as evidenced by [Goos et al. \(2014\)](#).¹¹

2.2 International Trade

There are N countries participating in international trade, and only final goods are traded. I assume that the final goods market is perfectly competitive, in which each country purchases each product from the lowest-cost supplier. The price of product e^j depends on the unit cost of the occupational input bundle c_i^j available in industry j as well as on the productivity $z_i(e^j)$. The Heckscher-Ohlin channel of this model is based on the relative type-level labor supply and endogenous occupational choices, which determines *across-industry* specialization patterns. The Ricardian force of trade, on the other hand, is active through productivity $z_i(e^j)$ and determines *within-industry* specialization patterns. The productivity $z_i(e^j)$ is drawn from a Fréchet distribution independently for each e^j :

$$H_i^j(z) = \exp(-A_i^j z^{-\nu^j}), \quad (2)$$

where the scale parameter A_i^j is connected to the absolute advantage of country i for industry j , and ν^j governs the dispersion of productivity across countries. The degree of dispersion is different across industries, as ν^j depends on the industry.

¹¹If $\gamma \rightarrow \infty$, the production function becomes linear in occupations, analogous to [Costinot and Vogel \(2010\)](#). In this limit case, country-level comparative advantage is exactly transferred to worker-level comparative advantage within countries, if comparative advantages are characterized by log-supermodularity. As a consequence, the model prediction becomes closer to the predictions of traditional trade theory which is mainly based on the positive assortative matching.

This framework is built on multi-industry extensions of the EK model by [Chor \(2010\)](#), [Costinot et al. \(2011\)](#), [Donaldson \(2012\)](#), and [Caliendo and Parro \(2015\)](#).¹²

Trade is subject to standard iceberg-type costs: $d_{in}^j \geq 1$ for any product in industry j produced in i and shipped to n . It is assumed that $d_{in}^j > 1$ for $i \neq n$, $d_{ii}^j = 1$ for every i , and $d_{in}^j = d_{ni}^j$. Trade costs are different across industries.¹³

2.3 Partial Equilibrium

Partial equilibrium results are derived separately for workers' occupational choices, production, and trade flows between countries. Each result is determined given the per-unit price $p_i^{j,o}$ of occupational input for each country, industry, and occupation.¹⁴ These prices are, in turn, determined in general equilibrium.

Occupational choice problem A potential wage of worker ω in country i with an idiosyncratic productivity $\epsilon_\omega^{j,o}$ for (j, o) is $w_{i,\omega}^{j,o} = p_i^{j,o} \epsilon_\omega^{j,o}$. The workers' occupational choice problem is to choose an industry and an occupation that maximize $w_{i,\omega}^{j,o}$. Using the Fréchet distribution of workers' productivity, the equilibrium probability that a worker ω of type τ works in industry j in occupation o is

$$\pi_{i,\tau}^{j,o} = \frac{T_{i,\tau}^{j,o} (p_i^{j,o})^{\theta_{i,\tau}}}{\sum_{j',o'} T_{i,\tau}^{j',o'} (p_i^{j',o'})^{\theta_{i,\tau}}}, \quad (3)$$

which determines the industry and occupation-level labor supply. Worker-level comparative advantage affects this labor supply function: workers are more likely to supply their labor to the industry and the occupation where they have a comparative advantage. In addition, the degree of within-type comparative advantage $\theta_{i,\tau}$ affects the responsiveness of type τ workers with respect to changes in $p_i^{j,o}$. The same change in $p_i^{j,o}$ may induce differential labor reallocation patterns across

¹²The parametrization in this paper is most closely related to [Caliendo and Parro \(2015\)](#) with industry-specific v^j . I generalize the labor supply side by considering workers' endogenous occupational choices but simplify the input-output linkage.

¹³This model can easily be extended to consider tariff and non-tariff parts of trade costs separately without much change to the main implication of the model.

¹⁴The per-unit price for occupational input varies both by industry and by occupation, because the labor supply curve is upward-sloping due to heterogeneous productivities. This variable is different from the actual wage observable in the data which includes unobservable efficiency units of labor.

different worker types because of the worker-level comparative advantage. A detailed derivation of (3) can be found in the Appendix.

Given workers' equilibrium choice of (j, o) , the probability distribution of the equilibrium wage of type τ workers is derived by

$$G_{i,\tau}^*(w) = \exp\left\{-\left[\sum_{j',o'} T_{i,\tau}^{j',o'} (p_i^{j',o'})^{\theta_{i,\tau}}\right] w^{-\theta_{i,\tau}}\right\}. \quad (4)$$

This is another Fréchet distribution with a scale parameter $\sum_{j',o'} T_{i,\tau}^{j',o'} (p_i^{j',o'})^{\theta_{i,\tau}}$ and a shape parameter $\theta_{i,\tau}$. It is important to have both a type-specific and country-specific parameter $\theta_{i,\tau}$, because the data show that the degree of wage dispersion within worker types varies significantly by worker type and country.¹⁵ This wage distribution gives the equilibrium average wage:

$$w_{i,\tau} = \left[\sum_{j',o'} T_{i,\tau}^{j',o'} (p_i^{j',o'})^{\theta_{i,\tau}}\right]^{\frac{1}{\theta_{i,\tau}}} \Gamma\left(1 - \frac{1}{\theta_{i,\tau}}\right), \quad (5)$$

where $\Gamma(\cdot)$ is a Gamma function. I assume $\theta_{i,\tau} > 1$ for all i and τ so that the average wage is well-defined. From (5), if type τ workers have a comparative advantage in the high-paying (j, o) , they have relatively higher wages on average. Industry- and occupation-level average wages are derived from the type-level average wage (5), employment allocation (3), and type-level labor supply $L_{i,\tau}$, where the first two depend on the endogenous variable $p_i^{j,o}$.¹⁶

Production and Trade Each firm solves a cost minimization problem by choosing the equilibrium demand for the efficiency units of labor with each occupational task, $y_i^{j,o}(e^j)$. The CES technology results in the following equilibrium unit cost function:

$$c_i^j = \left(\sum_o (\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma}\right)^{1/(1-\gamma)}. \quad (6)$$

The effective unit cost to produce a variety e^j in country i is $c_i^j/z_i(e^j)$. The price of

¹⁵For example, the data clearly show that better-educated workers are more dispersed in earned wages within their type than less-educated workers are.

¹⁶The average wage of industry j is $w_i^j = \sum_{\tau,o} w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o} / \sum_{\tau,o} L_{i,\tau} \pi_{i,\tau}^{j,o}$ and that of occupation o is $w_i^o = \sum_{\tau,j} \sum_{\tau,o} w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o} / \sum_{\tau,j} L_{i,\tau} \pi_{i,\tau}^{j,o}$. Thus, both w_i^j and w_i^o respond to any endogenous compositional shift of workers within types, which should be reflected in endogenous changes of $\pi_{i,o}^{j,o}$.

a product e^j in country n , if it were produced in country i is $P_{in}(e^j) = (\frac{c_i^j}{z_i(e^j)})d_{in}^j$. Due to perfect competition, the actual price of e^j in country n is given by $P_n(e^j) = \min_i P_{in}(e^j)$. Equilibrium price and trade flow are analogous to the results of the EK model. Details are provided in the Appendix.

Next, a gravity equation shows patterns of within-industry specialization. The probability that a country n buys a good in industry j from a country i is

$$\lambda_{in}^j = \frac{A_i^j (c_i^j d_{in}^j)^{-\nu^j}}{\Phi_n^j} = \frac{X_{in}^j}{X_n^j}, \quad (7)$$

where $\Phi_n^j \equiv \sum_{i=1}^N A_i^j (c_i^j d_{in}^j)^{-\nu^j}$. From this gravity equation, ν^j is the elasticity of imports with respect to trade costs, which is called the trade elasticity. An industry with less dispersion of productivity across countries has a higher trade elasticity, because trade flows respond more to changes in trade costs when countries are similar in productivity.

The exact price index P_i^j for industry j and country i is

$$P_i^j = \left(\Gamma\left(\frac{\nu^j + 1 - \eta_2}{\nu^j}\right) \right)^{\frac{1}{1-\eta_2}} (\Phi_i^j)^{-\frac{1}{\nu^j}}, \quad (8)$$

where $\Gamma(\cdot)$ is a gamma function. I assume $\nu^j + 1 > \eta_2$ so that the price index is well-defined. A country-level exact price index, $P_i = [\sum_j (P_i^j)^{1-\eta_1}]^{\frac{1}{1-\eta_1}}$ and the aggregate expenditure share λ_i^j are derived from the nested CES preference:

$$\lambda_i^j = \frac{(P_i^j)^{1-\eta_1}}{\sum_{j'} (P_i^{j'})^{1-\eta_1}}. \quad (9)$$

2.4 General Equilibrium

In general equilibrium, goods markets and occupation markets clear in all countries, and the trade balance condition holds. Final goods markets are cleared when

$$E_i^j = \sum_{n=1}^N \lambda_{in}^j X_n^j \quad (10)$$

holds for each i and j , where E_i^j is the value of gross output in industry j in country i . The total expenditure is $X_i^j = \lambda_i^j I_i$, where I_i is the total spending which is equal to the total income, $I_i = \sum_{\tau} w_{i,\tau} L_{i,\tau} + D_i$, with D_i being an aggregate trade deficit.

Since workers have heterogeneous productivities across industries and occupations, the occupation market clearing conditions are defined for each industry and occupation, making the total number of equations ($J \times O$) for each country $i \in \{1, \dots, N\}$,

$$(\mu_i^{j,o})^\gamma \left(\frac{p_i^{j,o}}{c_i} \right)^{1-\gamma} E_i^j = \sum_{\tau} w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o}. \quad (11)$$

Two market clearing conditions imply the trade balance condition for each country,

$$\sum_j \sum_{i=1}^N \lambda_{in}^j X_n^j - D_n = \sum_j \sum_{i=1}^N \lambda_{ni}^j X_i^j. \quad (12)$$

The equilibrium is solved for the per-unit occupational price $p_i^{j,o}$ for each i, j , and o that satisfies the equilibrium conditions (3), (5)-(12).

Equilibrium in proportional changes For more convenient comparative statics, another way to characterize the equilibrium is to solve the model for proportional changes of equilibrium variables. A proportional change of any variable x is denoted by $\hat{x} = x'/x$, where x' is a variable x at the counterfactual equilibrium. The so-called exact hat algebra (Costinot and Rodríguez-Clare (2014)) following Dekle et al. (2008) reduces the number of parameters that need to be determined and thus reduces data requirement for quantification.

I introduce two exogenous shocks in the counterfactual analysis: changes in bilateral trade costs (\hat{d}_{in}^j) and changes in sector-specific factor-neutral technology (\hat{A}_i^j).¹⁷ Specifically, I consider \hat{A}_{CHN}^j as one of trade shocks in the counterfactual analysis, given that changes in China's productivity are closely related to their exporting capability as it is pointed out in many papers in the literature; e.g., Autor et al. (2013). A counterfactual equilibrium in changes can be easily extended to incorporate the effect of changes in the other parameters such as $T_{i,\tau}^{j,o}$, $\mu_i^{j,o}$, and $L_{i,\tau}$,

¹⁷Factor-specific productivity parameter $T_{i,\tau}^{j,o}$ can be also time-varying. For the main counterfactual analysis, I consider \hat{A}_i^j as a technology shock in order to focus only on the effect of labor demand shocks. Unlike \hat{A}_i^j , $\hat{T}_{i,\tau}^{j,o}$ has first-order effects on labor supply and second-order effects on labor demand.

which is discussed in the [online appendix](#).

All equilibrium conditions (3), (5)-(12) can be re-written in terms of proportional changes. The counterfactual equilibrium determines $\hat{p}_i^{j,o}$ for each i, j , and o that satisfy the following equilibrium conditions.

- Labor supply: Assuming $\hat{T}_{i,\tau}^{j,o} = 1$,

$$\hat{\pi}_{i,\tau}^{j,o} = \frac{(\hat{p}_i^{j,o})^{\theta_{i,\tau}}}{\sum_{j',o'} (\hat{p}_i^{j',o'})^{\theta_{i,\tau}} \pi_{i,\tau}^{j',o'}} \quad (13)$$

- Type-level average wage:

$$\hat{w}_{i,\tau} = \left[\sum_{j,o} (\hat{p}_i^{j,o})^{\theta_{i,\tau}} \pi_{i,\tau}^{j,o} \right]^{\frac{1}{\theta_{i,\tau}}} \quad (14)$$

- Unit cost of production: Assuming $\hat{\mu}_i^{j,o} = 1$,

$$\hat{c}_i^j = \left[\sum_o \zeta_i^{j,o} (\hat{p}_i^{j,o})^{1-\gamma} \right]^{1/(1-\gamma)} \quad (15)$$

where $\zeta_i^{j,o} \equiv \frac{(\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma}}{\sum_{o'} (\mu_i^{j,o'})^\gamma (p_i^{j,o'})^{1-\gamma}}$ is a cost share of occupation o in industry j .

- Industry-level price index and expenditure share:

$$\hat{p}_n^j = \left[\sum_{i=1}^N \lambda_{in}^j \hat{A}_i^j (\hat{c}_i^j \hat{d}_{in}^j)^{-\nu^j} \right]^{-1/\nu^j}. \quad (16)$$

$$\hat{\lambda}_i^j = \frac{(\hat{p}_i^j)^{1-\eta_1}}{\sum_{j'} \lambda_i^{j'} (\hat{p}_i^{j'})^{1-\eta_1}} \quad (17)$$

- Bilateral trade flows:

$$\frac{\hat{X}_{in}^j}{\hat{X}_n^j} = \hat{A}_i^j \left(\frac{\hat{c}_i^j \hat{d}_{in}^j}{\hat{p}_n^j} \right)^{-\nu^j} = \hat{\lambda}_{in}^j \quad (18)$$

- Occupation market clearing condition: Assuming $\hat{L}_{i,\tau} = 1$,

$$\left(\frac{\hat{p}_i^{j,o}}{\hat{c}_i^j} \right)^{1-\gamma} \hat{E}_i^j = \sum_{\tau'} \left(\frac{w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau'} w_{i,\tau'} L_{i,\tau'} \pi_{i,\tau'}^{j,o}} \right) \hat{w}_{i,\tau} \hat{\pi}_{i,\tau}^{j,o} \quad (19)$$

where changes in the total gross output \hat{E}_i^j are derived by rewriting the final goods market clearing condition as

$$\hat{E}_i^j = \sum_{n=1}^N \frac{\lambda_{in}^j X_n^j}{\sum_{n'=1}^N \lambda_{in'}^j X_{n'}^j} \hat{\lambda}_{in}^j \hat{X}_n^j. \quad (20)$$

Change in the industry-level total expenditure is $\hat{X}_i^j = \hat{\lambda}_i^j \hat{I}_i$, where $\hat{I}_i = \frac{\sum_{\tau} w_{i,\tau} L_{i,\tau} \hat{w}_{i,\tau} + D_i'}{\sum_{\tau} w_{i,\tau} L_{i,\tau} + D_i'}$ is change in the total income in country i . I consider the aggregate trade deficit D_i as an exogenous variable which is fixed as a share of the world GDP, as in [Dekle et al. \(2008\)](#) and in [Caliendo and Parro \(2015\)](#).¹⁸ I normalize the sum of $p_i^{j,o}$ across all countries, industries, and occupations to remain constant between the base year and the counterfactual year.¹⁹

2.5 Model Mechanism

The model first captures the *labor demand channel*, which is the traditional channel by which trade shocks affect factor prices. A differential response is generated first across industries with industry-specific trade elasticities ν^j . Together with the differential pattern of the initial labor allocation, this industry-specific trade elasticity is the key parameter that captures differential impact of trade across workers.²⁰ The elasticity of substitution γ between occupations in production also carries weight in the labor demand channel, since demands for different occupations are interrelated. Despite the same industry-level trade shock, demands for different occupations may respond differentially. This channel engenders different gains

¹⁸Similarly to the equilibrium conditions in levels, two market clearing conditions imply the trade balance condition at the counterfactual equilibrium.

$$\sum_j \sum_{i=1}^N \lambda_{in}^j X_n^j - D_n' = \sum_j \sum_{i=1}^N \lambda_{ni}^j X_i^j.$$

¹⁹According to the Walras' law, one market clearing condition becomes redundant without additional normalization. Normalizing the sum of per-unit occupational prices is one way to add an independent equation to the system. Alternatively, we can assume that the world total output is kept constant as a normalization: $\sum_{i,j} E_i^j = \sum_{i,j} E_i^{j'} = E$. The main results of this paper are very robust to this alternative normalization.

²⁰[Ossa \(2015\)](#) points out that industry-specific trade elasticities magnify the aggregate welfare effect of trade. I focus on the relationship between industry-specific trade elasticities and the distributional effect of trade.

from trade depending on workers' initial occupation affiliation.

The second channel is the *labor supply channel* through which trade impacts the labor supply decisions of heterogeneous workers. This channel has not been widely studied in the literature. If workers of the same type are all homogeneous, $p_i^{j,o}$ entirely decides the labor allocation which should be same for all workers with the same type. In contrast, this model is based on workers' comparative advantage which generates a differential pattern of labor reallocation. The elasticity of industry- and occupation-level labor supply with respect to $p_i^{j,o}$ is $\theta_{i,\tau}(1 - \pi_{i,\tau}^{j,o})$. The parameter $\theta_{i,\tau}$ governs the responsiveness of type τ workers to changes in $p_i^{j,o}$. The self-selection of workers and compositional shift within worker types thus affect the distribution of gains from trade.

