

**Institute for International Economic Policy Working Paper Series  
Elliott School of International Affairs  
The George Washington University**

**Climate Change, Uncertainty, and Decision-Making**

**IIEP-WP-2010-24**

**Arun Malik, Jonathan Rothbaum, and Stephen C. Smith  
Department of Economics and  
Institute for International Economic Policy  
George Washington University  
Washington DC 20052**

**November 10, 2010  
Revised: May 17, 2011**

Institute for International Economic Policy  
1957 E St. NW, Suite 502  
Voice: (202) 994-5320  
Fax: (202) 994-5477  
Email: [iiep@gwu.edu](mailto:iiep@gwu.edu)  
Web: [www.gwu.edu/~iiep](http://www.gwu.edu/~iiep)

Climate Change, Uncertainty, and Decision-Making

Arun Malik, Jonathan Rothbaum, Stephen C. Smith

Elliot School of International Affairs  
George Washington University  
1957 E Street, N.W., Suite 502  
Washington, D.C. 20052

IIEP Working Paper 2010-24  
November 10, 2010  
Revised: May 17, 2011

Background paper for the Initiative on the Economics of Adaptation to Climate Change in Low-Income Countries at George Washington University. Malik is a Professor of Economics; Rothbaum is a PhD Candidate in Economics; and Smith is a Professor of Economics and International Affairs at the George Washington University. Address Correspondence to Stephen C. Smith at [ssmith@gwu.edu](mailto:ssmith@gwu.edu). Support from an anonymous donor and the Elliott School of International Affairs is gratefully acknowledged.

## **1. Introduction**

It is impossible to look at how individuals, communities, and governments adapt, proactively and reactively, to climate change without looking at how uncertainty affects decision-making. Uncertainty is a defining characteristic of climate change economics. There is broad consensus that anthropogenic warming is occurring. However, the obvious limitations to performing scientific experiments on the global climate system and its extremely complicated nature render our understanding incomplete.

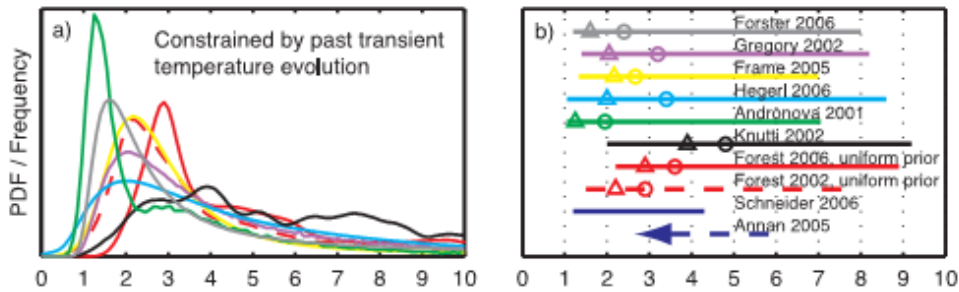
As a result, there are a myriad of uncertainties individuals confront when making decisions that affect, or are affected by, climate change. This survey provides a representative, though not exhaustive, review of those uncertainties. It is our goal to highlight the important issues involved in climate change uncertainty and provide the reader with an understanding of why these issues are important and how they have been approached in the literature. Additionally, the distinction between the notions of risk and ambiguity will prove important when analyzing climate change uncertainties. Irreversibilities and fat-tailed distributions for catastrophes also complicate climate change decision-making. After defining these concepts, we will look how their implications differ in the short and long terms. Theoretical and practical applications of decision theory given risk and ambiguity will be surveyed. The ultimate goal of this survey is to show how theories of decision-making under uncertainty can be used to help model adaptation and provide a framework for policy makers to address climate change and for researchers to analyze how individuals will make adaptation decisions in the face of an uncertain climate future.

Another important issue that is related to uncertainty and climate change economics, which is beyond the scope of this survey, is discounting. Due to the permanence of CO<sub>2</sub> in the atmosphere, climate change economics must consider very long time frames, making modeling and policy recommendations very sensitive to the discount rate used (Heal and Kriström 2002).

## **2. Climate Change Uncertainty**

The 2007 IPCC report on the physical science basis of climate change includes many models which show the wide range of temperature increase predictions. Figure 1 gives a sense for the uncertainties involved in climate change modeling. Each model attempts to take what we know about the climate system and determine the probability that the climate will stabilize with a global mean temperature increase from 0-10°C. While there is broad agreement across the models that temperature increases will occur, the distributions vary considerably.

**Figure 1: Probabilities of Equilibrium Temperature Increases in Sample of Different Climate Models<sup>1</sup>**



a) Probability of equilibrium temperature change (climate sensitivity) in different climate models.

b) Confidence interval (5%-95%) for temperature change

Circles represent the median temperature and triangles the maximum probability.

The purpose of this section is to show just how pervasive the uncertainties involved in climate change are. These uncertainties and issues can be broken down into broad categories to give a sense of how they might affect different decision-makers.

## 2.1 Environmental Uncertainties and Issues

### 2.1.1 Feedback Loops – Ecological and Physical Processes (IPCC 2007b)

Feedback loops arise in the global climate system when increased temperatures affect other natural systems which further increase temperatures (positive feedback loops) or decrease temperatures (negative feedback loop). The primary feedback loops are described below.

#### Carbon Cycle

One important positive feedback loop is the carbon cycle, or the ability of the planet to absorb emitted CO<sub>2</sub> in carbon sinks in the ocean or on land. The absorption of carbon diminishes the temperature increase from a given quantity of emissions. As more CO<sub>2</sub> is released, the ability of the planet to absorb CO<sub>2</sub> is expected to decrease. This means that a larger proportion of emissions will remain in the atmosphere, resulting in a higher equilibrium level of CO<sub>2</sub> in the atmosphere in the future, and more warming.

#### Atlantic Ocean Meridional Overturning Circulation (MOC)

An example of a negative feedback loop (for the North Atlantic) is the predicted slowing of the Atlantic Ocean Meridional Overturning Circulation (MOC) due to climate change. The result of this would be slower warming in the North Atlantic and Europe than would otherwise be the case as less cold air is circulated from the North Atlantic to warmer regions of the ocean. Again, there is uncertainty about the feedback loops involved and the magnitude of the ensuing changes in ocean currents.

<sup>1</sup> Source: IPCC 2007 Physical Science Basis – Chapter 10. This is just a sample to show the general uncertainty in climate change projections.

## Clouds

The feedback effects of clouds on climate change are also important but not well understood. Clouds both reflect solar radiation back into space (albedo effect) and trap heat emitted from below. Which effect predominates, and therefore whether clouds result in positive or negative feedback loops, depends on cloud elevation, latitude, temperature, optical depth<sup>2</sup>, and atmospheric environment. Figure 2 shows the wide range of estimates on the feedback effects between climate change and clouds. The IPCC physical science report states, “cloud feedbacks remain the largest source of uncertainty in climate sensitivity estimates.”

## Methane and Permafrost Melting

According to the Stern Report as well as the IPCC, another positive feedback loop of uncertain magnitude is melting of the permafrost.<sup>3</sup> Methane (a greenhouse gas) trapped in the permafrost would be released as temperature increases result in permafrost melting. For example, observed methane emissions have increased by 60% in northern Siberia since the 1970's (Stern 2007).