This model nests existing models by considering different values of $\theta_{i,\tau}$. In the extreme case when $\theta_{i,\tau} \rightarrow \infty$ and $T_{i,\tau}^{j,o} = 1$ for all (i, τ, j, o) , workers are homogeneous in their productivities within a type. If there is only one occupation, then this case collapses to the multi-industry EK model. If it is assumed that $\theta_{i,\tau} \rightarrow \infty$; $T_{i,\tau}^{j,o} = 1$ for all (i, τ, j, o) ; $\mu_i^{j,o} = \mu^{j,o}$ for all i ; and $z_i(e^j) = z$ for all i and e^j , then this model is equivalent to the multi-industry Heckscher-Ohlin model with CES production technology. In both multi-industry EK and multi-industry Heckscher-Ohlin cases, the labor demand side is a dominating factor determining industry-level labor reallocation. Another extreme case is where $\theta_{i,\tau}$ is equal to 1, and workers are extremely heterogeneous in their productivities. This case corresponds to the intuition of the specific factors model. Instead of assigning a specific value for the parameter $\theta_{i,\tau}$ *ex ante*, I estimate this parameter in the next section in order to take the model most closely to the data.

This model can also nest models that allow for worker heterogeneity only across industries; e.g., [Galle et al. \(2015\)](#). By assuming that there is only one occupation, thus $T_{i,\tau}^{j,o} = T_{i,\tau}^j$, workers of different types have different distributions for their industry-specific productivities, but each efficiency unit of labor that they supply is treated as equal within each industry. In the quantitative analysis, I compare this limiting case to the baseline case with both industry and occupation dimensions.

2.6 Aggregate and Type-level Welfare Effect

The model delivers both aggregate gains and type-level gains from trade. Given the same homothetic preference for workers, the proportional change in country i 's welfare is

$$\hat{W}_i = \hat{I}_i / \left[\sum_j \lambda_i^j \hat{A}_i^j (\hat{c}_i^j (\hat{\lambda}_{ii}^j)^{\frac{1}{\nu_j}})^{1-\eta_1} \right]^{\frac{1}{1-\eta_1}}, \quad (21)$$

where $\hat{\lambda}_{ii}^j$ is the change in domestic absorption, and \hat{I}_i is the change in total income, as previously derived. Once the model is solved for the counterfactual equilibrium $\hat{p}_i^{j,0}$, welfare changes are calculated accordingly.

This formula for welfare changes nests previous works with several simplifying restrictions to my model. If trade is balanced in all countries ($D_i = D_i' = 0$ for all i), and there is only one industry ($J = 1$), a single type of labor ($T = 1$) with a perfectly inelastic supply, and one occupation, then equation (21) exactly matches the welfare formula derived by [Arkolakis et al. \(2012\)](#) (ACR, hereafter): $\hat{W}_i = \hat{\lambda}_{ii}^{-\frac{1}{\nu}}$ for the EK model with a trade elasticity ν . If we consider a multi-industry EK model with ACR restrictions as well as the Cobb-Douglas structure across industries, but without the endogenous labor allocation, equation (21) collapses to $\hat{W}_i = \prod_j (\hat{\lambda}_{ii}^j)^{-\frac{\lambda^j}{\nu}}$, where λ^j is a Cobb-Douglas share of industry j .

As the main focus of my paper, I now derive the welfare effect for each worker type to discuss the distribution of trade-induced welfare changes. Assuming that each worker type shares the aggregate trade deficit based on the ratio of their total labor income, the change in type-level welfare is

$$\hat{W}_{i,\tau} = \hat{I}_{i,\tau} / \left[\sum_j \lambda_i^j \hat{A}_i^j (\hat{c}_i^j (\hat{\lambda}_{ii}^j)^{\frac{1}{\nu_j}})^{1-\eta_1} \right]^{\frac{1}{1-\eta_1}}, \quad (22)$$

where $\hat{I}_{i,\tau}$ is the counterfactual change of type-level income $I_{i,\tau} = w_{i,\tau} L_{i,\tau} + D_{i,\tau}$, and $D_{i,\tau}$ is type τ 's share of the aggregate trade deficit.²¹ The change in aggregate welfare (21) is then a simple weighted average of the change in type-level welfare (22), where the weight is type-level income share in the base year. Changes in between-type-inequality are discussed by comparing this type-level welfare change

²¹[Galle et al. \(2015\)](#) derive a similar formula for changes in type-level welfare. If I assume that there is only one occupation and that the preference follows a Cobb-Douglas, equation (22) matches their formula.

across worker types in counterfactual analyses. Between-type-inequality can be also measured by the skill premium. I define the skill premium by the wage premium of college graduates over non-college graduates, where its proportional change depends on equation (14).

2.7 Labor Reallocation and Average Wages of Industry and Occupation

The model shows the endogenous pattern of workers' sorting into industry and occupation. Based on the model-predicted $\hat{\pi}_{i,\tau}^{j,o}$ and the data on $\pi_{i,\tau}^{j,o}$, I calculate $\Delta\pi_{i,\tau}^{j,o} \equiv \pi_{i,\tau}^{j,o} - \hat{\pi}_{i,\tau}^{j,o}$ to capture the employment response within a type, since $\pi_{i,\tau}^{j,o}$ is defined as a share which is summed to 1 for each type. The employment shifts can be further aggregated up to the industry or the occupation level with the data on $L_{i,\tau}$ in order to quantify the aggregate patterns of labor reallocation induced by trade across industries and occupations, respectively. Industry- and occupation-level employment shifts can be compared to the macro data in order to quantify how much of actual industry- and occupation-level employment shifts can be explained by trade shocks of interest.

In addition, this model also predicts changes in industry- and occupation-level average real wages after taking compositional shifts into account. Those changes are defined by \hat{w}_i^j / \hat{P}_i and \hat{w}_i^o / \hat{P}_i , respectively, where

$$\hat{w}_i^j = \left[\sum_{\tau,o} \left(\frac{w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau',o'} w_{i,\tau'} L_{i,\tau'} \pi_{i,\tau'}^{j,o'}} \right) \hat{w}_{i,\tau} \hat{\pi}_{i,\tau}^{j,o} \right] / \left[\sum_{\tau,o} \left(\frac{L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau',o'} L_{i,\tau'} \pi_{i,\tau'}^{j,o'}} \right) \hat{\pi}_{i,\tau}^{j,o} \right] \quad (23)$$

$$\hat{w}_i^o = \left[\sum_{\tau,j} \left(\frac{w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau',j'} w_{i,\tau'} L_{i,\tau'} \pi_{i,\tau'}^{j',o}} \right) \hat{w}_{i,\tau} \hat{\pi}_{i,\tau}^{j,o} \right] / \left[\sum_{\tau,j} \left(\frac{L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau',j'} L_{i,\tau'} \pi_{i,\tau'}^{j',o}} \right) \hat{\pi}_{i,\tau}^{j,o} \right]. \quad (24)$$

These results are structural counterparts to the trade-induced change in the industry and the occupation wage premia studied in many reduced-form analyses.

3 Quantitative Analysis

In this section, I discuss the data, the estimation of parameters, the calibration of changes in bilateral trade costs, and the algorithm to solve the model. I quantify the distributional effects of changes in the trade environment between 2000 and 2007, with 2000 as the base year. From an international trade perspective, this time period is interesting, especially because China joined the WTO in 2001.

3.1 Data

I consider $N = 33$ countries which consist of 32 countries and a constructed rest of the world. These 32 countries account for 76.19% of the world total trade volumes in 2000. I also consider $T = 5$ worker types, $J = 4$ industries, and $O = 5$ occupations. Worker types are defined by educational attainment: high school dropouts (HD), high school graduates (HG), workers with some college education (SC), college graduates (CG), and workers with advanced degrees (AD).²² I assume that there are 4 industries: agriculture (AGR), mining (MIN), manufacturing (MFG), and service (SVC). Table 1 gives the occupation categories defined by aggregating the occupation classification by Dorn (2009) and the International Standard Classification of Occupations (ISCO) classification. The five categories are based both on the level of required skills and on the routineness of the occupational task, as used in Autor and Dorn (2013).²³ More details are described in the Appendix.

Table 1: List of Occupation Categories

-
1. Low-skill Occupations (LSO)
 2. Assemblers and Machine Operators (AMO)
 3. Precision Production and Crafts Occupations (PPC)
 4. Administrative, Clerical, and Sales Occupations (ACS)
 5. Managers, Professionals, and Technicians (MPT)
-

²²The definition of educational attainment varies by household survey in different countries. As summarized in the Appendix, I make the definition consistent within each country.

²³In his most aggregate categorization, Dorn (2009) distinguishes between ‘transportation, construction, and agricultural occupations’ and ‘low-skill service occupations’ for the U.S. However, the ISCO codes include agricultural laborers in low-skill (elementary) occupations. I thus aggregate all agricultural occupations and low-skill service occupations into low-skill occupations.

The Integrated Public Use Microdata Series (IPUMS) International database provides labor market information from household survey for the 22 countries in the sample for 2000. For the remaining countries, I proxy their labor market allocation with the lagged survey data or the data from other countries with a similar income level and adjust them with the data from ILOSTAT and LABORSTA.²⁴ As described in Figure A1, the household-level survey data show that patterns of labor allocation vary significantly by worker type and country. While many existing works in the literature focus only on the industry dimension, the data show that it is also important to consider occupations to explain the full scope of the distributional effect of trade. In fact the industry-level pattern of labor allocation does not vary much by worker type. By contrast, different worker types show very different patterns of occupation-level labor allocation, which suggests that workers' skills have higher complementarity with occupation-specific tasks than with industry-specific tasks. As it can be seen from equation (14), differential patterns of initial labor allocation across occupations are important for relative gains from trade between worker types.

I obtain bilateral trade flows for agriculture, mining, and manufacturing industries from the UN Commodity Trade (COMTRADE) database. In addition, the Trade in Services Database of the World Bank provides bilateral trade flows in the service industry. Aggregate variables are obtained from various sources: UN Statistical Division (UNSD) national accounts, OECD Structural Analysis (STAN), World Input-Output Database (WIOD), KLEMS, ILOSTAT and LABORSTA from the International Labor Organization (ILO), and the Occupational Wage around the World (OWW).²⁵ Detailed descriptions can be found in the Appendix.

3.2 Parameters

The model parameters are either estimated, calibrated to the base year, or based on previous work. The key parameter, the labor supply elasticity $\theta_{i,\tau}$, is estimated using data from base year 2000. The occupation intensity parameter $\mu_i^{j,0}$ is calibrated to match the share of occupation within each industry in the base year.

²⁴I also use the Barro and Lee (2013) dataset to supplement the information on the labor supply with workers' educational type. Detailed strategy is summarized in the Appendix.

²⁵The basic methodology used to obtain the input-output table in the WIOD is summarized by Timmer (2012). The OWW database are made publicly available by Oostendorp (2012).

Estimation of labor supply elasticity $\theta_{i,\tau}$ For notational simplicity, I denote $\bar{T}_{i,\tau}^{j,o} \equiv T_{i,\tau}^{j,o} (p_i^{j,o})^{\theta_{i,\tau}}$ for the estimation of parameters. The Fréchet scale parameter $\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'}$ and the shape parameter $\theta_{i,\tau}$ of the distribution of the equilibrium wage in (4) are jointly estimated using the maximum likelihood (ML) method.²⁶ Denoting individual worker ω 's equilibrium wage by w_ω conditional on the choice of (j, o) , then the log-likelihood function for worker type τ in country i is:

$$\ln \mathcal{L}(\theta_{i,\tau}, \sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} | w_1, \dots, w_L) = L(\ln \theta_{i,\tau} + \ln(\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'})) - (\theta_{i,\tau} + 1) \sum_{\omega=1}^L \ln w_\omega - (\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'}) \sum_{\omega=1}^L w_\omega^{-\theta_{i,\tau}},$$

where L is the number of workers in the sample out of the total $L_{i,\tau}$ workers with type τ in country i . The baseline estimation is done for countries with available individual wage profiles for the base year: Brazil, India, Mexico, and the U.S.

Table A1 summarizes the estimation result. The ML estimates of $\theta_{i,\tau}$ vary from 1.48 to 1.97 for the U.S., and better-educated workers have smaller estimates.²⁷ The result implies that better-educated workers are more dispersed in their productivities and wages within the type, which is consistent with the evidence in wage data. The result also shows that less skilled workers have a larger labor supply elasticity generating differential impacts of trade across worker types.

In addition, the estimated $\theta_{i,\tau}$ is larger in the U.S. on average. This result is consistent with the existing research pointing out the role of labor market rigidity in developing countries: e.g., [Goldberg and Pavcnik \(2003; 2005\)](#) and [Topalova \(2007\)](#). The baseline counterfactual result in the next section is derived with the actual estimates of $\theta_{i,\tau}$ for the U.S., Brazil, India, and Mexico. For the other OECD (non-OECD) countries, the average of the estimates for the U.S. and Mexico (Brazil and India) is used, respectively.²⁸ As shown in Figure A2, the predicted wage distribution fits the distribution of the actual wage data very well.

Other parameters Type-level labor supply $L_{i,\tau}$ and occupation intensity $\mu_i^{j,o}$ are

²⁶This method assumes that there is no correlation between idiosyncratic productivity draws. With correlation allowed, a further normalization is required to identify the scale parameter.

²⁷Using GMM, I get larger estimates of $\theta_{i,\tau}$ with an average of approximately 2.5 for the U.S. Compared to recent works by [Lagakos and Waugh \(2013\)](#), [Hsieh et al. \(2013\)](#), and [Burstein et al. \(2015\)](#), I get similar or slightly lower estimates. This is related to the definition of worker types and the independence assumption across productivity draws.

²⁸For the other countries where the wage data are available in different years from the base year, I estimate this parameter for available years, and the main counterfactual result is very robust.

calibrated to match the 2000 data. The trade elasticity ν^j is taken from the estimates in [Caliendo and Parro \(2015\)](#) for the agriculture, mining, and manufacturing industries (9.59, 14.83, and 5.5, respectively).²⁹ I use [Eaton and Kortum \(2002\)](#)'s main estimate, 8.28, for the service industry. The elasticity of substitution across occupations in production γ is set to 0.90 from [Goos et al. \(2014\)](#), which allows complementarity between occupations. The elasticity of substitution η_1 across industries in preference is set to 0.75 following [Comin et al. \(2015\)](#).³⁰ Results with different values of ν^j , γ and η_1 are discussed in the robustness section.

3.3 Measuring Trade Shocks

I examine the effect of two exogenous shocks: changes in bilateral trade costs (\hat{d}_{in}^j) and changes in productivity in China (\hat{A}_{CHN}^j) between 2000 and 2007. First, I calibrate changes in bilateral trade costs to match changes in industry-level bilateral trade flows in the data. Two standard assumptions are required for identification: 1) symmetry, i.e., $\hat{d}_{in}^j = \hat{d}_{ni}^j$ for all i and n , and 2) no domestic trade cost, i.e., $\hat{d}_{ii}^j = 1$ for all i and j . With these two identifying assumptions, I follow the [Head and Ries \(2001\)](#) approach to back out changes in trade costs from bilateral trade flow data – see also [Parro \(2013\)](#). The gravity equation from the model results in the following relationship between trade flows and trade costs:

$$\frac{\hat{\lambda}_{in}^j \hat{\lambda}_{ni}^j}{\hat{\lambda}_{ii}^j \hat{\lambda}_{nn}^j} = (\hat{d}_{in}^j)^{-2\nu^j}. \quad (25)$$

The change in trade costs \hat{d}_{in}^j is calibrated to exactly match equation (25) given ν^j from [Caliendo and Parro \(2015\)](#) and [Eaton and Kortum \(2002\)](#).

Table A2 and Figure A3 illustrate the results, showing bilateral trade costs decreasing most in the manufacturing industry between 2000 and 2007. Bilateral trade costs declined by 12.4% on average in the manufacturing industry, while calibrated declines are 7.24%, 6.75%, and 6.57% in the agriculture, mining, and service

²⁹[Caliendo and Parro \(2015\)](#) estimate the sector-level trade elasticities for 20 sectors including agriculture, mining, and 18 2-digit International Standard Industrial Classification (ISIC) manufacturing sectors. I take an average of their estimates across 18 manufacturing industries.

³⁰This value is the estimate when considering three industries (agriculture, manufacturing, and services) and trade controls in [Comin et al. \(2015\)](#). [Buera et al. \(2015\)](#) and [Cravino and Sotelo \(2016\)](#) consider a much lower elasticity of 0.2 between the two aggregate sectors of goods and services.

industry, respectively. Changes in trade costs depend on a partner country as well. For instance, trade costs with China have decreased by 23.5% on average across all industries, which is the largest decline among all countries in the sample as trade partners. In a relation to this observation, trade costs have fallen more with low-income trade partners. For example, manufacturing trade costs with OECD partners have declined by 15.35% on average, while they have fallen by 18.87% with non-OECD partners. This biased trade liberalization pattern between 2000 and 2007 is expected to have induced a major structural change in all countries engaged in international trade.

I then use the result in [Hsieh and Ossa \(2016\)](#) for changes in productivity in China. The baseline shock I use for counterfactual simulation is an 11.2% increase of A_{CHN}^j during the time period of interest, which is the median result of [Hsieh and Ossa \(2016\)](#).