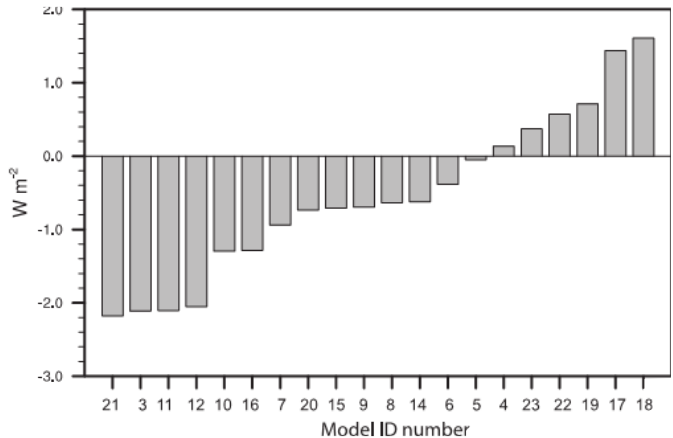
### 2.1.2 Thresholds and Irreversibilities (IPCC 2007b)

Another area of concern are thresholds that exist in the global climate system that once crossed result in irreversible changes to the climate. The primary thresholds and irreversibilities are described below.

## Sea Level Rise

Thermal expansion will result in sea level rise, though there is some uncertainty as to how far and fast sea levels would rise per degree increase in temperature.<sup>4</sup> Also, melting in the Antarctic and especially Greenland ice sheets will determine how much sea levels rise above the levels predicted by thermal expansion. Climate models suggest that if temperatures increase past a threshold of 1.9°C to 4.6°C, depending on the model, after 2100 and slowly over centuries, the complete melting of the Greenland ice sheet would result. This would cause a 7m increase in sea levels (IPCC 2007b). However, Oppenheimer, et al. (2007) suggest that the IPCC models do not include the possibility that this melting could happen much more rapidly.

**Figure 2: Effect of Cloud Changes on Climate Change in Sample of Models**



Source: IPCC 2007 Physical Science Basis, Chapter 10. Radiative forcing ( $\text{W/m}^2$ ) is a measure of warming due to clouds. From IPCC Chapter 2, the radiative forcing of all anthropogenic  $\text{CO}_2$  released in the atmosphere between 1750 and 2005 is less than  $2 \text{ W/m}^2$

2 Optical depth depends on the cloud's thickness, ice or water content, and the size and distribution of ice and water crystals.

3 Permafrost is soil whose temperature is below the freezing point of water.

4 Thermal expansion of oceans will take a considerable amount of time to occur. The IPCC report states that “for a reduction to zero emissions at year 2100 the climate would take on the order of 1 kyr to stabilise. At year 3000, the model range for temperature increase is 1.1°C to 3.7°C and for sea level rise due to thermal expansion is 0.23 to 1.05 m,” under Earth System Models of Intermediate Complexity.

## **MOC (IPCC 2007b)**

The MOC also may be subject to threshold effects. There is considerable uncertainty as to whether the MOC could shut down completely or what the threshold might be. Additionally, the question of whether a shutdown is irreversible is also open.

## **Vegetation Cover**

Slight changes in temperature and precipitation can have significant effects on the ability of plants and animals to survive in a region. Claussen, et al. (1999) develop a simulation model of the desertification of the Sahara 4000-6000 years ago as an example of how relatively small changes in environmental conditions can push plants past the temperature and precipitation limits under which they can survive. In cases where an entire ecosystem is destroyed (or species rendered extinct), these changes are irreversible. Again, the exact nature and levels of these thresholds are not completely known .

## **CO<sub>2</sub> Persistence in the Atmosphere (IPCC 2007b)**

CO<sub>2</sub> emissions are essentially irreversible for the following reasons. First, CO<sub>2</sub> remains in the atmosphere for approximately 100 years. Second, changes to a lower emissions future will likely be gradual and/or costly. Therefore, barring the development of a cheap carbon capture technology,<sup>5</sup> any emissions scenario must include a long time horizon to account for the long-term effects of irreversible CO<sub>2</sub> emissions.

### **2.1.3 Precipitation**

Climate models tend to agree on the aggregate changes in precipitation with climate change. Overall precipitation will increase, both over land and sea, however, in some areas, precipitation is expected to decrease, including Mediterranean Europe, Southern Africa, Southwest United States, Central America, Andean South America, and parts of Australia. However, climate models disagree on the boundaries between areas that will see increased or decreased precipitation. Additionally, the level of precipitation increase or decrease and the hydrological effects of precipitation and evaporation rate changes are uncertain.

### **2.1.4 Extreme Weather Events**

The IPCC report (2007b) and Easterling et al. (2000) agree that extreme weather events such as droughts, floods, and heat waves are projected to be more common even with relatively small changes in mean temperature and precipitation levels. In addition, climate change is expected to result in drier summers and wetter winters in the northern middle and high latitudes. The Stern report discusses how the European summer of 2003, the hottest in 500 years, is an example of the extreme high temperature events expected to be more common in the future . The likely future extreme weather events include hot days, single and multi-day heavy rains, as well as heat waves and droughts. High temperature extremes are more likely to occur, with low temperature extremes less likely.

Both the IPCC And Easterling et al. indicate that the average number of category 4 and 5 hurricanes per year has increased over the past 30 years, and the severity of hurricanes and cyclones is expected to continue to increase. The geographic range in which these storms are likely to occur will shift a few degrees of latitude toward the poles. The IPCC report states that although the number of intense tropical storms may increase, there will be fewer tropical storms, due to a decrease in weak tropical storms. However, Easterling et al. note the ability of current climate models to predict future changes

<sup>5</sup> While carbon capturing generally refers to removing CO<sub>2</sub> from the emissions from large sources such as power plants, it can also refer to removing CO<sub>2</sub> from the ambient air in the atmosphere.

in tropical storm frequency is under debate.

### **2.1.5 Increased Variability**

The IPCC report and Easterling et. al, agree that in addition to more extreme weather events, research suggests that monthly precipitation will become more variable (Easterling 2000; IPCC 2007a). Temperature variability is expected to change as well: decreasing during the cold season in the Northern Hemisphere, increasing in low latitudes and in the warm season in northern mid-latitudes (IPCC 2007b).

### **2.1.6 Regional Projections**

Individual responses to climate change will depend on how climate change affects the region in which they live and not on how it affects global means. Therefore, predictions of climate change impacts on regional and local conditions are important. Climate models generally agree on global changes, but can differ greatly at the regional level, according to the IPCC and a World Bank report (Margulis and Narain 2010; IPCC 2007; IPCC 2007b).

## **2.2 Economic Uncertainties**

In addition to the scientific uncertainties, there are a number of uncertainties that arise as part of the economics of climate change.

### **2.2.1 Impact Uncertainty**

Heal and Kriström (2002) note that even if future climate change were known, transforming greenhouse gas concentrations, temperatures, precipitation levels, etc. into economic impacts can be difficult. For example, according to Cline's study on agricultural impacts of climate change by country, there is uncertainty about how much agricultural productivity will be lost (or gained in some regions) due to climate change (Cline 2007).

### **2.2.2 Technological Uncertainty**

The role of technology is of central importance in climate modeling. Existing technology determines the cost of emissions abatement and adaptation. The development of future technology may be central to lowering future abatement and adaptation costs. For example, if a nearly costless method for carbon capture were developed and implemented in the near future, anthropogenic climate change caused by CO<sub>2</sub> would no longer be a problem, and emissions abatement today would be unnecessary. However, understanding the cost of implementing current technologies is not straightforward. For example, Fischer and Morgenstern (2006) find that estimates of the cost of reducing emissions to Kyoto Protocol levels varied by as much as a factor of five across different studies. Pizer and Popp (2008) state, "Technological change is at once the most important and least understood feature driving the future cost of climate change mitigation."