3.4 Solving for the World Equilibrium

With the model in proportional changes, I only need to obtain the data on E_i^j , λ_i^j , λ_{in}^j , D_i^j , and $\zeta_i^{j,0}$ for the base year 2000. To take the model to the data, E_i^j is first measured by the value of gross output by industry and country. Bilateral trade flows X_{in}^j are then used to calculate the domestic absorption $X_{ii}^j = E_i^j - \sum_{n \neq i} X_{in}^j$, bilateral trade shares λ_{in}^j , and trade deficits D_i^j . After that, I compute the total expenditure $X_i^j = \sum_{n \neq i} X_{ni}^j + X_{ii}^j$ to construct the expenditure share λ_i^j . Lastly, $\zeta_i^{j,0}$ is measured by the share of hourly wage paid to a certain occupation relative to the hourly wage paid to all occupations in industry j .

The computation strategy to solve the model for the equilibrium $\hat{p}_i^{j,0}$ is based on [Caliendo and Parro \(2015\)](#) and the step-wise method of [Alvarez and Lucas \(2007\)](#). I first guess the initial $\hat{p}_i^{j,0}$ and then solve for the change in the industry-level price \hat{P}_i^j . After that, I calculate corresponding equilibrium quantities derived in the model. The counterfactual equilibrium is $\hat{p}_i^{j,0}$ which eliminates excess demands of occupations for both base and counterfactual years. I repeat these steps with the updated initial guess of $\hat{p}_i^{j,0}$ until the system of equations (19) is satisfied. The technical details of the solution strategy are described in the Appendix.

4 Counterfactuals

The main advantage of this model is to be able to quantify the interplay of trade liberalization, inequality, and labor reallocation for a large number of countries. Another advantage of this model is the ability to easily test any specific counterfactual trade shocks. In this paper, I consider \hat{d}_{in}^j for $j = AGR, MIN, MFG$ and \hat{A}_{CHN}^j as two separate types of trade shocks.³¹ Parameters outside these two are assumed to be time-invariant.

The baseline counterfactual results are derived with the previously estimated $\theta_{i,\tau}$. Then, the importance of having a correct specification for the degree of worker heterogeneity is argued by comparing results with different values of $\theta_{i,\tau}$. Given $\hat{p}_i^{j,o}$ solved at the counterfactual equilibrium, corresponding equilibrium quantities of interest are derived: changes in aggregate welfare, type-level welfare, skill premium, within-type labor reallocation patterns, industry- and occupation-level real wages, as well as employment shares across industries and occupations.

4.1 Effect of Changes in Bilateral Trade Costs

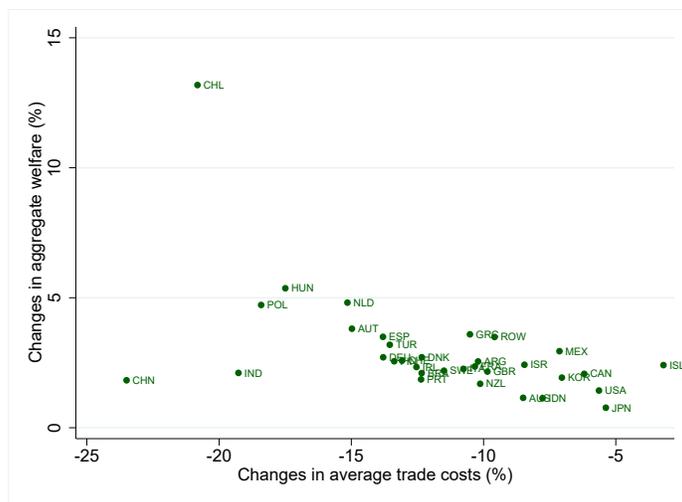
Calibrated changes of bilateral trade costs in three goods industries between 2000 and 2007 are first introduced holding other parameters fixed. The calibration shows that the decline of trade costs is twice as large in the manufacturing industry as in agriculture or mining industries. Thus, this exercise investigates the effect of an actual but biased trade liberalization especially toward manufacturing.

Figure 1 describes the counterfactual changes in aggregate welfare of each country against average declines of bilateral trade costs between 2000 and 2007 for each country. Numbers are provided in Table A3. Aggregate welfare increases in all countries in the sample, and countries with a larger decline of trade costs gain more on average.

Changes in between-type inequality The model predicts an unequal distribution of those aggregate welfare gains across worker types, which is the main focus

³¹Changes in bilateral trade costs in the service industry are also calibrated in the previous section, but for the counterfactual exercise, I focus only on the effects of trade liberalization in goods industries.

Figure 1: Counterfactual Changes in Aggregate Welfare from Changes in Trade Costs (%)



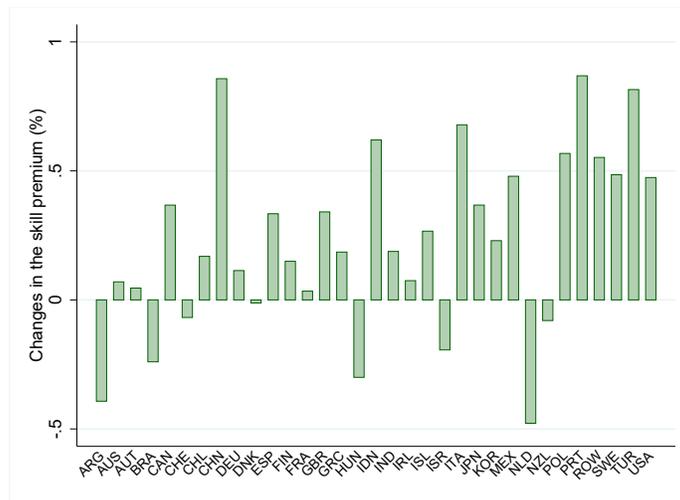
of this paper. Table A3 and Figure A4 show counterfactual changes in the type-level welfare for all countries in the sample. Between-type inequality measured by relative changes in the type-level welfare increases in most high-income countries due to changes in trade costs. Given that trade costs have declined on average during the time period of interest, this result is consistent with the Stolper-Samuelson prediction. Even though no worker group in any country loses from trade in absolute terms, better-educated workers tend to gain significantly more from trade in many developed countries.

On the other hand, the model predicts mixed results for relatively low-income countries. One of striking results is for manufacturing-oriented low-income countries such as China, India, and Indonesia who have become one of major exporters especially during the time period of interest. In those countries, trade liberalization increases between-type inequality, which is the opposite to what is expected by traditional trade theories. In Latin American countries such as Brazil and Argentina, between-type inequality decreases, which is in line with the empirical facts that those countries have experienced a decrease of inequality in recent 10-15 years. However, Figure A4 also shows that among workers with a at least high school education in those countries, better-educated workers gain more from trade.

A trade-induced change in between-type inequality is also captured by counterfactual changes in the skill premium. Figure 2 shows the similar pattern to

what was described based on counterfactual changes in type-level welfare in Figure A4. Counterfactual changes in the skill premium from the decline in trade costs explain 11.42% and 17.07% of the actual changes in the skill premium in the U.S. and in China, respectively.³² While the skill premium measure has been increasing sharply in most countries around the world in the late 1990s, it is well-documented that the pattern has become modest or even reversed in many countries after 2000.³³ This pattern is mostly from the increased skill supply in many countries in recent time periods. This model predicts that trade liberalization moves the skill premium toward the same direction as an increase of skill supply does in some countries such as Brazil and Argentina. In other countries such as Denmark, Italy, and Spain, on the other hand, trade liberalization alone increases the skill premium even though the increased skill supply depresses the overall skill premium in data during the same time period.

Figure 2: Counterfactual Changes in the Skill Premium from Changes in Trade Costs (%)



³²The skill premia in data are from the author’s calculation based on the U.S. ACS database for the U.S. and from Ge and Yang (2014) for China.

³³Even though the skill premium has surged in most European countries before 2000, some high-income European countries show decreasing skill premium after 2000 due to increased skill supply—e.g., Denmark, Italy, and Spain. Calculation is based on the EU-SILC database and the EU KLEMS. However, the model predicts that the decline in trade costs increases the skill premium in those countries, while the overall skill premium in data decreases mainly from increased skill supply.

Labor reallocation across industries and occupations The baseline model can also quantify the endogenous employment reallocation within worker types, which is another channel by which trade impacts domestic labor markets. This is equivalent to asking: “who goes where”? Figure A5 shows within-type labor reallocation for the U.S., China, and Brazil to represent high-income countries, rising countries with a comparative advantage in manufacturing, and low- or middle-income countries who used to have a comparative advantage in manufacturing before China rose. Figures show results only for two worker types, high school dropouts and college graduates, as a comparison.³⁴

The result first shows the importance of occupation-level labor reallocation. Since the labor reallocation pattern across industries is relatively similar between worker types, it captures only a limited trade effect on different patterns of worker reallocation. The main reason behind this result is because industry-level employment shift depends mostly on industry-level structural change which is not much related with the degree of complementarity between workers’ skills and industry-specific tasks. For example, as the manufacturing industry in the U.S. is hit by import competition from low-income countries following trade liberalization, both high school dropouts and college graduates tend to move out of the manufacturing industry and go to the service industry.

On the contrary, the occupation-level labor reallocation varies significantly by worker type. In high-income countries, changes in trade costs are more likely to force less-educated workers to switch from routine to low-skill occupations, while better-educated workers relocate into high-skilled occupations. For example in the U.S., even though both high school dropouts and college graduates tend to move from the manufacturing industry to the service industry, high school dropouts are likely to have low-skill occupations, while college graduates are likely to be managers in the same service industry. Second, in low-income and manufacturing-oriented countries such as China, while all worker types are likely to move to the manufacturing industry due to country-level comparative advantage, less-educated workers are likely to have lower-skilled occupations such as production jobs, while better educated workers tend to have high-skilled jobs. This differential reallocation pattern will further create discrepancy in gains from trade between

³⁴The full results for all worker types and for all other countries in the sample are also available upon request.

worker types.

Lastly, for Latin American countries who had a comparative advantage in manufacturing before 2000, e.g., Brazil and Argentina, China's surge now gives them a comparative disadvantage in manufacturing. As a result, agriculture and service industries relatively expand, with a larger expansion for the agriculture industry due to their comparative advantage over high-income countries. In response to this structural change, different worker types show different labor reallocation patterns: less-educated workers are better able to move into the agriculture industry due to their comparative advantage. Better-educated workers are more likely to head to the service industry instead and have managerial and professional jobs.

Aggregating the within-type labor reallocation up to industry and occupation levels, this model quantifies the macro implication of decline in trade costs on industry- or occupation-level employment shifts. Figures on the left column of Figure A6 compare the industry- and occupation-level employment shifts in the U.S., China, and Brazil caused by changes in trade costs as examples. Changes in trade costs significantly reduce the manufacturing employment and induce job polarization in the U.S. In recent years, this has been well-known in labor markets of many high-income countries.³⁵ Compared to the actual change in employment share in data, these numbers explain roughly 20% actual industry- or occupation-level employment shifts in the U.S. In low-income countries such as China, on the other hand, employment shifts mainly from the agriculture industry to the manufacturing industry. The polarized occupation-level reallocation patterns in high-income countries are exactly reversed. Since the rise of China puts middle-income countries such as Brazil in a comparative disadvantage in the manufacturing industry and gives a comparative advantage in the agriculture industry instead, employment moves toward the agriculture industry and low-skilled occupations in those countries.

Industry- and occupation-level wage effects Depending on countries' comparative advantage and occupation intensities, changes in trade costs first induce structural change across industries and occupations. This trade-induced change in labor demand then results in labor supply responses. These two forces in turn affect industry- and occupation-level wages through a compositional shift of labor

³⁵In this paper, job polarization is defined as a relative contraction of employment in middle-skilled occupations.

within industries or occupations. In other words, it matters who remains in a certain industry or with a certain occupation for counterfactual changes in industry- or occupation-level average wages in response to trade shocks.

Figures on the right column of Figure A6 show counterfactual changes in industry- and occupation-level average real wages for the U.S., China, and Brazil as examples. One pattern to note from the U.S. result is that an average real wage increases slightly more in the manufacturing industry than in the service industry, even though demand for U.S. manufacturing goods plummets with a decline in trade cost. Worker-level comparative advantage can explain this pattern and complement the argument provided by Autor et al. (2013) and Ebenstein et al. (2014). Following decreased labor demand in the manufacturing industry, only workers who are most productive in the manufacturing industry stay. The selection based on workers' comparative advantage increases average real wages in the manufacturing industry despite the negative manufacturing labor demand shock from trade liberalization.

In China, a similar pattern to the U.S. emerges at the occupation level, and selection also plays a role for this result. While a reduction of trade costs leads to an increased demand for middle-skill occupations in China, less-skilled workers tend to take those jobs, while better-educated workers tend to have higher-skilled occupations. This compositional change increases real wages most for managers and professionals. In Brazil, the pattern is the exact opposite compared to the China, since China and Brazil—and two sets of countries represented by each—interchange their comparative advantages between agriculture and manufacturing during the time period of interest.

In summary, this model quantifies various effects of trade liberalization on between-type inequality and labor market reallocation in many countries around the world. Country-level comparative advantage and worker-level comparative advantage interact with each other to affect domestic labor markets at the general equilibrium.

4.2 China Effects

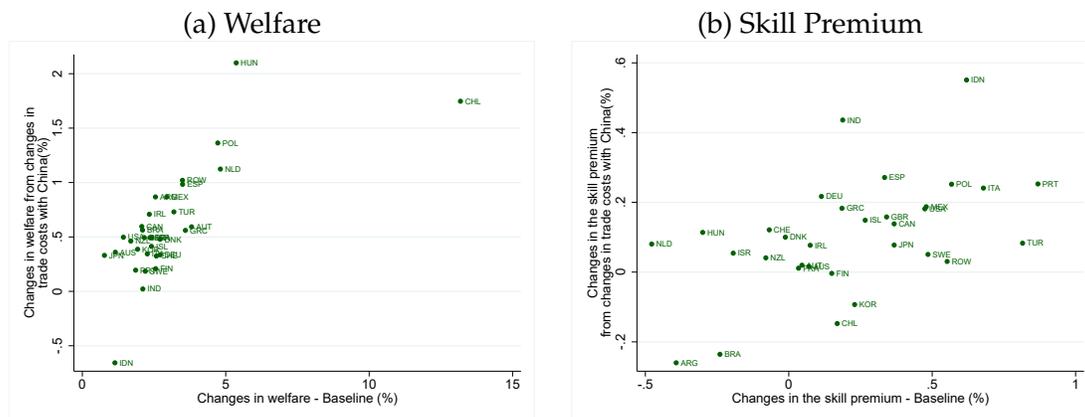
Based on a dramatic increase in trade flows to and from China since 2001, it is reasonable to expect that trade with China has significant effects on both aggregate

welfare and inequality in partner countries. The rise of China can be investigated in two ways. First, changes in bilateral trade costs with China matter. The importance of China is confirmed by the calibration result in Subsection 3.3. China's joining the WTO mainly affects this part. The model framework of this paper can decompose the effect of total changes in bilateral trade costs into the effect of China-involved trade and the effect of non-China-involved trade. Changes in trade costs with China, $\hat{d}_{i,CHN}^j$ and $\hat{d}_{CHN,i}^j$ for $j = AGR, MIN, MFG$, are introduced to the model as counterfactual shocks for this exercise. Figure 3 first shows that changes in trade costs with china account for about 22% of counterfactual changes in country-level aggregate welfare from the baseline trade liberalization \hat{d}_{in}^j for all i, n and for $j = AGR, MIN, MFG$.

When only changes in trade costs with China are introduced as counterfactual shocks, there are more countries in the sample that show an increase of the skill premium. Especially in high-income countries where the baseline shock \hat{d}_{in}^j for all country pairs decrease the skill premium such as Finland, Netherlands, New Zealand, and Switzerland, changes in trade costs only with China in fact increase the skill premium. This result suggests that the overall decrease of the skill premium is driven mainly by changes in trade costs with other trade partners than China in those four countries. For countries where the direction of changes in the skill premium is the same in both scenario, changes in trade costs with China account for about 61% of counterfactual changes in the skill premium calculated from the baseline case. Therefore, China has been a major player in the world market, shaping domestic inequality in many partner countries.

Another path to investigate the distributional effect of trade with China is to quantify the effect of changes in China's technology. In a simple trade model setup, Autor et al. (2013) show that import competition from China is connected to changes of productivity in China, which can be explained by the model of this paper. Changes in China's technology affect Chinese workers' productivity then impact China's cost advantage in the global market. As a result, within-industry trade flows with all China's trade partners change. Domestic labor demand is affected accordingly, with workers' reallocation across industries and occupations being the reaction from the labor supply side. The shock of interest is, therefore, $\hat{A}_{CHN}^j = 11.2\%$ taken from Hsieh and Ossa (2016). Figure A7 shows that aggregate

Figure 3: Effects of the Decline in Trade Costs with China - Changes in Welfare and the Skill Premium (%)



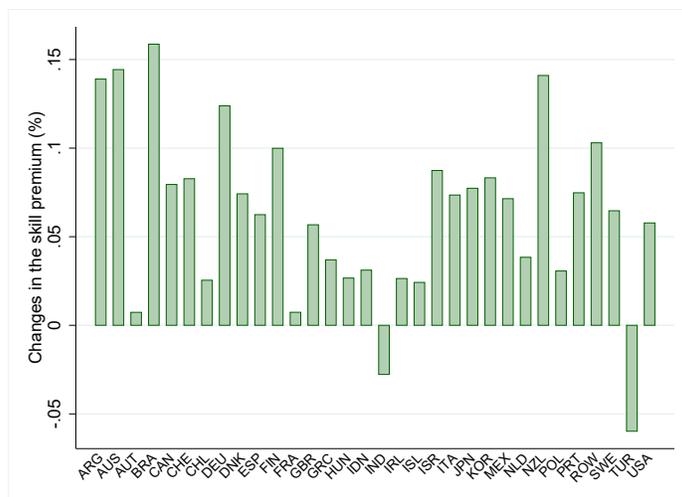
welfare increases due to the increase in China’s productivity in most countries in the sample. However, the result shows no significant relationship between welfare increases and changes in import shares from China. The magnitude is comparable to, but smaller than, welfare gains from decline in trade costs, which is mainly because \hat{A}_{CHN}^j indirectly affects prices of traded goods, while \hat{d}_{in}^j has a direct effect on them.

More importantly, welfare gains from an increase in China’s productivity are distributed unequally across workers. The pattern in Figure 4 is similar to the result from the trade cost experiment with $\hat{d}_{i,CHN}^j$: the skill premium increases in almost all countries in the sample. To isolate the effect of changes in own productivity, China is excluded from the figure. While overall changes in trade costs decrease between-type inequality in some countries in the sample, China-related shocks, especially if measured with increase in China’s productivity, tend to increase the skill premium in most countries. An increase in China’s productivity also induces employment shifts across industries and occupations and changes industry and occupation wage premia in partner countries with a similar pattern to what is predicted from the trade cost experiment.³⁶

In summary, this counterfactual exercise shows the importance of considering both productivity changes in partner countries and changes in trade costs

³⁶Detailed numbers for all countries in the sample are available upon request.

Figure 4: Counterfactual Changes in the Skill Premium from Changes in China's Productivity (%)



to provide a more precise prediction for the distributional effect of trade. While changes in China-involved trade costs are included in changes in all \hat{d}_{in}^j for $j = AGR, MIN, MFG$ that are considered in the first counterfactual exercise, changes in China's productivity are independent shocks. Combining effects of total changes in trade costs and effects of changes in China's productivity, for the U.S. as an example, counterfactual changes in the skill premium explain on average 12.83% of actual changes in the skill premium in data.

4.3 Limit Cases and Alternative Specifications

The model clearly nests two extreme cases in the literature: the specific factors model and the model with homogeneous workers. As discussed in Section 2.5, different values of $\theta_{i,\tau}$, which governs the degree of within-type worker heterogeneity, are expected to generate different patterns of welfare and distributional effects. Instead of pre-committing to a specific assumption regarding the degree of worker heterogeneity *ex ante*, I do counterfactuals with the actual estimates of $\theta_{i,\tau}$.

In this section, the earlier counterfactual scenario is reassessed with different values of $\theta_{i,\tau}$ to discuss the importance of endogenizing workers' sorting with a correct degree of worker heterogeneity. For this exercise, I consider only changes in trade costs, \hat{d}_{in}^j , as trade shocks, as in Subsection 4.1. The baseline result is

compared to the results with five alternative specifications of $\theta_{i,\tau}$. Case 0 assumes $\theta_{i,\tau} = 1$ for all i and τ for a case where workers are extremely heterogeneous in their productivities, which is in line with the specific factors model. Case 1 takes an average of the estimated $\theta_{i,\tau}$ across worker types and across countries. Case 2, 3, and 4 take larger values of $\theta_{i,\tau}$ than the estimates for all types and countries, 5, 10 and 50, respectively, in order to consider the cases with extremely homogeneous workers as in traditional trade models. As I consider larger values for $\theta_{i,\tau}$, the standard labor demand channel dominates in the distributional effect of trade.

Counterfactual changes in the skill premium vary significantly by value of $\theta_{i,\tau}$. As described in Figure A8, the trade effect becomes negligible as the specification moves closer to standard trade models with an unrealistically large $\theta_{i,\tau}$, which makes workers homogeneous within types. A cross-country variation of a trade-induced change in the skill premium also vanishes as we shift to the cases with homogeneous workers, which is inconsistent with empirical evidence. As it has been pointed out in many quantitative trade models in the literature, the Stolper-Samuelson effect is quantitatively very small when a model is taken to the data: e.g., Parro (2013). As $\theta_{i,\tau}$ becomes larger, the Stolper-Samuelson effect dominates for the effect of trade liberalization on the skill premium, so it becomes quantitatively negligible. These results support the importance of having precise country- and type-specific estimates of $\theta_{i,\tau}$, especially when the main focus is on the effect of trade shocks on inequality at a disaggregate level.

Another interesting limit case is the case when workers have the same degree of within-type comparative advantage as in the baseline case but choose only industries. This limit case can be formulated easily under this framework by assuming $O = 1$. This case is isomorphic to the model discussed by Galle et al. (2015), but with worker types based on education not based on regions as in that paper. With this limiting assumption, there are $(N \times J)$ endogenous variables, per-unit wage \hat{p}_i^j , and the same number of labor market clearing conditions. The same iterative algorithm can be applied to solve the model. If the same counterfactual shock \hat{d}_{in}^j for $j = AGR, MIN, MFG$ is introduced to this limit case with no occupation-level heterogeneity or occupational choices, counterfactual changes of the skill premium are on average 11% smaller in absolute terms compared to the baseline case with both industry- and occupation-level heterogeneity and endogenous choices. For example, the occupation channel alone explains 12% of the effect of changes in

trade costs on the skill premium in the U.S. The percentage is largest in Finland, where the occupation channel explains 26% of counterfactual changes in the skill premium derived for the baseline case.

5 Robustness Check

In this section, I repeat the same counterfactual experiment for the effect of changes of trade costs in non-service industries (\hat{d}_{in}^j for $j = AGR, MIN, MFG$) with different parameter values of the elasticity of substitution between occupations in production (γ), the elasticity of substitution across industries in preference (η_1), and the trade elasticity (ν^j).³⁷ These parameters are closely connected to the traditional labor demand channel by which trade impacts inequality. The main counterfactual results are robust across different values of labor demand channel parameters.

Elasticity of substitution between occupations in production In the baseline specification, I assign $\gamma = 0.90$ following [Goos et al. \(2014\)](#) to account for the complementarity between occupations in production. I consider alternative values of $\gamma = 0.1, 1, 3, \text{ and } 10$. [Figure A9](#) compares counterfactual changes in the skill premium with alternative values of γ to the result obtained from the baseline case with $\gamma = 0.90$. Changes in the skill premium show very similar patterns to the baseline result as γ takes different values. As it is evident from the figures, having different elasticities of occupation in production generates almost no relative effect across worker types nor level effects.

Elasticity of substitution between industries in preference The elasticity of substitution between product varieties (η_2) does not affect equilibrium outcomes in this model, except that $\nu^j + 1 > \eta_2$ is required for the price level to be well defined. On the contrary, the elasticity of substitution for the upper nest of the utility function across industries (η_1) does affect the equilibrium, since the industry expenditure shares λ_i^j changes endogenously. I use $\eta_1 = 0.75$ from [Comin et al. \(2015\)](#) for the baseline results. I consider alternative values of $\eta_1 = 0.2$ from [Buera et al. \(2015\)](#) and [Cravino and Sotelo \(2016\)](#), $\eta_1 = 1$ (Cobb-Douglas), and $\eta_1 = 1.5$ from [Backus et al. \(1994\)](#) and [Chari et al. \(2002\)](#).

³⁷Results of the sensitivity analysis for other counterfactual shocks are available upon request.

The main finding remains unchanged. Changes in trade costs increase between-type inequality with alternative values of η_1 in most high-income countries and countries with a comparative advantage in manufacturing such as China, while decreasing inequality in some Latin American countries such as Brazil and Argentina. However, Figure A10 shows that the case with a lower η_1 tend to predict larger increases in inequality. The intuition is that if goods from different industries are more substitutable, then the relative demand for the importing industry increases even more due to the reduction of trade costs. This in turn increases labor demand in those industries and for occupations that are intensive there, offsetting the negative effect on the labor in import-competing industries.

Trade elasticity There are a number of papers estimating trade elasticity using different estimation methods. The trade environment in this paper is most closely related to [Caliendo and Parro \(2015\)](#) who consider a multi-industry EK model with industry-specific trade elasticities. I thus derive baseline results with their industry-specific estimates of trade elasticities. I consider $\nu = 4$ ([Simonovska and Waugh \(2014\)](#)), 6.9 (intermediate value of [Head and Ries \(2001\)](#)'s estimates that [Anderson and van Wincoop \(2004\)](#) consider in their survey), and 8.28 (EK) as alternative values of trade elasticity and assume that these alternative values do not vary by industry.³⁸ Compared to the estimates of [Caliendo and Parro \(2015\)](#) which are used in the baseline counterfactual exercise, these alternative specifications assume relatively lower trade elasticities in agriculture and the mining industries.

The main finding about the distributional effect of trade liberalization is robust across different values of trade elasticities, as documented in Figure A11. However, a model with smaller trade elasticities predicts smaller changes in the skill premium on average. This result is due to smaller trade elasticities for the agriculture and mining industries compared to the baseline case. If trade flows are less elastic in those two industries which are relatively unskilled-labor intensive, unskilled workers are more insulated from negative labor demand shocks following trade liberalization. However, the overall prediction on inequality is not affected much due to their relatively small shares in the total economy.

³⁸There are many other existing papers that estimate trade elasticity with different methods. Most results find trade elasticities overall ranging from 4 to 20: e.g., [Simonovska and Waugh \(2014\)](#), [Broda and Weinstein \(2006\)](#), [Head and Mayer \(2014\)](#), [Bergstrand et al. \(2013\)](#), [Hertel et al. \(2007\)](#), and [Romalis \(2007\)](#).

6 Conclusion

In this paper, I present a general equilibrium trade model featuring worker heterogeneity and endogenous sorting of workers based on worker-level comparative advantage in order to explore the distributional effect of trade in many countries. The model shows the mechanism by which trade affects between-educational-type inequality and patterns of endogenous labor reallocation across industries and occupations.

I quantify the model to examine the effect of changes in the trade environment on those labor market outcomes between 2000 and 2007, where these changes are captured by the reduction of bilateral trade costs, as well as the increase in China's productivity. In order to take the model to the data, I use the household-level survey data for a large number of countries that encompass detailed labor market information. The quantitative result shows that trade shocks lead to increases in between-educational-type inequality between 2000 and 2007 in most high-income countries and low-income countries with a comparative advantage in the manufacturing industry such as China, India, and Indonesia, while this pattern is reversed for some Latin American countries such as Brazil and Argentina. The paper also shows that China effects are sizable. Changes in trade costs with China account for 61% of changes in the skill premium from changes in trade costs with all partner countries. Increase in China's productivity also has significant effects on between-educational-type inequality in all partner countries.

I also quantify patterns of trade-induced labor reallocation across industries and occupations. Workers' self-selection into industries and occupations based on their comparative advantage is important, especially for the occupational dimension. The model also shows that international trade can help explain many stylized facts in labor markets, such as changes in industry and occupation wage premia, employment shifts across industries, and job polarization in high-income countries.

The general, but still tractable model of this paper can easily nest many existing trade models, depending on the labor supply elasticity parameter. Instead of pre-committing to a specific model framework, I estimate the labor supply elasticity, which brings the model most closely to the data. Comparing the distributional effect of trade predicted in this paper to the predictions of existing models

shows the importance of correctly introducing worker-level comparative advantage. Since this paper allows one to test any trade shocks consisting of changes in trade costs or changes in partner countries' productivity, it provides a new tool for future research to investigate the distributional effect of various trade shocks for a large number of countries.

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A Tables and Figures

Table A1: Maximum Likelihood Estimates of $\theta_{i,\tau}$

Worker type	High school Dropouts	High school Graduates	Some College Education	College Graduates	Advanced Degrees
U.S. (2000)	1.97 (0.033)	1.86 (0.042)	1.74 (0.043)	1.61 (0.049)	1.48 (0.055)
Brazil (2000)	1.09 (0.093)	1.24 (0.129)	1.17 (0.158)	1.05 (0.231)	1.04 (0.362)
India (1999)	1.26 (0.172)	1.09 (0.180)	1.00 (0.259)	1.03 (0.346)	1.05 (0.335)
Mexico (2000)	1.18 (0.225)	1.28 (0.372)	1.23 (0.409)	1.19 (0.510)	1.19 (0.545)

Notes: For the U.S., N=10,000 for each type. For the other three countries, N=5000 for each type. Standard errors are displayed in parentheses.

Table A2: Summary of Calibrated Changes in Bilateral Trade Costs (%)

	All industries	Agriculture	Mining	Manufacturing	Service
Argentina	-10.21	-7.86	-2.26	-12.53	-6.15
Australia	-8.50	-2.60	-6.84	-9.50	-8.28
Austria	-14.98	-9.88	-18.89	-19.53	-7.65
Brazil	-12.34	-6.63	-8.63	-13.47	-11.83
Canada	-6.19	-7.18	-7.19	-5.84	-7.00
China	-23.50	-11.32	-9.69	-26.20	-15.14
Chile	-20.82	-8.87	-11.24	-25.76	-11.96
Denmark	-12.34	-8.18	0.75	-17.54	-8.17
Finland	-13.38	-7.04	-4.21	-15.86	-9.11
France	-10.33	-6.75	-6.23	-13.32	-4.34
Germany	-13.80	-8.08	-8.77	-16.35	-9.26
Greece	-10.52	-7.21	-1.80	-17.47	-6.05
Hungary	-17.50	-14.10	-17.08	-20.29	-9.56
Iceland	-3.20	-9.68	-2.41	-14.09	8.50
India	-19.27	-12.63	-9.04	-23.73	-16.61
Indonesia	-7.77	-8.79	-6.72	-9.26	-5.24
Ireland	-12.54	-5.54	-8.87	-12.07	-13.37
Israel	-8.45	-5.10	-3.82	-9.63	-4.51
Italy	-10.76	-7.46	-7.86	-14.07	-4.44
Japan	-5.38	-3.46	-7.13	-6.62	-2.47
Republic of Korea	-7.04	-3.47	-1.67	-7.12	-8.88
Mexico	-7.13	-6.21	-5.85	-7.25	-7.04
Netherlands	-15.15	-9.17	-4.38	-23.83	-3.03
New Zealand	-10.13	-5.74	-7.07	-11.40	-8.30
Poland	-18.41	-15.21	-5.50	-25.54	-6.91
Portugal	-12.36	-10.57	-8.98	-14.25	-8.89
Spain	-13.80	-8.45	-6.40	-17.12	-9.79
Sweden	-11.50	-7.12	-3.06	-14.30	-7.27
Switzerland	-13.08	-4.29	-12.51	-15.39	-10.24
Turkey	-13.55	-8.03	-8.34	-21.25	-5.03
United Kingdom	-9.86	-5.73	-6.10	-11.09	-8.38
United States	-5.64	-5.93	-6.93	-6.79	-3.30
ROW	-9.58	-8.20	-6.76	-13.42	-6.95
Average	-9.95	-7.24	-6.75	-12.40	-6.57

Notes: Numbers are in %. Changes in trade costs are weighted by the volume of trade, when being aggregated up to industry, country, country group, and the world level.

Table A3: Counterfactual Changes in Aggregate and Type-level Welfare from Changes in Trade Costs(%)

	Aggregate Welfare	HD	HG	SC	CG	AD
Argentina	2.55	2.98	2.20	2.12	2.12	2.33
Australia	1.15	1.14	1.12	1.13	1.22	1.22
Austria	3.81	3.83	3.79	3.89	3.83	3.83
Brazil	2.10	2.38	1.73	1.81	1.88	1.97
Canada	2.07	1.69	1.97	2.08	2.36	2.36
China	1.80	1.45	1.92	2.53	2.62	2.62
Chile	13.15	12.97	13.19	13.27	13.38	13.09
Denmark	2.71	2.64	2.79	2.78	2.67	2.67
Finland	2.56	2.38	2.63	2.66	2.66	2.65
France	2.36	2.21	2.40	2.23	2.44	2.51
Germany	2.71	2.55	2.75	2.58	2.88	2.88
Greece	3.59	3.58	3.53	3.49	3.76	3.78
Hungary	5.36	5.84	5.37	5.66	5.06	5.06
Iceland	2.41	2.16	2.50	2.55	2.62	2.62
India	2.11	2.04	2.18	2.23	2.31	2.31
Indonesia	1.14	1.02	1.41	1.68	1.76	1.76
Ireland	2.33	2.08	2.40	2.42	2.37	2.37
Israel	2.42	2.76	2.34	2.26	2.33	2.35
Italy	2.27	1.76	2.36	2.60	2.95	2.95
Japan	0.77	0.37	0.72	0.80	1.06	1.05
Republic of Korea	1.93	1.56	1.92	1.99	2.13	2.12
Mexico	2.94	2.88	2.83	3.19	3.37	3.34
Netherlands	4.81	5.40	4.86	4.67	4.22	4.69
New Zealand	1.69	1.79	1.68	1.66	1.65	1.65
Poland	4.72	4.04	4.88	5.02	5.27	5.26
Portugal	1.86	1.44	2.30	2.51	2.69	2.78
Spain	3.50	3.44	3.38	3.72	3.73	3.86
Sweden	2.19	1.36	2.21	2.36	2.59	2.58
Switzerland	2.58	2.64	2.57	2.57	2.51	2.51
Turkey	3.19	2.90	3.45	3.67	3.99	3.99
United Kingdom	2.16	1.93	2.17	2.36	2.44	2.43
United States	1.43	1.15	1.15	1.49	1.62	1.81
ROW	3.49	3.31	3.56	3.87	4.01	4.04

Notes: Numbers are in %. Worker types are: high school dropouts (HD), high school graduates (HG), workers with some college education (SC), college graduates (CG), and workers with advanced degrees (AD).