A number of studies have been conducted to model the effects of technology on future climate scenarios, especially on abatement costs to lower greenhouse gas emissions. One of the main difficulties these studies have faced is the lack of an empirical foundation, which makes it difficult to know which model representation more accurately expresses the underlying reality (Jaffe, Newell, and Stavins 2003). Given the open empirical questions, interpreting technological change studies is not always straightforward as modelers are not always transparent about what assumptions they are making (Pizer and Popp 2008). Also, the type of model (for example, general equilibrium vs. partial equilibrium) can have a large impact on the predicted technological development and associated costs

(Edenhofer et al. 2006). Another difficulty in modeling technological change in climate change is that technologies that do not currently exist, such as cheap carbon capture, could significantly improve the situation if developed. However, how to model this set of technologies is not clear given the obvious absence of empirical data about them (Nordhaus 2002). The difficulty in modeling technical change generates considerable uncertainty about the future costs and benefits of abatement, mitigation, and adaptation (Edenhofer et al. 2006).

### **2.2.3 Policy Uncertainty**

Policy uncertainty is intertwined with technological uncertainty. Technology and investment decisions are not made in a vacuum, but instead are influenced by incentives. Present and future policy decisions can have a large role in determining the technologies that are developed and greenhouse gas concentrations in the atmosphere (Heal and Kriström 2002; Fischer and Newell 2008; Jaffe et al. 2005). Therefore, uncertainty about the nature and timing of future policy generates uncertainty about technological progress and future emission levels. One effect of policy uncertainty is that the private sector may have difficulty planning investments given the policy risk involved (Sullivan and Blyth 2006).

### **2.2.4 Adaptation Uncertainty**

Linked to policy uncertainty is adaptation uncertainty. When making policy decisions, government officials are basing those policies on the adaptation decision they are likely to induce from individuals and firms. This issue is not addressed directly in the literature. Ulph and Ulph (1997) construct a simple model with an adaptation parameter under climate uncertainty. Kelly et al. (2005) model adaptation decisions of farmers in the Midwestern US to climate shocks under uncertainty to estimate their adjustment costs and adaptation decisions. Adaptation uncertainty is also linked to impact uncertainty. The impact of climate change on welfare, for example on individual wealth and health, depends on how individuals adapt to climate change.

## **2.3 Model and Parameter Uncertainty**

While models provide a window to understanding the global climate system, they do not completely and accurately represent those systems. Due to many of the aforementioned uncertainties, there is reason to be cautious in interpreting climate change models and basing policy decisions on their conclusions. Uncertainties introduced by modeling can be separated into two categories, model uncertainty and parameter uncertainty.

### **Model Uncertainty**

Due to the complex nature of the global climate and the economic systems involved, it is nearly impossible to determine which model is the correct one. Matching possible models to historic data allows us to narrow the focus to ones that better match the data. It is important to note that even if a model perfectly matches the data, there may be errors in the model specification that limit its ability to predict future events (Edenhofer et al. 2006). In some cases, when analyzing the climate system, individual models are combined, given some weighting scheme, to generate multi-model projections of climate impacts (IPCC 2007b). However, given that the uncertainties in the individual models may be unknown and unquantifiable, the uncertainties in the multi-model combination may be unquantifiable as well (Dessai and Hulme 2004; Hall et al. 2007).

### **Parameter Uncertainty**

Even if the functional forms that describe the economy were known, the parameters to use in those



functions are still estimates that we would only be able to calibrate to their 'real' values. For example, different parameters in climate models yield different results on the feedback effects between cloud cover and climate change. Different parameters on risk aversion affect how much abatement should be done now to lower future risks (Dessai and Hulme 2004; Edenhofer et al. 2006; Heal and Kriström 2002; Held et al. 2009; IPCC 2007; Lösschel 2002).

Parameter uncertainty is also linked to model uncertainty. When attempting to calibrate parameters to a model through some statistical technique, an incorrect model will generally result in apparent parameter uncertainty, so it may not be clear if the model is incorrect or the parameter is just difficult to identify from the noise in the data (Refsgaard et al. 2007).

### 3. Uncertainty Issues in Climate Change

#### 3.1 Risk vs. Ambiguity

##### 3.1.1 Risk

In most economic analysis of decisions under uncertainty, by assumption the probabilities of outcomes are known, or can be inferred with some confidence. With known probabilities, expected utility maximization, a simple and useful decision framework, has been very widely used in economics. For simplicity, in discrete terms, with  $n$  possible outcomes  $\mathbf{x}=(x_1, x_2, \dots, x_n)$ , where each outcome has a probability  $p_i$  of occurring, and given some utility function  $u$ , expected utility of  $\mathbf{x}$  is:

$$U(\mathbf{x}) = \sum_{i=1}^n p_i u(x_i) \quad (1)$$

Expected utility is weighted average of the utility in each possible state, with the probability of ending up in that state as the weight. In a simple example, suppose you were drawing balls from an urn which contains 50 red balls and 50 black balls.<sup>6</sup> If you draw a red ball, you receive \$100, and if you draw a black ball you receive \$0. After normalizing  $u(\$0)=0$ , your expected utility would be:

$$U(\mathbf{x}_{urn}) = p_{red} u(x_{red}) + p_{black} u(x_{black}) = 0.5 u(100) + 0.5 u(0) = 0.5 u(100) \quad (2)$$

Now suppose that to draw a ball (and have the possibility of earning \$100), you had to wager \$50. In expected value terms, drawing from the urn is worth \$50.<sup>7</sup> If you prefer keeping the \$50, to an uncertain wager with an expected value of \$50, you are considered *risk averse* (Pratt 1964). Using the notation above, this can be expressed:

$$u(50) > 0.5 u(100) \quad (3)$$

This framework has been extremely useful, with applications in many areas, including insurance and financial markets. The intuition behind risk aversion is that there are diminishing returns to getting more income. Individuals get more happiness out of their first \$100 than out of the \$100 that raises their income to \$100,000 (by say, avoiding starvation with the first \$100, as opposed to buying slightly better food at \$100,000).

However, this analysis depends critically on the assumption that the probabilities of the different

<sup>6</sup> This example and the discussion on ambiguity is a simplified version from Ellsberg's seminal paper on risk and ambiguity.

<sup>7</sup>  $EV(\mathbf{x}_{urn}) = p_{red} x_{red} + p_{black} x_{black} = 0.5(100) + 0.5(0) = 50$

outcomes are known, or can be estimated reasonably (Ellsberg 1961). In this framework, risk represents uncertainty over which outcome will be realized *given the probability* that each outcome will occur.