Figure A1: Within-type Labor Allocation across Industries and Occupations in 2000 by Country Group

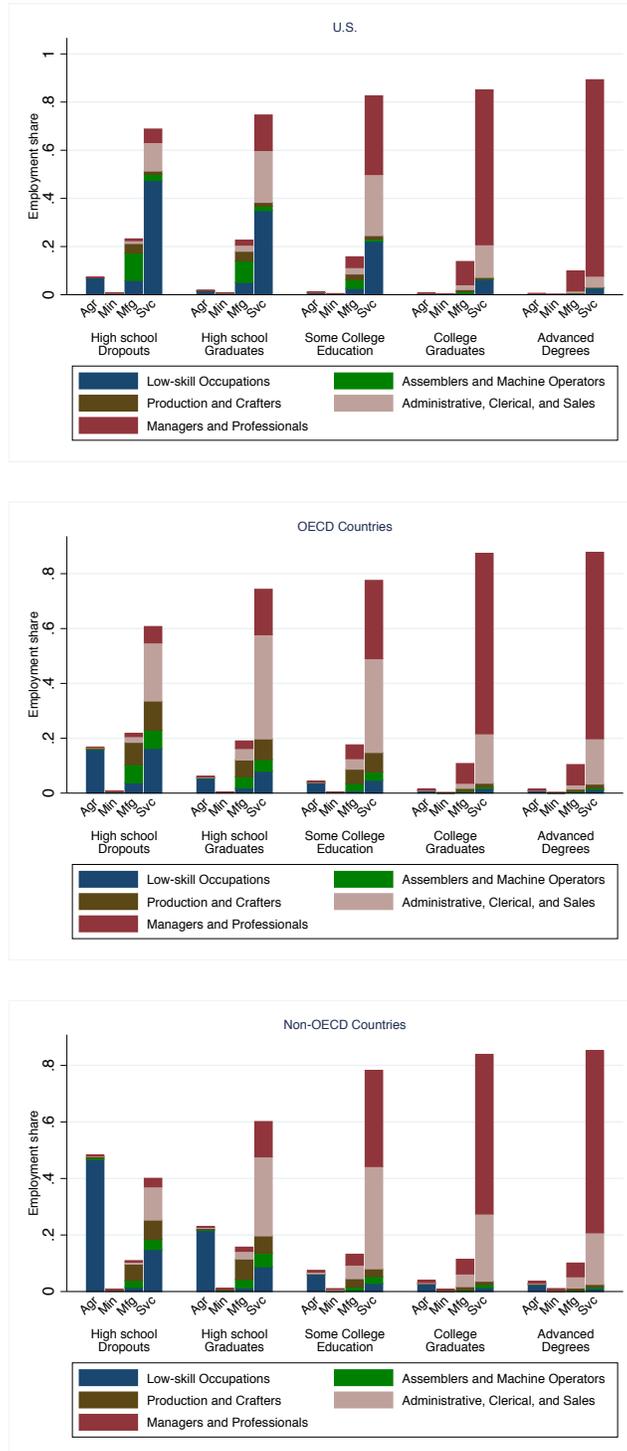
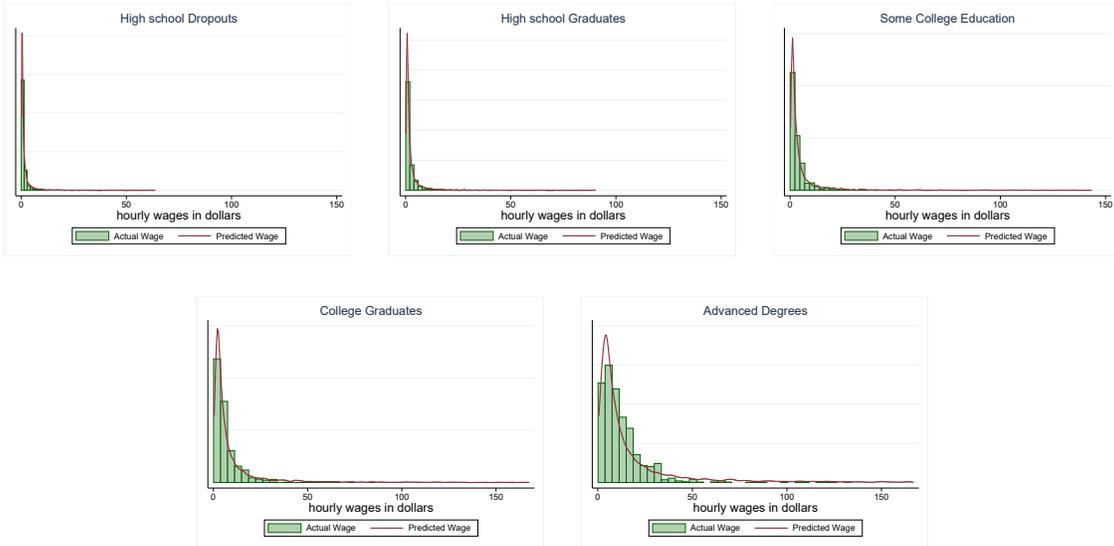


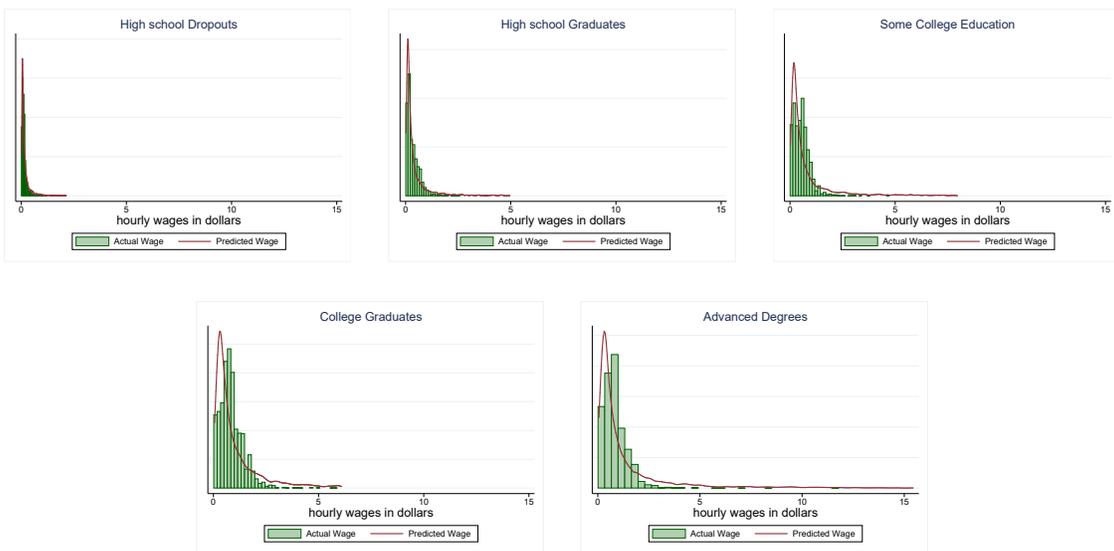
Figure A2: Model Fit with the ML Estimates of $\theta_{i,\tau}$ for Within-type Wage Distribution

(a) Brazil (2000)

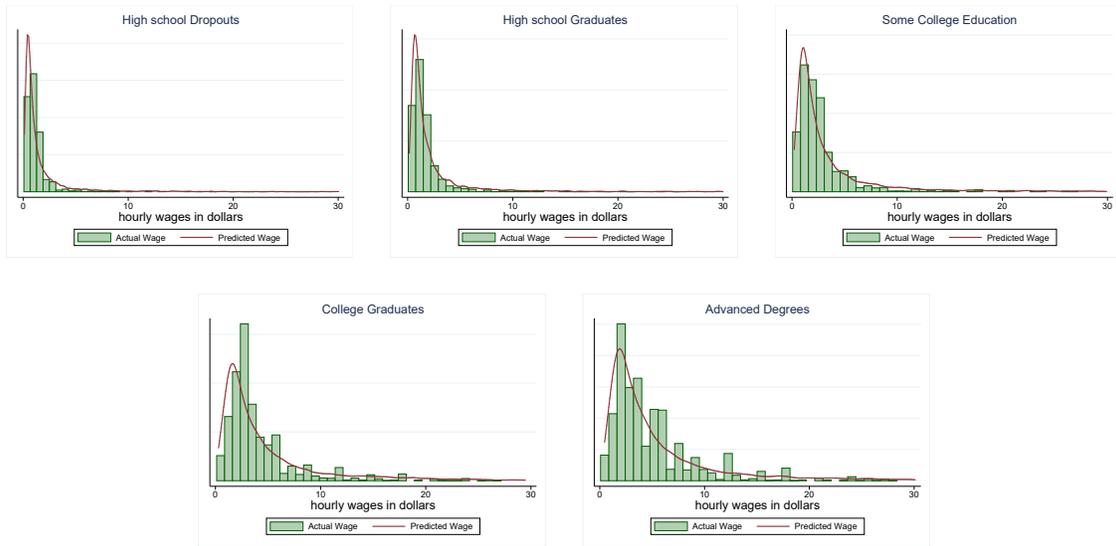


Note: The estimation for non-U.S. countries is in the local currency. Fits are drawn in terms of the U.S. dollar converted with the exchange rates of the corresponding year for expositional purposes.

(b) India (1999)



(c) Mexico (2000)



(d) U.S. (2000)

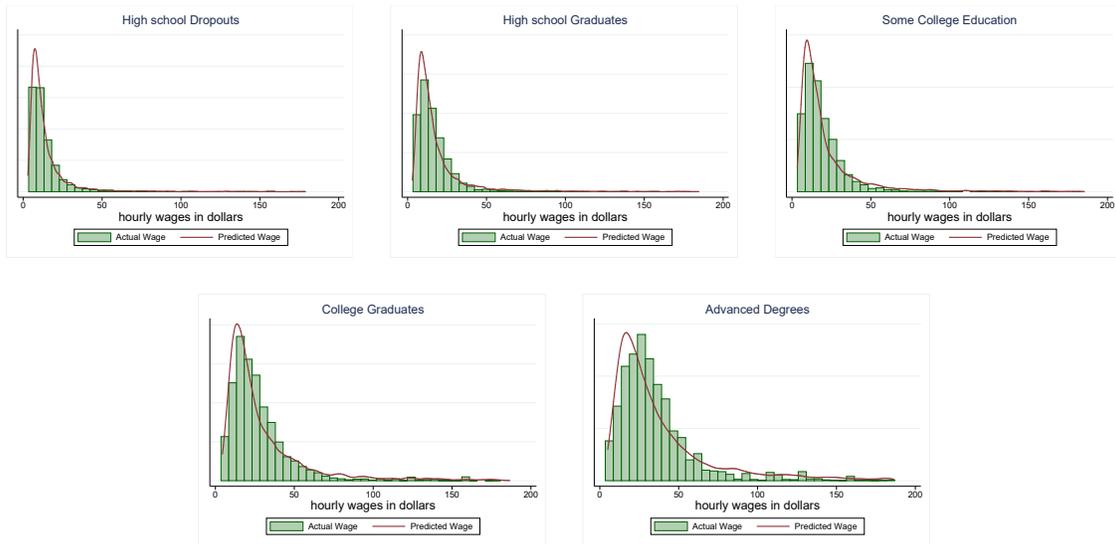
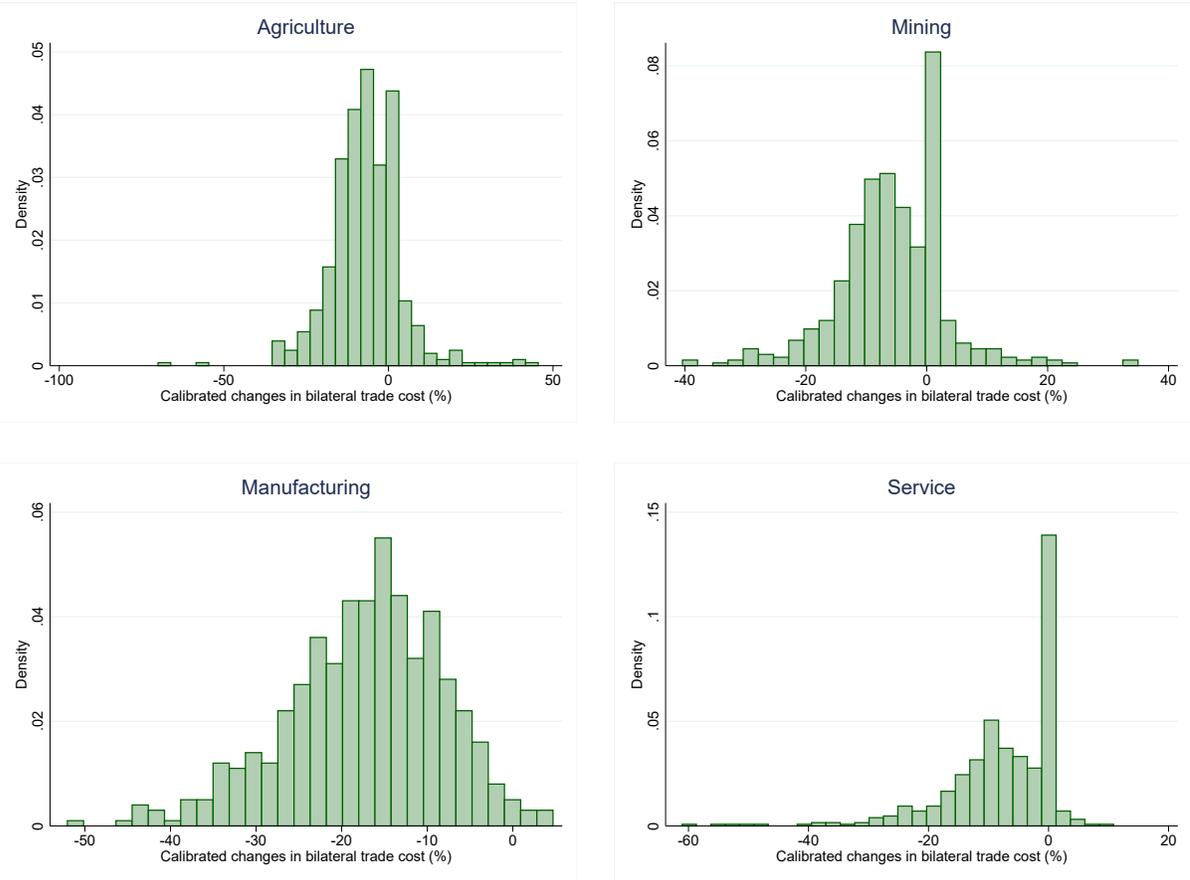
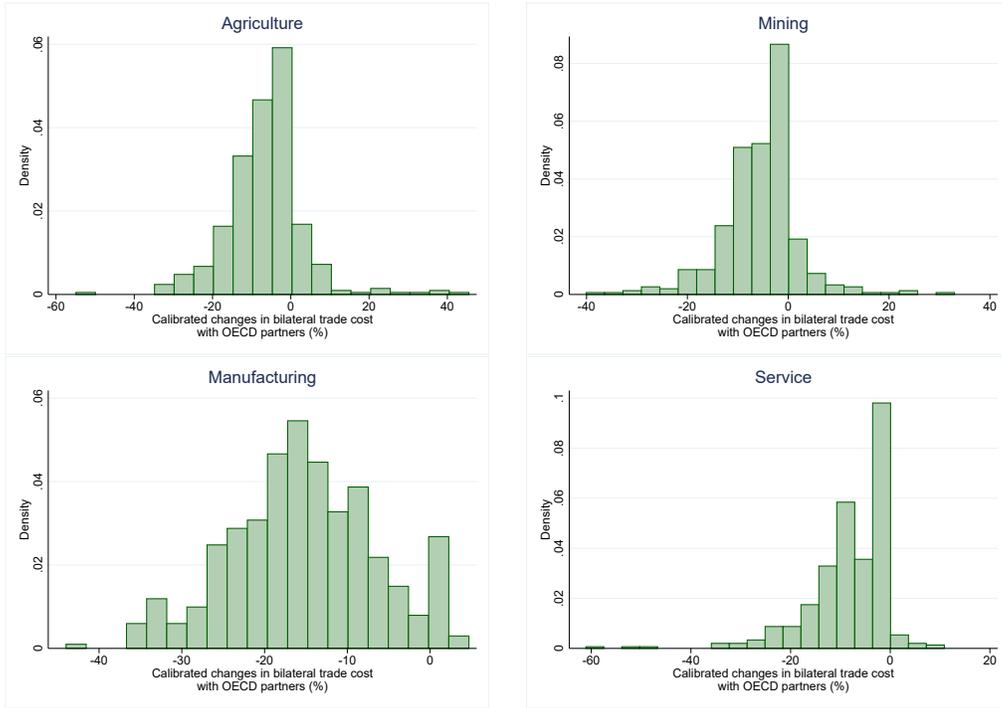


Figure A3: Calibrated Changes in Bilateral Trade Costs by Industry

(a) With All Trade Partners



(b) With OECD Trade Partners



(c) With non-OECD / Latin American Trade Partners

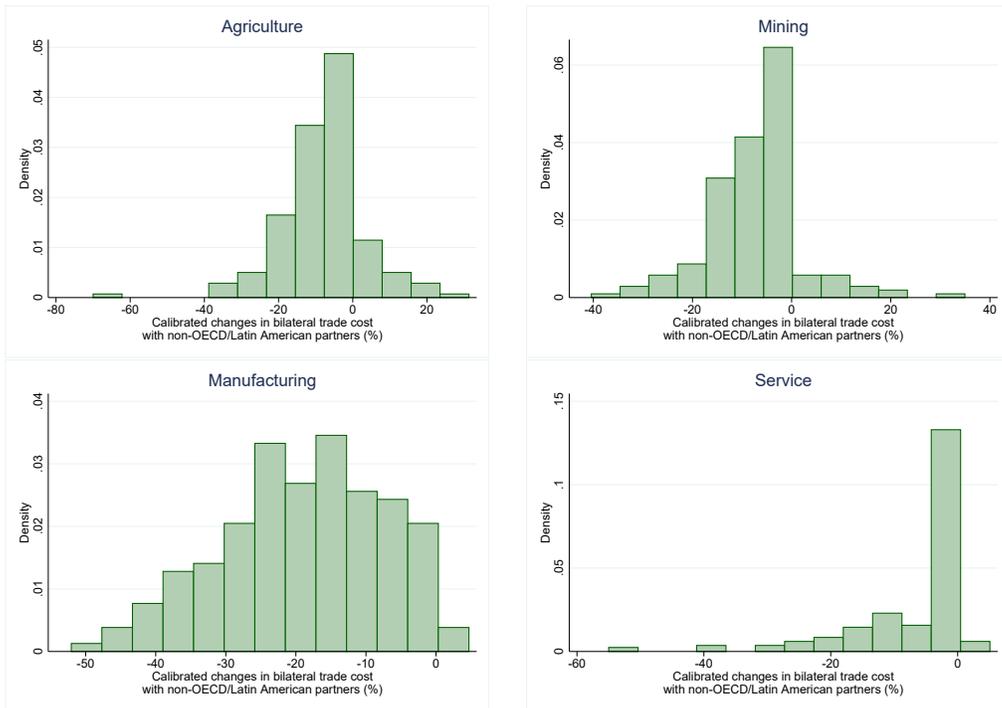
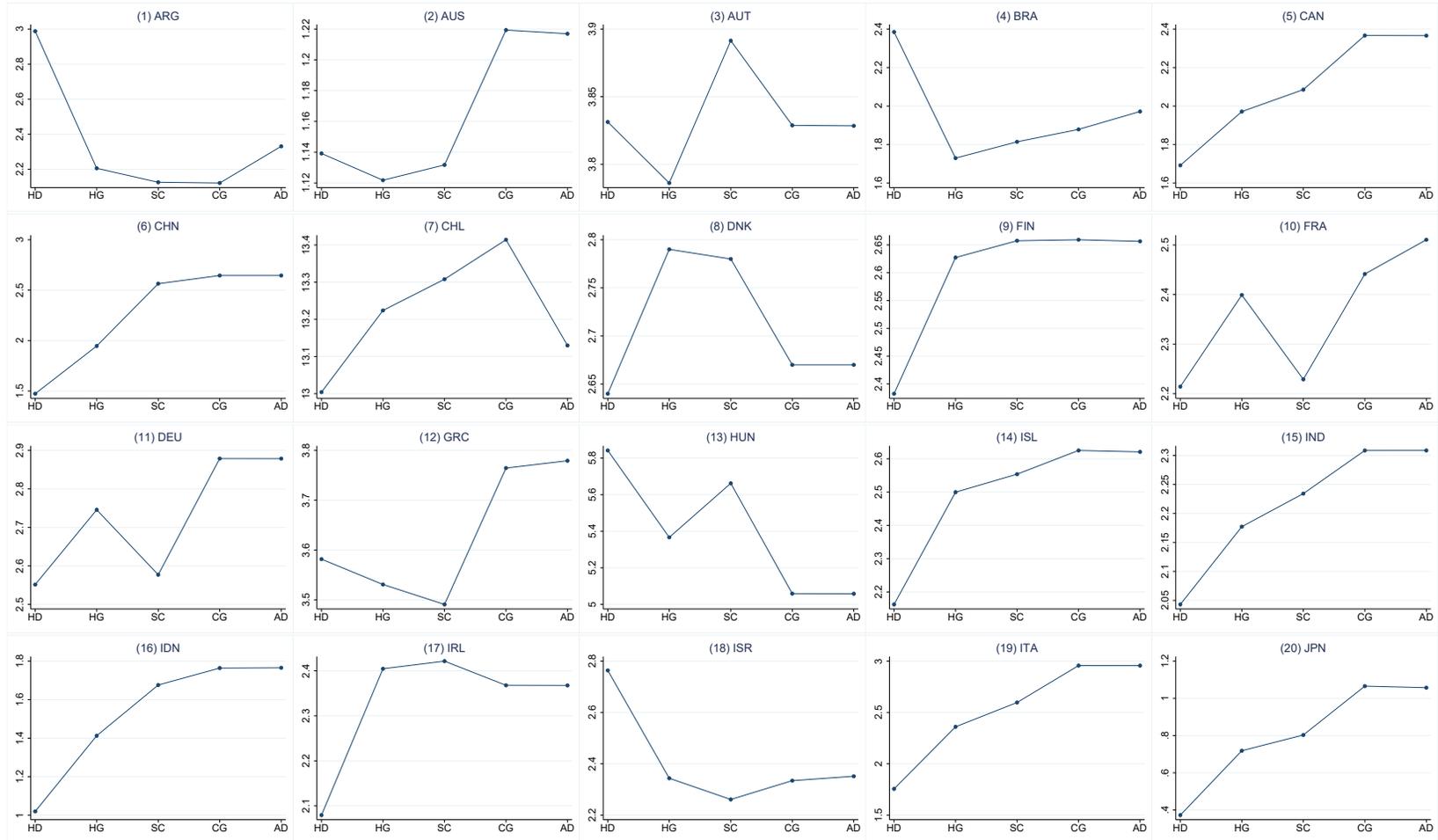


Figure A4: Changes in Type-level Welfare from Changes in Trade Costs (%)

Note: The x-axis describes five worker types as defined previously based on the educational attainment. The y-axis shows the percentage change in type-level welfare.



(Figure A4 continued)

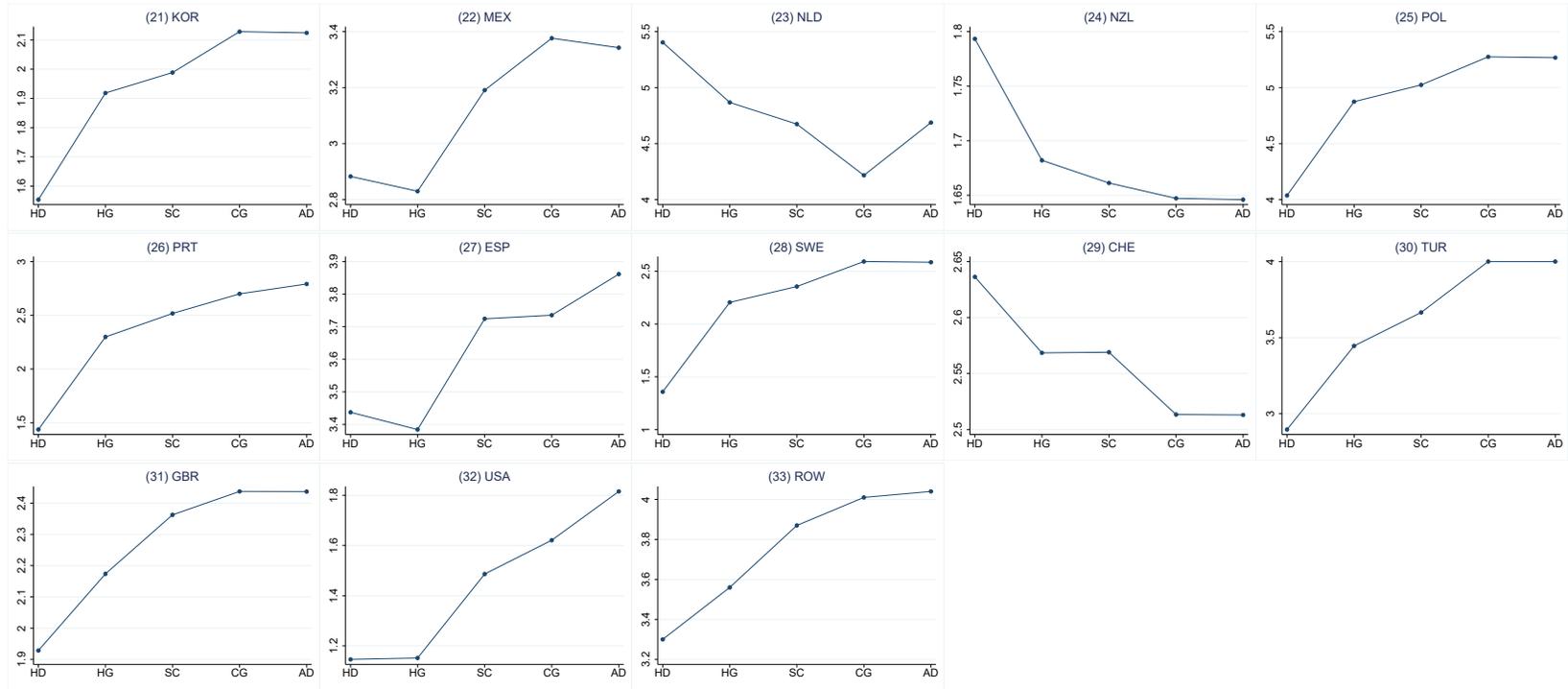
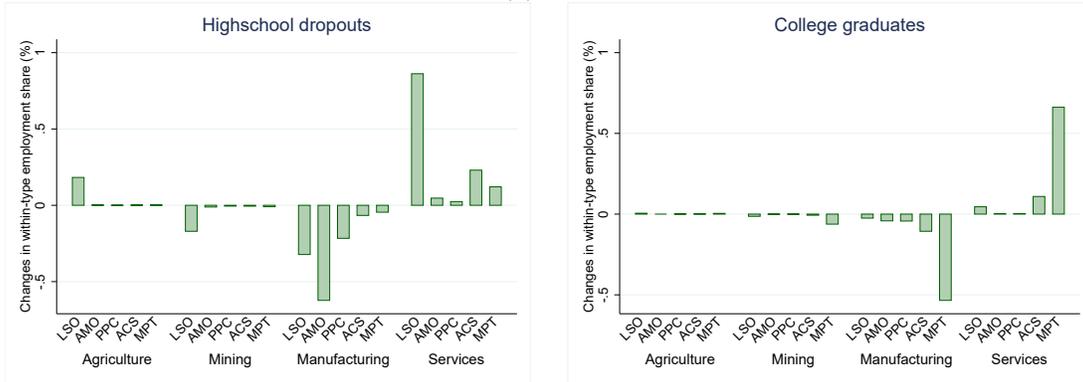
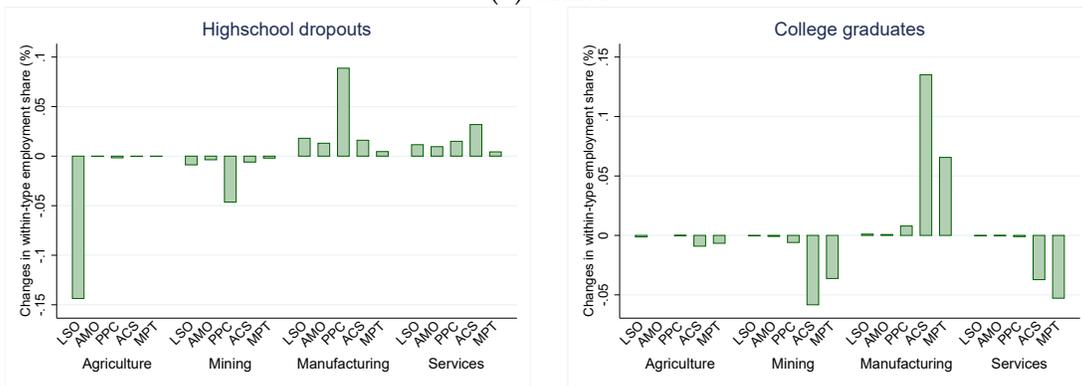


Figure A5: Changes in Within-type Employment Shares Resulting from Changes in Trade Costs (%)

(a) U.S.



(b) China



(c) Brazil

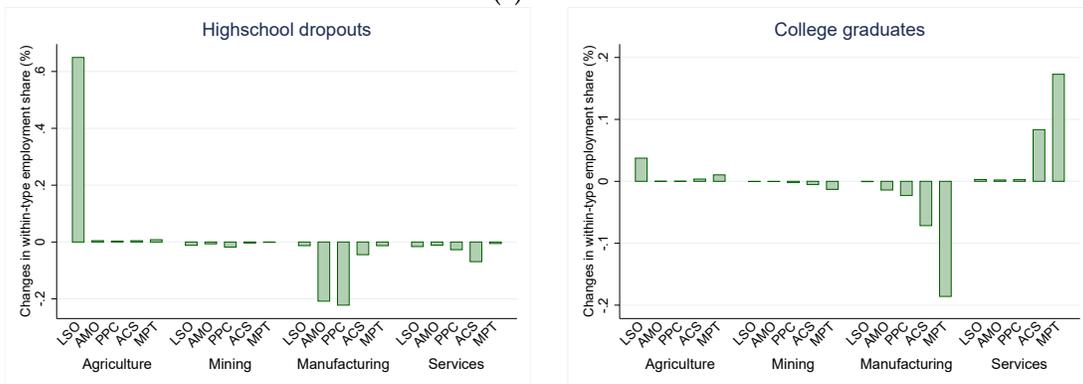
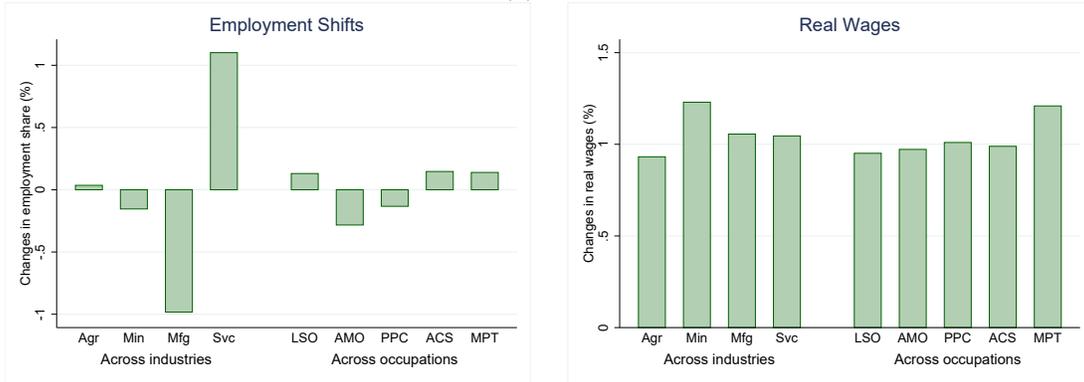
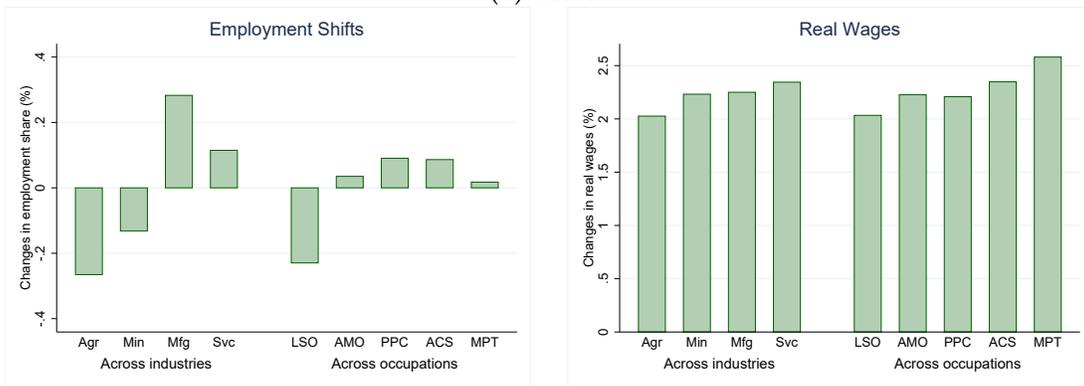


Figure A6: Changes in Employment Shares and Real Wages Resulting from Changes in Trade Costs (%)

(a) U.S.



(b) China



(c) Brazil

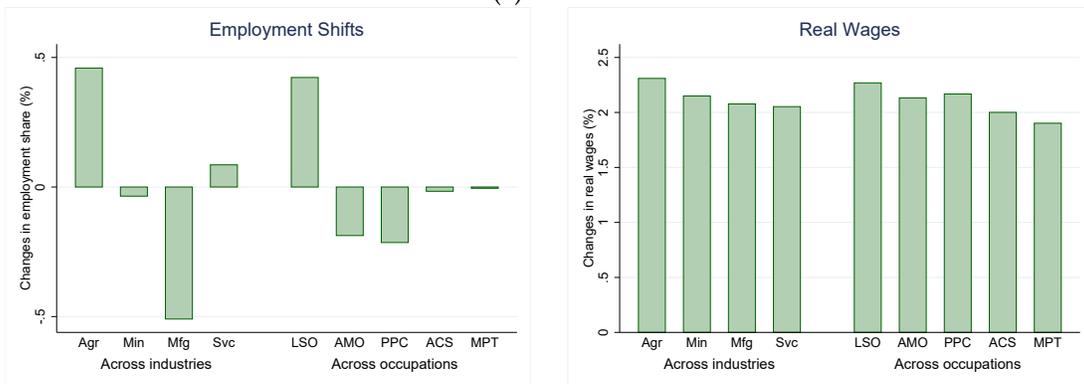
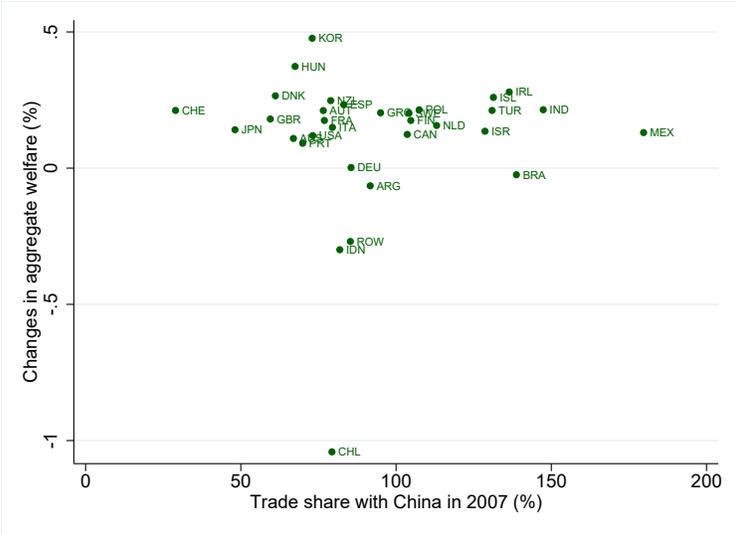


Figure A7: Counterfactual Changes in Welfare from Increases in China's Productivity (%)



Note: In order to not consider the effect of changes to its own productivity, this figure excludes China.