### 3.1.2 Ambiguity

Now consider the same situation: an urn with 100 balls, either red or black. However, this time you do not know how many are red and how many are black. Additionally, you have no reason to believe that any distribution of balls is any more likely than any other. Any possible distribution from 100 black balls and 0 red ones to 0 black balls and 100 red ones is possible, and you have no way of distinguishing which one is more likely. Now the decision to make the \$50 wager is a different one. If, in the absence of better information, you assume all possibilities are equally likely, the problem reduces to the one above, and your decision would depend on your risk aversion as before.<sup>8</sup>

However, in the absence of any information about the balls, some decision-makers may prefer to be more cautious. Some risk-loving people who would have bet in the previous case would choose not to wager in the ambiguous case (Ellsberg 1961). In this case, the person can be considered *ambiguity averse* as opposed to *risk averse*. It is the ambiguity, defined as *uncertainty about the probabilities themselves*, that prevents this person from placing the wager. This notion of ambiguity has also been called deep or Knightian uncertainty. The term ambiguity was chosen here because it represents a more general notion of uncertainty about probabilities. In deep or Knightian uncertainty, the probability distributions are unknown whereas ambiguity, as used in decision theory, better encapsulates the idea that we may or may not have some information about the probabilities distributions, and only in the extreme are completely ignorant of them.

In the previous case, the person chose not to place the wager because of the risk, or *uncertainty about the outcome that will be realized given those probabilities*. Ambiguity aversion means that an individual prefers a situation with clear probabilities to those ones with uncertain probabilities (even if expected utility were the same in both cases) (Camerer and Weber 1992).

In this stylized example, the difference between risk and ambiguity is clear. This example is an extreme case of complete ambiguity, where there is no information about the probabilities involved. However, different degrees of ambiguity can exist, where the decision-maker has some knowledge or confidence about the likelihood of different probability distributions.

### Ambiguity Attitude in Empirical Studies

After the Ellsberg paper, a number of experimental studies were done to test ambiguity aversion. Camerer and Weber (1992) conducted a comprehensive survey of prior experiments. They concluded that individuals are willing to pay a premium of 10-20% on average to avoid ambiguity. Not all participants in these studies were ambiguity averse, and differences in experimental design and the stakes involved make comparisons across studies difficult. They also note that ambiguity loving behavior was common given a high probability for losses and a low probability for gains. Also, Camerer and Weber note that the correlation between ambiguity and risk aversion appears low, however, due to problems with study designs and implementation, they are not convinced by this result.

More empirical work has expanded on Camerer and Weber's findings. Bossaerts, et al. (2010) conduct

---

<sup>8</sup> By the reduction of compound lotteries, this problem reduces to the risk problem with 50 red balls and 50 black balls.

an experiment on portfolio choice in asset markets and find heterogeneity of ambiguity attitudes and that ambiguity neutrality and ambiguity aversion are most common. Additionally, their data suggests a positive correlation between risk aversion and ambiguity aversion. Halevy (2007) conducted an Ellsberg-type experiment to assess the performance of different decision-theoretic frameworks. He found that 15-20% of subjects were ambiguity neutral and that the majority of the remaining participants exhibited ambiguity aversion. He also found considerable heterogeneity in individual attitudes toward (and processing of) ambiguity. Ahn et al. (2009) conduct a portfolio experiment to test ambiguity aversion models. They also find considerable heterogeneity in the experimental population and a large percentage of individuals who show ambiguity aversion. Chakravarty and Roy (2008) conduct an experiment to test whether ambiguity aversion behavior is affected by whether individuals are likely to experience losses or gains. They found that individuals were more risk averse over gains and more risk and ambiguity seeking in losses. However, aversion to both risk and ambiguity is the most common trait. Budescu and Templin (2008) and Di Mauro and Maffioletti (2004) found that decision-makers were ambiguity loving in gains and ambiguity averse over losses. Cabantous (2007) surveyed actuaries on insurance pricing and found that ambiguity was more costly to insure than risk.

Many of the studies since Camerer and Weber in 1992 seek to explain some aspect of ambiguity aversion rather than testing directly for its presence. Due to their different experimental methodologies, the results of these studies cannot always be compared. However, they agree with earlier studies that there is considerable heterogeneity of ambiguity attitude in the population. It should be noted that considerable heterogeneity has also been observed in risk aversion (Choi et al. 2007). Also, many studies suggest that ambiguity attitudes are not symmetric across gains and losses.

### 3.2 Fat-Tailed Distributions

Weitzman (2009a) considers the problem of a “fat-tailed” distribution of losses and gains, where a fat tail “assigns a relatively much higher probability to rare events in the extreme tails than does a thin-tail.” The idea is that a fat-tailed distribution has a much higher (though still low) probability of catastrophic events. Weitzman argues that if the costs of a catastrophe are sufficiently great and the probability of one occurring is not sufficiently small, expected utility and cost benefit analysis are not capable of informing our decisions about managing the risks. During a catastrophe, marginal utility becomes extremely high.<sup>9</sup> If marginal utility in increasingly catastrophic outcomes increases faster than their probabilities decrease given fat tails, the possibility of a catastrophe dwarfs all other considerations in expected utility/cost-benefit analysis (Aldy et al. 2009). In this case, decision-makers should take extremely strong actions (if possible) to lower the probability of a catastrophe. For example, if climate change has catastrophic consequences, this could justify drastically cutting back on or eliminating CO<sub>2</sub> emissions.

This analysis depends on events being sufficiently catastrophic. Weitzman mentions the possibility of a climate change causing damages of 99% of current welfare-equivalent consumption. To put that into perspective, China's per capita GDP in 1978 is about equal to 1% of current US GDP per capita.<sup>10</sup> Aldy et al. (2009) cite studies which place damages, even in extreme scenarios, at under 3% of consumption. If catastrophes are not sufficiently catastrophic (far more so than 3% of consumption), then fat-tails concerns would not invalidate expected utility analysis. Another argument against Weitzman's fat-tail

---

9 Extremely high marginal utility under starvation (Weitzman's example) can be understood by considering that people would forgo almost everything today to avoid mass starvation tomorrow.

10 Source: World Bank World Development Indicators. GDP measured at PPP. 1978 per capita Chinese GDP is 1.2% of 2008 per capita US GDP.

argument is that if learning allowed us to discover we were on a course toward a catastrophic outcome, we could cut emissions drastically then, assuming no catastrophe threshold had been crossed and such learning were possible (Aldy et al. 2009).

Weitzman (2009b) counters that modeling results are sensitive to the damage function<sup>11</sup> chosen, and that as a result, fat-tailed considerations cannot be ruled out. He also notes that due to the permanence of CO<sub>2</sub> emissions in the atmosphere, mid-course corrections may not be possible because “by the time we learn that a climate-change disaster is impending it may be too late to do much about it.”

Therefore, even if damages were low enough to allow for the use of expected utility, given the uncertainties involved, decision-makers must incorporate fat-tail uncertainty over catastrophic situations into their analyses (Nordhaus 2009).

### 3.3 Option Value/Irreversibilities

Two linked issues also arise in the economics of climate change, the irreversibility of some damages, which have been enumerated above, and the option value of future emissions. Given irreversible CO<sub>2</sub> emissions, if we choose not to pollute now, we are preserving the “option” to pollute later. Paraphrasing Arrow and Fisher's (1974) seminal paper on irreversibility in environmental economics: given an ability to learn from experience, under-pollution can be remedied in the future (by increasing production and pollution) if we learn that climate change is less harmful or easier to mitigate than we thought, whereas mistaken over-pollution cannot be remedied, and the consequences of over-pollution will persist irreversibly. In this way, emissions abatement now preserves the option value of future pollution. An important additional consideration is that this option value is only present given uncertainty and irreversibility. If emissions are reversible (for example, given carbon capture technology), then pollution today can be removed returning us to low greenhouse gas concentrations. If there is little cost to removing greenhouse gases from the atmosphere, there is little value to paying now to pollute less and preserve the option of polluting more in the future. Additionally, without uncertainty, the optimal pollution path can be determined beforehand given the known irreversible threshold, and there is no option value to delaying pollution, just the normal tradeoff between the marginal benefits and costs of present vs. future emissions (Pindyck 2007).