Figure A8: Counterfactual Changes in the Skill Premium from Changes in Trade Costs for Different $\theta_{i,\tau}$

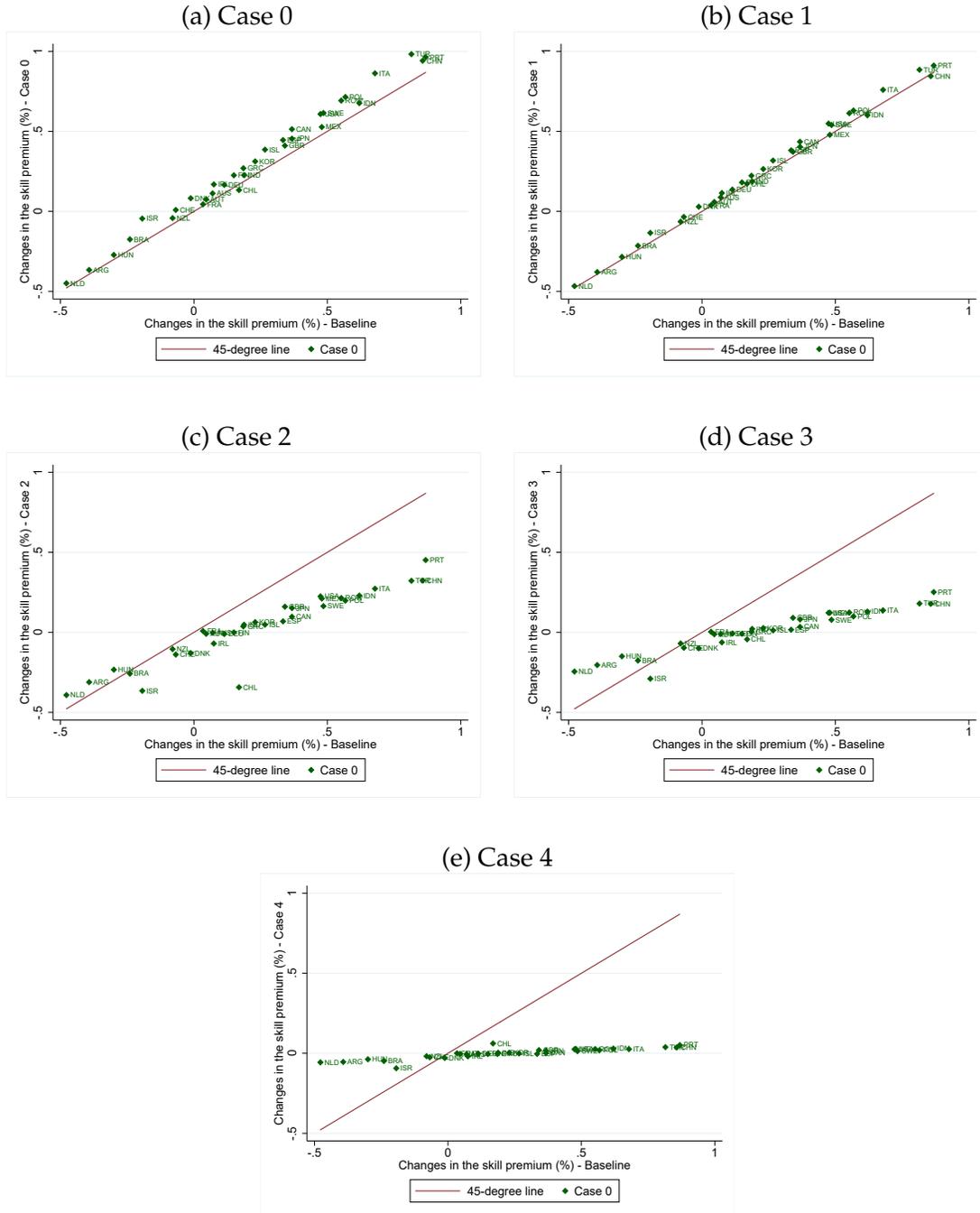


Figure A9: Changes in the Skill Premium for Different γ

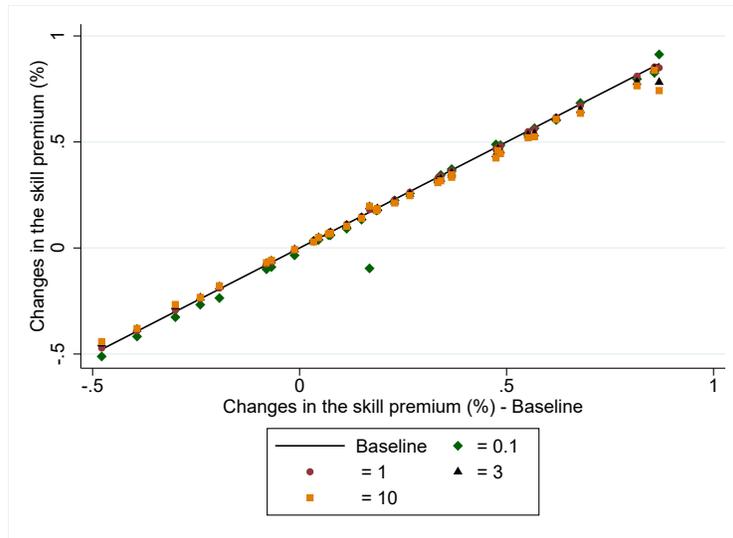


Figure A10: Changes in the Skill Premium for Different η_1

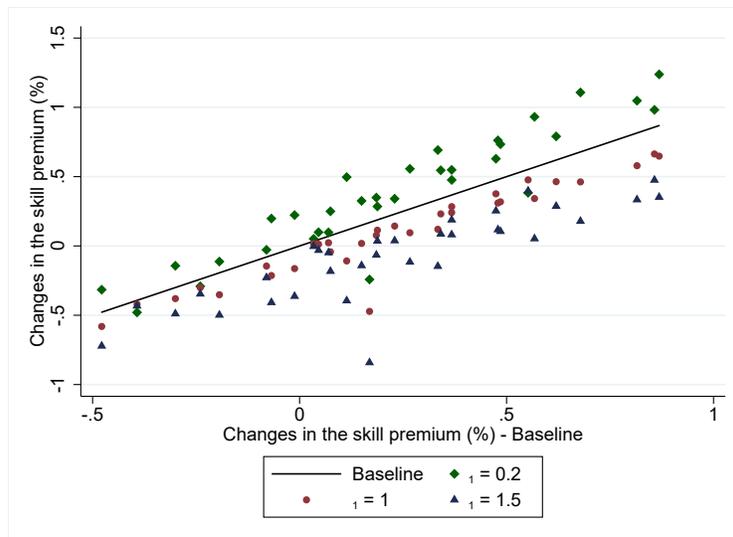
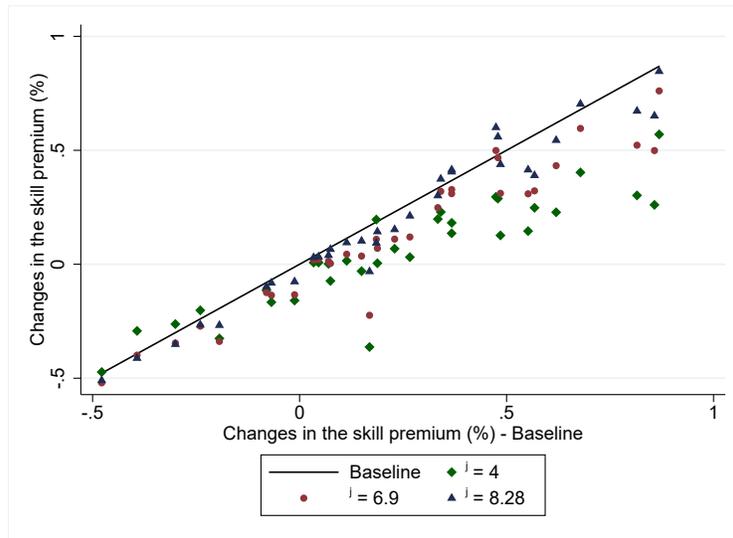


Figure A11: Changes in the Skill Premium for Different ν^j



B Derivations of Equations in the Model

B.1 Occupational Choice Problem

First, each worker ω in country i solves the following occupational choice problem

$$\max_{j,o} w_{i,\omega}^{j,o} = p_i^{j,o} \epsilon_{\omega}^{j,o},$$

where $p_i^{j,o}$ is a per-unit price for labor input and $\epsilon_{\omega}^{j,o}$ is an idiosyncratic productivity of worker ω for (j,o) . In the partial equilibrium analysis, $p_i^{j,o}$ is given.

Within-type labor allocation $\pi_{i,\tau}^{j,o}$ The equation (3) is derived using a Fréchet property.

$$\begin{aligned} \pi_{i,\tau}^{j,o} &= \Pr[w_{i,\omega}^{j,o} > w_{i,\omega}^{j',o'} \quad \forall j' \neq j \text{ and } \forall o' \neq o] \\ &= \Pr[p_i^{j,o} \epsilon_{\omega}^{j,o} > p_i^{j',o'} \epsilon_{\omega}^{j',o'} \quad \forall j' \neq j \text{ and } \forall o' \neq o] \\ &= \Pr[\epsilon_{\omega}^{j',o'} < (\frac{p_i^{j,o}}{p_i^{j',o'}}) \epsilon_{\omega}^{j,o} \quad \forall j' \neq j \text{ and } \forall o' \neq o] \\ &= \prod_{\substack{j' \neq j \\ o' \neq o}} \Pr[\epsilon_{\omega}^{j',o'} < (\frac{p_i^{j,o}}{p_i^{j',o'}}) \epsilon_{\omega}^{j,o}] \quad , \text{ from independence assumption} \\ &= \int F_{i,\tau}^{j,o}((\frac{p_i^{j,o}}{p_i^{j_1,o_1}})\epsilon, \dots, (\frac{p_i^{j,o}}{p_i^{j_l,o_l}})\epsilon) d\epsilon \\ &\quad \text{where } F_{i,\tau}^{j,o}(\epsilon) \text{ is a marginal distribution of } F_{i,\tau}(\epsilon) \text{ with respect to } (j,o)\text{-th} \\ &\quad \text{dimension of } (J \times O)\text{-dimensional vector } \epsilon \\ &= \int \theta_{i,\tau} T_{i,\tau}^{j,o} \epsilon^{-\theta_{i,\tau}-1} \exp(-\sum_{j',o'} T_{i,\tau}^{j',o'} (\frac{p_i^{j,o}}{p_i^{j',o'}})^{-\theta_{i,\tau}} \epsilon^{-\theta_{i,\tau}}) d\epsilon \\ &= \frac{\bar{T}_{i,\tau}^{j,o}}{\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'}} \int \theta_{i,\tau} (p_i^{j,o})^{-\theta_{i,\tau}} \sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} \epsilon^{-\theta_{i,\tau}-1} \exp(-(p_i^{j,o})^{-\theta_{i,\tau}} \sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} \epsilon^{-\theta_{i,\tau}}) d\epsilon \\ &\quad \text{where } \bar{T}_{i,\tau}^{j,o} \equiv T_{i,\tau}^{j,o} (p_i^{j,o})^{\theta_{i,\tau}} \text{ is an effective productivity.} \\ &= \frac{\bar{T}_{i,\tau}^{j,o}}{\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'}} \int d\tilde{F}_{i,\tau}(\epsilon) \\ &\quad \text{where } \tilde{F}_{i,\tau}(\epsilon) = \exp(-(p_i^{j,o})^{-\theta_{i,\tau}} \sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} \epsilon^{-\theta_{i,\tau}}) \text{ is another Fréchet distribution.} \\ &= \frac{\bar{T}_{i,\tau}^{j,o}}{\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'}} \end{aligned}$$

Average wage for each type $w_{i,\tau}$ The average wage for each type τ in each country i is an expectation of distribution of equilibrium wage of each type τ conditional on workers' equilibrium choice of industry and occupation. The unconditional distribution of type τ workers' potential wage for a certain pair (j, o) is

$$\begin{aligned} G_{i,\tau}^{j,o}(w) &= \Pr[w_{i,\omega}^{j,o} \leq w] \\ &= \Pr[\epsilon_{\omega}^{j,o} \leq \frac{w}{p_i^{j,o}}] \\ &= \exp[-\bar{T}_{i,\tau}^{j,o} w^{-\theta_{i,\tau}}] \end{aligned}$$

from the distributional assumption for the idiosyncratic productivity $\epsilon_{\omega}^{j,o}$. This distribution is again a Fréchet distribution with a location parameter $\bar{T}_{i,\tau}^{j,o}$ and a shape parameter $\theta_{i,\tau}$. I derive the equilibrium distribution of wage of type τ workers conditional on the choice of industry and occupation in worker's occupational choice problem by simply deriving the distribution of the maximum of potential wages. From the property of the extremum distribution, the distribution $G_{i,\tau}^*(w)$ is again a Fréchet distribution.

$$G_{i,\tau}^*(w) = \exp[-\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} w^{-\theta_{i,\tau}}]$$

Since the distribution of equilibrium wage only depends on the type, within-type heterogeneity is summed out once the equilibrium occupational choice is given. The average wage for each type is straight-forward by taking an expectation of the distribution function $G_{i,\tau}^*(w)$, which gives

$$w_{i,\tau} = \left(\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} \right)^{\frac{1}{\theta_{i,\tau}}} \Gamma\left(1 - \frac{1}{\theta_{i,\tau}}\right),$$

where $\Gamma(\cdot)$ is a Gamma function. Also, the variance of wage within each type is given b

$$var_{i,\tau}(w) = \left(\sum_{j',o'} \bar{T}_{i,\tau}^{j',o'} \right)^{\frac{2}{\theta_{i,\tau}}} \left(\Gamma\left(1 - \frac{2}{\theta_{i,\tau}}\right) - \left(\Gamma\left(1 - \frac{1}{\theta_{i,\tau}}\right) \right)^2 \right).$$

B.2 Production

Assume that there is an intermediate labor-input-producing unit in each industry which produces the labor input using workers' labor supply and sells it to final goods producers with zero profit. Final goods producers choose the equilibrium demand for occupational input $y_i^{j,o}(e^j)$ to minimize their costs. The cost minimization problem of a final good producer for product e^j of industry j in country i is given by

$$\min_{y_i^{j,o}(e^j)} \sum_o p_i^{j,o} y_i^{j,o}(e^j) \quad \text{s.t.} \quad Y_i(e^j) = z_i(e^j) \left(\sum_o \mu_i^{j,o} (y_i^{j,o}(e^j))^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}},$$

with the CES production technology and a factor-neutral productivity $z_i(e^j)$. The first-order conditions of this problem are

$$p_i^{j,o} = \lambda z_i(e^j)^{\frac{\gamma-1}{\gamma}} \mu_i^{j,o} \frac{\gamma-1}{\gamma} (y_i^{j,o}(e^j))^{-\frac{1}{\gamma}} \quad \text{for } o = 1, \dots, O \quad (26)$$

$$Y_i(e^j)^{\frac{\gamma-1}{\gamma}} = z_i(e^j)^{\frac{\gamma-1}{\gamma}} \left(\sum_o \mu_i^{j,o} (y_i^{j,o}(e^j))^{\frac{\gamma-1}{\gamma}} \right), \quad (27)$$

where λ is a Lagrange multiplier. Rearranging (26) and (27) gives a conditional demand function for occupational labor input $y_i^{j,o}$.

$$y_i^{j,o}(e^j) = z_i(e^j)^{-1} (\mu_i^{j,o})^\gamma (p_i^{j,o})^{-\gamma} \left(\sum_o (\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma} \right)^{\frac{\gamma}{1-\gamma}} Y_i(e^j)$$

The total cost function is thus given by

$$TC_i(e^j) = z_i(e^j)^{-1} \left(\sum_o (\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma} \right)^{\frac{1}{1-\gamma}} Y_i(e^j),$$

which gives an effective unit cost of $z_i(e^j)^{-1} \left(\sum_o (\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$. An industry-level unit cost function for occupational input bundle is

$$c_i^j = \left(\sum_o (\mu_i^{j,o})^\gamma (p_i^{j,o})^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

B.3 International Trade

Equilibrium results for the international trade part of the model are generalizations of [Eaton and Kortum \(2002\)](#) to the multi-industry and multi-factor setting. The distribution of the final good price can be derived from a Fréchet property, as the productivity parameter $z_i(e^j)$ follows a Fréchet distribution which is country- and industry-specific as in equation (2). The equilibrium bilateral trade flows are thus a multi-industry generalization of the EK results.

Distribution of final good price The distribution of the final good price can be derived from a distributional assumption for factor-neutral productivity $z_i(e^j)$ for each within-industry product variety e^j produced in country i . Given each country's equilibrium per-unit price of occupational task $p_i^{j,o}$ and iceberg trade cost, a price of product in industry j produced in country i purchased by country j follows the following distribution.:

$$\begin{aligned} H_{in}^j(p) &= \Pr\left[\frac{c_i^j d_{in}^j}{z_i(e^j)} \leq p\right] \\ &= 1 - \Pr\left[z_i(e^j) < \frac{c_i^j d_{in}^j}{p}\right] \\ &= 1 - \exp\left(-\left(A_i \left(\frac{c_i^j d_{in}^j}{p}\right)^{-\nu_j}\right)\right) \end{aligned}$$

A country buys e^j from the lowest-cost supplier in a perfectly competitive market, thus the distribution of the price of a good e^j in industry j that a country n actually buys is

$$\begin{aligned} H_n^{*j}(p) &= 1 - \prod_{i=1}^N \Pr[P_{in}(e^j) > p] \\ &= 1 - \exp[-\Phi_n^j p^{v^j}] \end{aligned}$$

where $\Phi_n^j \equiv \sum_{i=1}^N A_i (c_i^j d_{in}^j)^{-v^j}$ is an effective price parameter for industry j in country n . Since this model follows a multi-industry EK framework, the effective price parameter depends on the state of technology around the world, input costs around the world, and the geographic barriers which are industry-specific in this case.

Exact price index First, a corresponding probability density function of the distribution function $H_n^{*j}(p)$ is

$$h_n^{*j}(p) = \Phi_n^j v^j p^{v^j-1} \exp(-\Phi_n^j p^{v^j}).$$

From the nested CES preference of consumers, the exact price index for industry j in country j is derived as follows.

$$\begin{aligned} (P_n^j)^{1-\eta_2} &= \int p^{1-\eta_2} dH_n^{*j}(p) \\ &= \int \Phi_n^j v^j p^{v^j-\eta_2} \exp(-\Phi_n^j p^{v^j}) dp \\ &\quad \text{Define } x \equiv \Phi_n^j p^{v^j} \text{ to have} \\ &= (\Phi_n^j)^{\frac{\eta_2-1}{v^j}} \int x^{\frac{1-\eta_2}{v^j}} \exp(-x) dx \\ &= (\Phi_n^j)^{\frac{\eta_2-1}{v^j}} \Gamma\left(\frac{1-\eta_2+v^j}{v^j}\right), \end{aligned}$$

where $\Gamma(\cdot)$ is a gamma function, and $v^j + 1 > \eta_2$.

Bilateral trade flows Given the previous results, the gravity equation for each industry j is derived as follows. A probability that a country n buys a good in industry j from a country i is

$$\begin{aligned} \lambda_{in}^j &= \Pr[P_{in}(e^j) \leq \min_{i' \neq i} \{P_{i'n}(e^j)\}] \\ &= \int \prod_{i' \neq i} [1 - H_{i'n}^j(p)] dH_{in}^j(p) \\ &= \int A_i (c_i^j d_{in}^j)^{-v^j} v^j p^{v^j-1} \exp(-\Phi_n^j p^{v^j}) dp \\ &= \frac{A_i (c_i^j d_{in}^j)^{-v^j}}{\Phi_n^j} \int dH_n^{*j}(p) \\ &= \frac{A_i (c_i^j d_{in}^j)^{-v^j}}{\Phi_n^j}. \end{aligned}$$

This is equal to the expenditure share $\lambda_{in}^j = \frac{X_{in}^j}{X_n^j}$, where X_n^j is a total expenditure for industry j in country n , and X_{in}^j is an expenditure made by country n for all industry- j products made in country i . This equality holds because $X_{in}^j = \Pr[P_{in}(e^j) \leq \min_{i' \neq i} \{P_{i'n}(e^j)\}] X_n^j$ in a perfectly competitive market.

C Data Description

C.1 Size of the Model

As explained in Section 3.1, there are $N = 33$ countries, $T = 5$ worker types, $J = 4$ industries, and $O = 5$ occupations. Detailed list and classification are as follows.

List of Countries The sample consists of the following 32 countries: Argentina, Australia, Austria, Brazil, Canada, Chile, China, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, UK, and US. The last country is the rest of the world (ROW) which takes up all the remaining outputs, expenditures, and trade flows. Among the total 33 countries, Argentina, Brazil, Chile, China, India, Indonesia, Israel, and ROW are classified as non-OECD members during the sample period from 2000 to 2007.

Worker Types Any observable worker characteristics such as educational attainment, age, gender, or race technically can be used to define worker types. In this paper, worker types are defined solely by educational attainment to restrict the total dimension of the model as well as to relate the trade effect and productivity effect to the skill premium. Workers are categorized into five types defined by the educational attainment level: high school dropouts (HD), high school graduates (HG), workers with some college education (SC), college graduates (CG), and workers with advanced degrees (AD). The definition of educational attainment is specific to each country and each household-level survey. Details are discussed in the next subsection with the explanation on the dataset used for each country.

Industries I consider an aggregate sector-level definition of industries based on the major divisions (1-digit level) of the International Standard Industrial Classification (ISIC) Revision 3 classification: Agriculture, Mining, Manufacturing, and Service industry which aggregates the major divisions from D to U. Household-level survey, industry-level macro data, and trade data are all aggregated up to these four industry classifications based on corresponding crosswalks.

Occupations Dorn (2009) provides a new occupation classification based on the skill levels that are required for each occupation's task and the routineness of required tasks. At the most disaggregate level, this system consists of 330 occupations which are consistent over time for the U.S. Census data. I aggregate these occupation categories into 5 upper-level groups and reorder them based on the required skill levels similar to Autor and Dorn (2013).

1. “Low-skill Occupations (LSO)” include two broad occupation groups that engage mostly in manual tasks in their classification; low-skill service occupations and transportation/construction/mechanic/mining/agriculture occupations. To expand the analysis to many other countries as well, I combine these two occupation groups into one category, since these two groups are not distinguished in the International Standard Classification of Occupations (ISCO) which most household-level survey in the other countries are based on. ISCO 06 and 09 occupations belong to this category. Thus, this occupation category describes occupations that do low-skill and manual tasks, which are distinguished from routine tasks.
2. “Assemblers and Machine Operators (AMO)” include relatively middle-skilled and routine occupations. Operators of any kind of equipment or machines such as textile cutting or sewing machines and drilling machines belong to this occupation group. Assemblers of equipment are also included in this category. (ISCO 08)
3. “Precision Production and Crafts Occupations (PPC)” also include relatively middle-skilled and routine occupations. General production workers as well as workers engaging in production that requires precision all belong to this category: e.g., precision grinders and fitters, furniture/wood finishers, shoemakers, and bookbinders. (ISCO 07)
4. “Administrative, Clerical, and Sales Occupations (ACS)” are also classified as middle-skilled and routine occupations, but they require relatively higher-skilled tasks than the last two routine occupations do. This category includes sales and administrative support occupations such as salespersons, cashiers, secretaries, and bank tellers. (ISCO 04 and 05)
5. “Managers, Professionals, and Technicians (MPT)” include the most high-skilled occupations that engage in abstract tasks. For example, this occupation category includes CEOs, engineers, doctors, and professors. (ISCO 01, 02, and 03)

C.2 Labor Market Information from the IPUMS - International

The Integrated Public Use Microdata Series (IPUMS)-International database provides the detailed labor allocation information for around 2000 for the following countries in the sample: Argentina (2001), Austria (2001), Brazil (2000), Canada (2001), Chile (2002), France (1999), Greece (2001), Hungary (2001), India (1999), Indonesia (2000), Ireland (2002), Italy (2001), Mexico (2000), Netherlands (2001), Portugal (2001), Spain (2001), Switzerland (2000), Turkey (2000), UK (2001), USA (2000). For China, Germany, and Israel where only the data for earlier periods are available, I use the household survey for the years of 1990, 1987, and 1995, respectively, and then adjust them to 2000 with the variable ‘Employment by economic activity and occupation’ in the ILOSTAT database. For the other countries where the household-level survey data are not available, the OECD and non-OECD averages are applied depending on a country’s membership to the OECD and also adjusted with the ILOSTAT data. I supplement type-level labor supply $L_{i,\tau}$ with the variable ‘Working-age

population by sex, age, geographical coverage, school attendance status and education' in the ILOSTAT database and Barro and Lee (2013).

Since the information on worker's educational attainment in household-level surveys is collected based on different definitions of the education level in different countries, it is important to have a consistent definition of educational attainment across countries. The baseline definition follows years of schooling in the U.S. Census data. People with strictly less than 12 years of schooling are considered high school dropouts, exactly 12 years as high school graduates, 13 to 15 years as workers with some college education, exactly 16 years as college graduates, and strictly more than 16 years as workers with advanced degrees.

Countries for which the years of schooling variable is available in their household-level survey, worker types are defined by this rule. For the other countries where the years of schooling variable is not available but the more detailed categorical variable for the educational attainment is available, the educational attainment information – especially, distinction between college graduates and workers with advanced degrees – is defined by this detailed categorical variable. For the remaining countries where only a coarse level of categorical variable for educational attainment is available – Austria, Switzerland, and Turkey –, I assume that the ratio of workers with advanced degrees within the total workers with bachelor's degrees is the same as that of the U.S. The other three less-educated workers types are all well-defined with the available variable for the educational attainment for all countries in the sample. Based on this information on the educational attainment, I define 5 worker types.

Individual worker's industry affiliation is recoded in all household-level surveys I use in this paper. The information roughly conforms the ISIC classifications at 2-digit level, thus it can be exactly aggregated to four industry classification in the quantitative analysis of this paper without any additional adjustment. Worker's occupation affiliation information is gathered as described in the previous subsection using the ISCO information available in the survey data for each country except for Argentina and the U.S. For Argentina, the occupation information is not recoded to match the ISCO classification, so I manually classify the four-digit level occupation information of the survey into five occupation categories. I use Dorn (2009)'s crosswalk to categorize the U.S. census occupation codes into five categories. Collecting all this information, I measure $\pi_{i,\tau}^{j,o}$ as described in the online appendix.

Individual wage or earned income profiles for around the base year of 2000 are available in Brazil, India, Mexico, and the U.S. For all four countries, I consider only workers older than the age of 15 and also only workers whose employment status, educational attainment, industry affiliation, and occupation affiliation are available. Hourly wage data are available for the U.S. I multiply by 1.5 for top-coded observations. For Brazil and Mexico, I use monthly earned income profiles and divide them by the usual working hours. Once the hourly wages are derived, top-coded observations in Mexico are multiplied by 1.5. Weekly wage and salary income are available for India in 1999, so I again use the usual working hours to derive hourly wages.

C.3 Macro Variables

The industry-level gross output of each country is obtained mostly from the UN National Accounts by Industry database and the OECD STructural ANalysis (STAN) database for the base year 2000. For countries where the industry-level gross output data are not available in either source, I use the WIOD table (Australia, Brazil, China, Indonesia, and Mexico) or the data from the respective national statistics bureau (Iceland and Turkey.) For ROW, I calculate the industry-level gross output by re-defining it as the rest of the world in the WIOD and the other countries not included in my sample.

To measure the occupation share in the CES production function for each industry $\mu_i^{j,o}$, I use the variable ‘Employment by economic activity and occupation’ from the ILOSTAT database.³⁹ For countries where the data are not available for the base year, I again use the OECD or non-OECD average depending on a country’s OECD membership. The cost share $\zeta_i^{j,o}$ of occupation o in the unit cost of production in industry j is calculated with the industry- and occupation-specific average hourly wage available in the Occupational Wages around the World (OWW) database. For countries where the data are not available for 2000, I proxy the measure with the data for 1999 (Argentina, Brazil, Chile, Denmark, and Poland.) For countries where the data are not available for around 2000, I use the OECD or non-OECD average.

C.4 Bilateral Trade Data

I obtain bilateral trade flows for agriculture, mining, and manufacturing industries from the UN Commodity Trade (COMTRADE) database for 2000 and 2007. Trade flows are in HS 6-digit level, which I aggregate up to three industries. In addition, bilateral trade flows for the service industry are collected from the Trade in Services Database from the World Bank. Bilateral trade flows between ROW and each partner country are calculated by subtracting the total trade flow between corresponding partner countries and the other countries in the sample from the total trade flow of that partner country.

D Technical Details of the Algorithm to Solve for the Equilibrium

The system of equations is solved for the unknowns $\hat{p}_i^{j,o}$ at the equilibrium.⁴⁰ I denote the vector of unknowns by $\hat{\mathbf{p}} = (\hat{p}_1^{1,1}, \dots, \hat{p}_1^{1,O}, \dots, \hat{p}_1^{J,1}, \dots, \hat{p}_1^{J,O}, \dots, \hat{p}_N^{1,1}, \dots, \hat{p}_N^{1,O}, \dots, \hat{p}_N^{J,1}, \dots, \hat{p}_N^{J,O})'$, which is a $(N \times J \times O)$ -dimensional vector. First, guess the initial $\hat{\mathbf{p}}$; e.g., $\hat{\mathbf{p}} = (1, \dots, 1)'$.

³⁹All results are very robust to the alternative measure of $\mu_i^{j,o}$ with the total payment, instead of the employment count.

⁴⁰The algorithm to numerically solve for the equilibrium is based on Alvarez and Lucas (2007) and Caliendo and Parro (2015). This paper is without intermediate inputs in the model but has multiple industries and multiple factors. Alvarez and Lucas (2007) consider a single industry, and both papers consider only a single type of labor as a production factor.

Given $\zeta_i^{j,o}$ and λ_{in}^j from the data in the base year 2000, parameter values for γ and ν^j , and counterfactual changes in bilateral trade costs \hat{d}_{in}^j which are calibrated to the data, solve for changes in the unit cost \hat{c}_i^j and changes in the industry-level price index \hat{P}_i^j using equations (15) and (16). Next, solve for changes in the outcomes of the occupational choice problem using equations (13) and (14), given $\pi_{i,\tau}^{j,o}$ from the data in 2000, the estimated parameter $\theta_{i,\tau}$, and the counterfactual changes in the industry-level labor productivity \hat{T}_i^j calibrated to the data. Therefore, $\hat{c}_i^j(\hat{\mathbf{p}})$, $\hat{P}_i^j(\hat{\mathbf{p}})$, $\hat{\pi}_{i,\tau}^{j,o}(\hat{\mathbf{p}})$, and $\hat{w}_{i,\tau}(\hat{\mathbf{p}})$ are all derived as functions of $\hat{\mathbf{p}}$ given the initial guess.

Counterfactual changes in the total expenditure are solved as functions of $\hat{\mathbf{p}}$ as well. First, counterfactual changes in the industry-level expenditure share $\hat{\lambda}_i^j(\hat{\mathbf{p}})$ are derived given λ_i^j from the data in 2001 and $\hat{P}_i^j(\hat{\mathbf{p}})$ using the equation (17). Second, changes in the total income in country i , \hat{I}_i , are solved as functions of $\hat{\mathbf{p}}$ as well from $\hat{I}_i = \frac{\sum_{j,o} \psi_i^{j,o} + D_i'}{\sum_{j,o} \psi_i^{j,o} + D_i}$, where $\psi_i^{j,o} = \sum_{\tau} w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o} \hat{w}_{i,\tau}(\hat{\mathbf{p}}) \hat{\pi}_{i,\tau}^{j,o}(\hat{\mathbf{p}})$. Therefore, changes in the industry-level expenditure are solved from $\hat{X}_i^j(\hat{\mathbf{p}}) = \hat{\lambda}_i^j(\hat{\mathbf{p}}) \hat{I}_i(\hat{\mathbf{p}})$. With counterfactual changes in bilateral trade costs \hat{d}_{in}^j and the trade elasticity parameter ν^j , counterfactual changes in the total industry-level output are derived also as functions of $\hat{\mathbf{p}}$ using equation (20) which is the final goods market clearing condition in proportional changes. Therefore, the final goods market clearing conditions and the occupation market clearing conditions are reduced to the following system of independent equations plus $\sum_{i,j} E_i^j = \sum_{i,j} E_i^j = E$ as a normalization, given that $\mu_i^{j,o}$ and $L_{i,\tau}$ do not change over time.

$$\left(\frac{\hat{P}_i^j}{\hat{c}_i^j(\hat{\mathbf{p}})}\right)^{1-\gamma} \hat{E}_i^j(\hat{\mathbf{p}}) = \sum_{\tau} \left(\frac{w_{i,\tau} L_{i,\tau} \pi_{i,\tau}^{j,o}}{\sum_{\tau'} w_{i,\tau'} L_{i,\tau'} \pi_{i,\tau'}^{j,o}}\right) \hat{w}_{i,\tau}(\hat{\mathbf{p}}) \hat{\pi}_{i,\tau}^{j,o}(\hat{\mathbf{p}}) \quad (28)$$

These equations directly imply the trade balance condition for each country. Therefore, I have $(N \times J \times O)$ independent equations and the same number of unknowns in $\hat{\mathbf{p}}$. I check if the initial guess of $\hat{\mathbf{p}}$ satisfies (28). If not, update the initial guess and repeat until (28) is satisfied.