Just as with fat tails, learning is extremely important when considering option values given uncertain irreversibilities. For example, with slow or no learning, there is a risk that irreversible thresholds will be crossed without decision-makers being aware of it (Aldy et al. 2009). Pindyck also highlights the flip side of irreversibility in climate change. Cutting emissions now to preserve the option of future pollution can be costly (for example, given high costs of transitioning to alternative energy sources). This leads to an option value to polluting and waiting to see if it is necessary to incur the costly and irreversible investments in abatement.

A number of studies have been done to assess how irreversibilities affect climate change decisions, though with mixed prescriptions, which depend on modeling assumptions. In some models, investment irreversibilities dominate emissions irreversibilities, and the optimal emissions path is to pollute more now and invest in abatement in the future (Fisher and Narain 2003; Keller et al. 2004; Ulph and Ulph 1997; Heal and Kriström 2002). Gollier and Treich (2003) construct a model that suggests the opposite, that precautionary emissions reductions should be adopted to preserve the option of future

---

11 A function of the damage caused by temperature increases.

pollution. Baker (2005) shows that assumptions about risk and learning are crucial in determining how emissions levels should be set over time. One issue that these studies ignore is the ability of firms and individuals to adapt to climate change, which should affect policy makers' decision on abatement costs. Ingham et al. (2007) construct a simple model with adaptation to show that given learning and uncertainty, adaptation decreases the amount of mitigation and abatement we should engage in today.

Irreversibilities also affect adaptation decision-making, as many adaptations require upfront investments. For example, given the irreversible fixed cost of investing in irrigation, there may be an option value to waiting to see how exactly local conditions are affected by climate change before investing in a new irrigation system. Switching crops could also entail an upfront investment in acquiring crop-specific knowledge. This could affect the timing of adaptation decisions under ambiguity.

### **3.4 Unknown Unknowns**

In addition to the preceding discussion of uncertainty as risk or ambiguity, the long time frames involved and the our imperfect knowledge of the world climate system and future economic and technological development result in “unknown unknowns.” With unknown unknowns, “we find ourselves in total ignorance, unable to even know what uncertainty exists” (Baroang, Hellmuth, and Block 2009). In the case of unknown unknowns, the outcome space itself may be unknowns or there may be no way of assigning likelihoods to outcomes or probability distributions. The IPCC also highlights the difficulty of modeling given unknown unknowns. They note that in the context of structural modeling choices, these concerns can be “attenuated when convergent results are obtained from a variety of different models using different methods, and also when results rely more on direct observations (data) rather than on calculations” (Halsnæs et al. 2007).

## **4. Uncertainty in Climate Change**

The distinction between risk and ambiguity is important in the discussion of climate change. When attempting to design policy or model individual decision-making in climate change economics, those policies and decisions must be conditioned on the inherent uncertainties involved.

### **Long vs Short Term**

Depending on the time frame and scope, different uncertainties are relevant to the decision-making process. When looking at ambiguous information, fat-tailed distributions, or irreversibilities, the time frame of the decision is crucially important.

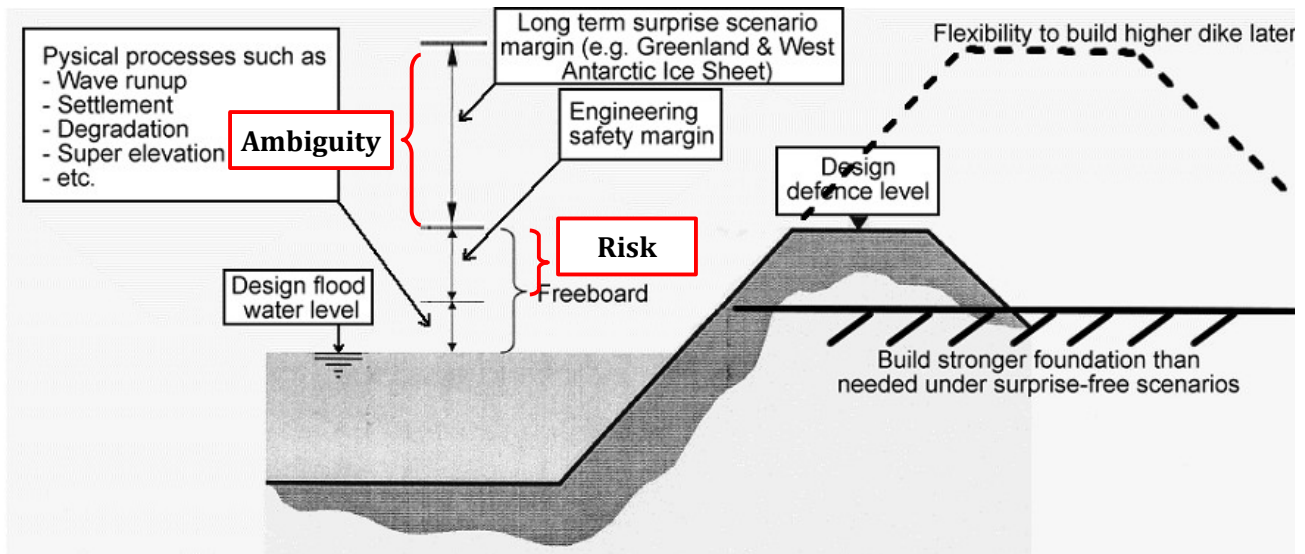
#### **Long-Term**

Long-term decisions are those that have high fixed costs and are difficult to change or reverse. Infrastructure investments, including roads, electrical infrastructure (power plants, transmission lines), water and sewer systems, etc. clearly qualify as long-term decisions. These investments require planners to account for both risk and ambiguity in climate change predictions. Private individuals make long-term decisions on investments, such as real estate and capital investment or migration decisions.

An illustrative example of a long-term public infrastructure decision would be the design of dikes or levies to protect coastal or river areas from flooding due to extreme weather events or sea level rise. It may be necessary to account for the ambiguity inherent in climate change by including flexibility or

some additional safety margin to account for the fact that future water levels may be very different than they are expected to be. An example of this was a plan put forth for the dike system in the Netherlands. Rather than building dikes with a safety margin only adequate to handle the risk of sea level rise in the likely scenarios, a more flexible option was proposed. This entailed building dikes to the height necessary given the expected risk and also including a stronger foundation that allows a larger dike to be built much more quickly and easily in the case of a larger than expected sea level rise. This type of planning would result in higher cost now, but greater resilience of the dike in case the Greenland and Antarctic ice sheets melt much faster than expected. Figure 3 is adapted from a figure in a study on adaptation to climate change in the Netherlands (Dessai and Van Der Sluijs 2007). This example shows how both risk and ambiguity can be incorporated into long-term decision-making.

**Figure 3: Flexible Dike Design to Address Ambiguity of Sea Level Rise**



*Adapted from Dessai and Van Der Sluijs 2007*

Policy uncertainty also plays a role long-term decision-making. When planning long-term investments, such as electrical infrastructure, companies must account for the risk of higher emissions prices, which depends in part on future government policy decisions. For example, large uncertainties could lead utility companies to delay investments in new plants leading to higher emissions than would have otherwise been the case (Sullivan and Blyth 2006). In this case the the uncertainty is over future government action. This uncertainty would be considered risk if companies believe they can reasonably assign probabilities to future policy outcomes. However, if the uncertainty is great enough, it may be impossible to do so; in which case, this would be an example of ambiguity.

### Short-Term

Short-term decisions are those that can be changed easily or at low cost or whose effect disappears over the long-term. A farmer deciding which crop to plant this year would be an example of a short-term decision, as it can be revisited again each year. For short-term decisions, risk uncertainty may be more important than ambiguity. When deciding which crop to plant, a farmer can generally ignore the ambiguity in temperature and precipitation patterns 20 years from now. For these types of decisions, the long-term ambiguity of climate change is less relevant. Instead, the current variability of climate is

the important risk factor. While over long periods, the probability distribution governing droughts, extreme weather events, floods, etc. are likely to change significantly, the probabilities in any one year are unlikely to differ significantly from the year before.

In addition, it may be possible to insure against the short-term risks. Weather index insurance has the potential to help farmers hedge against the risk inherent in increased weather variability. The principle of index insurance is that insurance payments are made based on some independent measure that is correlated with outcomes but not under the control of individual policy holders. For example, weather conditions are correlated with farm yield, but are outside the control of farmers (Skees, Barnett, and Hartell 2005).

An important advantage when considering insurance is that it gives a clear price signal to farmers about the expected climate risk. Price signals could alert farmers to the need to consider alternative agricultural strategies or even abandon agriculture for other economic activities. However, there are limitations to the applicability of index insurance, especially given the ambiguity over changes to regional climates. When the variance, and especially, the mean of weather conditions is unknown due to climate ambiguity, it may be difficult to accurately price insurance (Skees, Barnett, and Collier 2008).

Many of the major ambiguities inherent in climate change, such as the feedback loops, irreversibilities, and economic uncertainties are uncertainties that primarily affect long-term decision-making. Additionally, as available data, climate science, and modeling improve, precipitation changes, weather variability, extreme events, and regional variations in climate change will all be subject to less ambiguity in the short term.

## **5. Theories of Decision-Making under Uncertainty**

There is a considerable literature on decision-making under uncertainty. This survey does not attempt to provide a comprehensive review of this literature. Instead, the goal is to provide a summary of some of the most important theoretical developments. Also, as the goal is to use these decision theories to model how individuals make decisions in the face of climate change, we will also try to address how these theories can be applied as well as some possible hurdles to doing so.

### **5.1 Expected Utility / Subjective Expected Utility**

The basic expected utility model was explained in Section 3. The major shortcoming of this model in the context of climate change is the requirement of a known probability distribution. One way to incorporate imperfect knowledge of probabilities into the expected utility is to include subjective probabilities, or the decision-maker's prior judgment about the likelihood of different outcomes. In this framework, the subjective probabilities are updated as new information comes to light. In cases where the probabilities are known or where the extreme risks are bounded or reasonably well understood, the expected utility or subjective expected utility models provide an excellent framework for analysis.

### **Expected Utility in Climate Decision-Making**

In climate change, much of the uncertainty is ambiguity and choosing a prior distribution is both arbitrary and extremely important for the results of economic models. The existence of low (but unknown) probability, high cost events (the melting of the Greenland ice sheets or warming of more than 10°C) do not lend themselves well to expected utility analysis given the tremendous uncertainty

about their probabilities (Shaw and Woodward 2008). The main criticism of standard expected utility is that in a problem with ambiguity, the model does not accurately predict how people make decisions. The Ellsberg paradox (1961), which has been demonstrated in repeated experiments, violates expected utility theory and shows that people generally exhibit ambiguity aversion (Camerer and Weber 1992). The applied literature on ambiguity attitude in the population was discussed in Section 3.1.2.

## 5.2 Precautionary Principle (PP)

The need to include ambiguity aversion in decision-making has been recognized in policy-making circles for decades. In this context, ambiguity aversion has been referred to as the Precautionary Principle (PP). There have been numerous definitions of the PP, and it has been included in many areas of environmental regulation and policy (for example, in the Second World Climate Conference in 1992) (Harding and Fisher 1999). While some formulations are stronger than others, most definitions of the PP state that precaution should be taken in choosing a course of action given that there is uncertainty about the level of damage that will be caused. One characterization states that the PP should entail a “willingness to take action in advance of scientific proof, or in the face of fundamental ignorance of possible consequences, on the grounds that further delay or thoughtless action could ultimately prove far more costly than the ‘sacrifice’ of not carrying on right now” (O’Riordan and Jordan 1995). Thus the PP can be thought of as similar to an option value of not taking a possibly environmentally costly course of action. However, the PP is too vague to in itself guide policy. Gollier and Treich (2003) even go so far as to say, “the common formulation of the [Precautionary Principle] has no practical content and offers little guidance for conceiving regulatory policies.” In order for the PP to be useful as a decision criterion, a formal definition is needed (Basili, Chateauneuf, and Fontini 2008).

Therefore, to incorporate the PP into decision-making, the expected utility framework can be extended to include ambiguity aversion. This can be achieved with the inclusion of subjective probabilities and non-additive (nonlinear) expected utility. Allowing utility to follow a form other than the function in Section 3 complicates the theory, but also allows decision theory to be applied to more types of problems (Gilboa 1987; Schmeidler 1989). The inclusion of ambiguity aversion has the added benefit of more accurately representing how people actually make choices as ambiguity aversion has been shown to be important in decision-making (Camerer and Weber 1992).

## 5.3 General Ambiguity Aversion Theories

### 5.3.1 Maximin Expected Utility and Choquet Expected Utility (MEU and CEU)

Two formal models of decision-making with ambiguity aversion are Maximin Expected Utility (MEU) and Choquet Expected Utility (CEU). In both, there are multiple possible probability distributions, and there is ambiguity about which probability distribution is valid. For example, each climate model could represent a possible probability distribution. In these models, the decision-maker exercises extreme caution and chooses a policy that maximizes their utility in the *worst* case that they consider possible given the ambiguity. The name maximin comes from the maximization of the minimum utility (Schmeidler 1989; Gilboa and Schmeidler 1989).

An example may help explain these models given an ambiguity averse decision-maker. Assume there are three possible climate models,  $A$ ,  $B$ , and  $C$ . I believe  $A$  has a 75% chance of being the correct model, and I believe there is a 25% chance that either  $B$  or  $C$  is correct. But due to ambiguity, I cannot divide the 25% probability that either model is correct and assign it to models  $B$  and  $C$ . This situation is summarized in Table 1. It is clear from the table that this represents non-additive probability, as  $0\% = P(B) + P(C) \neq P(B \text{ or } C) = 25\%$ . This non-additivity is what separates CEU



and MEU from subjective expected utility. The decision-maker does not have enough information to assign a probability to all possible models, and therefore does not. In subjective expected utility, it would likely be assumed that each model had a 12.5% chance of being correct, in a sense, ignoring the ambiguity.

<b>Table 1: Example of MEU/CEU</b>			
<b>Probabilities</b>		<b>Utilities</b>	
P( A )	75%	U( A )	2
P( B )	0%*	U( B )	1
P( C )	0%*	U( C )	3
P( A or B )	75% (Due to ambiguity of B , same as P( A ))		
P( A or C )	75% (Due to ambiguity of C , same as P( A ))		
P( B or C )	25%		
P( A , B , or C )	100% (encompasses all possible outcomes)		
* P( B ) and P( C ) are perceived as 0% due to the decision-maker's inability to assign a precise probability to the likelihood of the outcome occurring.			
<b>Expected Utility</b>			
When calculating utility in this case, the ambiguity averse decision-maker assumes the worst for ambiguous possibilities so expected utility is:			
$EU = P(A)U(A) + P(B \text{ or } C) \min\{U(B), U(C)\} = 0.75 * 2 + 0.25 * 1 = 1.75$			

By always assuming the worst given ambiguous information, the ambiguity averse decision-maker therefore exercises extreme caution (Mukerji 1997).<sup>12</sup>

### MEU/CEU in Climate Decision-Making

In discrete terms, for simplicity, assume there are  $m$  climate models<sup>13</sup>, each with  $n$  possible climate outcomes. Let  $Y$  represent the set of policies being analyzed, where  $y$  is an individual policy. In each model  $j$ ,  $p_{i,j}(y)$  is the probability that outcome  $i$  will occur given policy  $y$ . For example, given a specific tax on carbon emissions as the policy  $y$ , in model 3, outcome 4 is that the temperature will rise 3°C with a probability of 20%, then  $p_{4,3}(y) = 0.2$ . Each  $x_{i,j}(y)$  is the payoff (for example in income), under outcome  $i$  in model  $j$  for policy  $y$ , and  $u(x_{i,j}(y))$  is the utility given payoff  $x_{i,j}(y)$ .

We will make a simplifying assumption about the distinction between ambiguity and risk. There is complete ambiguity about which model is correct. However, if a given model is correct, the

<sup>12</sup> This example is a simplification of one provided in Mukerji's paper. It is not meant to completely characterize CEU or MEU preferences, but to give an intuitive sense for how they work.

<sup>13</sup> The  $m$  models could also include combinations of different models and parameterizations of different models to reflect the model and parameter uncertainty mentioned previously.

probabilities are known for each possible outcome, the uncertainty within the model is only risk uncertainty. For model  $j$ , the expected utility of policy  $y$  is:

$$EU_j(y) = \sum_{i=1}^n p_{i,j}(y) u(x_{i,j}(y)) \quad (4)$$

For each set of policies  $y$ , MEU would be represented as:

$$\max_{y \in Y} EU(y) = \min(EU_1(y), \dots, EU_m(y)) \quad (5)$$

To apply MEU to public policy, each policy option would be evaluated by determining the welfare under the climate model with the lowest expected utility given that policy. Then, the decision-maker would choose the policy that was least bad in its worst case. Therefore, MEU preferences represent an extreme form of the PP.

While there may be cases where MEU decision criteria should be used, it seems overcautious to incorporate only the worst-case scenario into a decision-making process (Quiggin 2005). However, given the ambiguity involved in climate change, some form of ambiguity aversion does seem appropriate. The MEU and standard expected utility frameworks provide bounds on the ambiguity aversion. The MEU decision-maker exhibits complete aversion to ambiguity and the expected utility decision-maker shows no ambiguity aversion.

A more flexible approach is needed to address ambiguity aversion and model climate change decisions. Several models have been proposed to provide a framework for decision-making under ambiguity, including models using  $\alpha$ -Maxmin Expected Utility (Ghirardato, Maccheroni, and Marinacci 2004), Smooth Ambiguity (Klibanoff, Marinacci, and Mukerji 2005), Multiplier Preferences (Hansen and Sargent 2001), and Case-Based Decision Theory (Gilboa and Schmeidler 1995; Guerdjikova 2008). All of these theoretical models use mathematical axioms to define what they mean by ambiguity and ambiguity aversion. However, because they are all theoretical, they may not all be easy to incorporate into climate change decision-making.

### 5.3.2 $\alpha$ -Maxmin Expected Utility ( $\alpha$ -MEU)

The  $\alpha$ -MEU model is very similar to the MEU model with a slight changes to generalize MEU to allow for differing levels of ambiguity aversion. The  $\alpha$ -MEU includes a parameter  $\alpha$  that represents the decision maker's ambiguity aversion. The value of  $\alpha$  is between 0 and 1, and it does not depend on the distribution of possible outcomes. Maximizing expected utility in  $\alpha$ -MEU could be represented discretely as:

$$\max_{y \in Y} EU(y) = \alpha \min(EU_1(y), \dots, EU_m(y)) + (1 - \alpha) \max(EU_1(y), \dots, EU_m(y)) \quad (6)$$

where  $\max(EU_1(y), \dots, EU_m(y))$  represents the expected utility of the model with the *highest* expected utility.

Together, the best and worst possible models provide bounds on what the decision-maker can expect because the “true” expected utility must by definition be between the best and worst possible cases. When  $\alpha=1$ , the decision-maker is completely ambiguity averse and assumes the worst (and the formula reduces to MEU). If  $\alpha=0$ , the decision-maker is completely ambiguity loving and assumes

the best. In this way, different  $\alpha$  represent different levels of ambiguity aversion.<sup>14</sup>

### **$\alpha$ -MEU in Climate Decision-Making**

Hayashi and Wada (2008) highlight an important limitation of the  $\alpha$ -MEU model. A decision maker in the  $\alpha$ -MEU focuses only on the best and worst outcomes and ignores all information about other possible outcomes. They conducted an experiment to compare different ambiguity-based decision theories and found that subjects care about more than just the best and worst cases when making decisions. In climate change decision-making, it seems reasonable to include these non-extreme future scenarios as well as the best and worst possible outcomes. For example, if new information came to light that rendered “good” climate models more likely, but did not affect the best or worst case models, under  $\alpha$ -MEU models our decisions would be unchanged as we ignore merely “good” models, because they are neither the best nor the worst case.

### **5.3.3 Smooth Ambiguity**

Klibanoff, Marinacci, and Mukerji (2005) develop an ambiguity framework that is very similar to standard expected utility over risk. For this purpose we would define a function  $\phi$  which converts the expected utilities of each possible probability distribution into an ambiguity-sensitive utility when summed over all possible probability distributions. This is a direct analog of how the utility function  $u$  in standard utility theory converts the utility of each possible outcome (given a probability distribution) into risk-sensitive-utility.  $q_j$  can be defined as the likelihood that that probability distribution is the correct one.

### **Smooth Ambiguity in Climate Decision-Making**

In terms of climate change, as before, assume there are  $m$  climate models, each with  $n$  possible climate outcomes. Depending on how likely we believe a particular model is correct, we assign it a probability  $q_j$ . The Smooth Ambiguity model in this case is:

$$\max_{y \in Y} EU(y) = \sum_{j=1}^m q_j \phi(EU_j(y)) \quad (7)$$

This model has the advantage of maintaining the same intuition that generates risk aversion in standard expected utility models. This model also provides a more simple way to compare and test how individuals make decisions under ambiguity. In this way, it provides a framework for determining plausible functions for  $\phi$ , which could then be incorporated into an applied decision-making framework to guide policy choices when faced with ambiguity. The specification of  $\phi$  can be done much as risk aversion is specified. For example, a constant absolute ambiguity attitude could be represented by any positive affine transformation of  $\phi(x) = \frac{-1}{\alpha} e^{-\alpha x}$ .

Epstein (2010) uses several thought experiments that extend Ellsberg's urn example to critique Smooth Ambiguity's applicability to some decision problems. Klibanoff, Marinacci, and Mukerji (2009a) counter that Smooth Ambiguity performs reasonably well, and that all economic models, by their very definition, are simplifications which allow economists to focus on the relevant issues while not necessarily encompassing all possible scenarios.

<sup>14</sup> If there is no ambiguity, for any  $\alpha$ , the problem reduces to the standard expected utility case, because the best and worst possible models are the same. The “true” model is known and is therefore also the best and worst possible one.

By extending the intuition used in standard expected utility under risk, Smooth Ambiguity provides a clear and more simple framework to analyze decision-making under ambiguity at the cost of possibly not accurately encompassing how people make decisions under uncertainty in all circumstances.

### 5.3.4 Robust Control/Multiplier Preferences

The Multiplier Preferences framework was developed for use in macroeconomic and financial decision-making under uncertainty. This model uses the notion of relative entropy<sup>15</sup> as a measure of the perceived ambiguity. The entropy measures the decision-maker's confidence in the accuracy of his prior probability distribution (his model, for example) as opposed to the other possible probability distributions. This model was specifically developed to handle problems where decision-makers were facing ambiguity due to parameter or model uncertainty (Hansen and Sargent 2001).

A simple form of Robust Control is, where  $w$  represents model uncertainty and  $\theta$  represents the decision-maker's sensitivity to ambiguity (large  $\theta$  indicates low sensitivity). As a thought experiment, imagine a malevolent nature that chooses  $w$  to minimize the decision-maker's utility. There is a penalty to nature to increasing ambiguity (in this example  $w^2$ ). Nature minimizes the expected utility of the decision-maker subject to this penalty:

$$U(x) = \min_w \left\{ \sum_{i=1}^n p_i u(x_i, w) + \theta w^2 \right\} \quad (8)$$

Therefore with larger  $\theta$ , the penalty to nature's introduction of ambiguity is greater, which leads to smaller optimal values for  $w$ . An entropy constraint is imposed on the model, which determines which specifications are ignored by the decision-maker as being too implausible, and the remaining possibilities are considered indistinguishable from the correct model. Similarly to MEU preferences, the agent considers the range of “plausible” models given the entropy constraint, and maximizes his utility subject to the worst possible model (Backus, Routledge, and Zin 2004). Hennlock (2009) phrases the robust control problem as one where “a hypothetical minimizer that resides in the head of our household making her to think ‘what if the worst about climate sensitivity turns out to be true.’”

### Robust Control/Multiplier Preferences in Climate Decision-Making

One weakness of Multiplier Preferences is that the model is not based on observed decision-making behavior (Maccheroni et al. 2006). Multiplier preferences also suffer from the weakness of MEU and  $\alpha$ -MEU in focusing on the extreme cases and maximizing utility under those circumstances without considering the non-extreme possibilities. Hennlock (2009) uses a two-period Integrated Assessment Model with Robust Control to analyze how ambiguity aversion affects carbon emissions decisions. He finds that ambiguity aversion results in a “shadow ambiguity premium” on carbon emissions.

### 5.3.5 Case-Based Decision Theory (CBDT)

Case-Based Decision Theory (CBDT) accounts for the fact that individuals make decisions based on the success of actions under similar circumstances in the past. In this model, the decision-maker faces uncertainty and the past circumstances the decision-maker has faced provide some, but imperfect, information about the likely outcome of different outcomes in the current situation. The decision-maker uses a “similarity” function to assess how similar past circumstances are to the current situation

---

<sup>15</sup> Relative entropy is a measure of the difference between probability distributions.

and perceives ambiguity when those past events are not similar enough to the current one (Gilboa et al. 2002). Gilboa et al. (2006) also consider statistical methods for estimating similarity functions based on empirical data.

### **CBDT in Climate Decision-Making**

In the context of climate change, CBDT can be applied to models individual decision-making as optimal policy exercises. For example, in the context of policy-making under adaptation uncertainty, CBDT models may provide insights into how an autonomously-responding agent assimilates and applies information from past experiences to current problems, for example, how farmers use past weather and precipitation conditions to inform their planting decisions as climate change progresses. Modeling adaptation decisions could help policy-makers reduce adaptation uncertainty when making decisions.

### **5.4 Learning and Dynamic Modeling of Ambiguity Aversion**

The next step to applying any of the above ambiguity aversion frameworks to climate change is to consider the decision-making processes over time. Climate change decisions are not necessarily static ones. As new information and new technologies become available, responses to climate change will likely vary. In this section, we will provide a brief survey of attempts to model the resolution of ambiguity, or one type of learning.<sup>16</sup> As more information becomes available, for example, scientific uncertainty over the affects of climate change is likely to be reduced. In some climate change decision-making exercises, it may be necessary to be able to accommodate this type of learning.

#### **5.4.1 Two-Period Models**

For exercises in how learning affects individual decision-making in a partial equilibrium context, some papers have adopted a simple two- or three- period model with exogenous learning. However, as these papers show, even simple models of ambiguity and learning do not always yield unambiguous conclusions about how ambiguity affects decision-making. Ulph and Ulph (1997) construct a two-period model to analyze how learning affects optimal abatement policy. Their model addresses risk given uncertain irreversible thresholds. This paper addresses learning given risk and does not include ambiguity. Ingham et al. (2007) extend on that paper by including a parameter for adaptation in their model. Baker (2005) incorporates a simple learning model in a strategic game theoretic framework to model how the correlation of climate change damages across different countries affects the equilibrium level of emissions. Lange and Treich (2008) introduce ambiguity and ambiguity aversion to a two-period decision framework.

#### **5.4.2 Recursive Models**

In order to embed learning in a model that looks into the indefinite future as opposed to two or three periods, additional modeling tools are needed. Epstein and Schneider (2003) develop a recursive framework where in each period the ambiguous prior distributions are updated according to Bayes' rule. Bayesian updating is both an intuitive method for updating subjective probabilities (or learning) and guarantees dynamic consistency.<sup>17</sup> In another paper, they extend this concept to accommodate another notion of learning based on likelihood functions. In this model, they use a likelihood ratio test

---

<sup>16</sup> Endogenous learning in recursive climate change through research is beyond the scope of this study. However, this could be an alternative to the exogenous learning or Bayesian updating models included here.

<sup>17</sup> Dynamic consistency means that if in all possible states of the world in the future, outcomes from one act are preferred to outcomes from another act, the first act should be preferred now (ex ante) as well. This is a main condition that allows recursive modeling with ambiguity.

to discard priors that are not sufficiently likely given the observed outcomes. This paper also highlights that although each new piece of information results in learning, it is possible that the decision-maker's understanding will not converge to one “true” probability distribution (for example, the correct parameterization and climate model), and that situations may exist where ambiguity is never completely resolved (Epstein and Schneider 2007).

### **Recursive MEU Models**

Epstein and Schneider then apply these theoretical techniques to modeling asset prices in financial markets given MEU preferences and ambiguous asset returns (Epstein and Schneider 2008; Epstein and Schneider 2010). Leopold et al. (2008) develop a similar model to analyze asset returns using ambiguity aversion and MEU preferences in an attempt to explain the equity premium puzzle.

### **Recursive Smooth Ambiguity**

Hanany and Klibanoff develop a general theoretical framework that is extended by Klibanoff, Marinacci and Mukerji to allow dynamic recursive modeling of ambiguity under Smooth Ambiguity preferences (Hanany and Klibanoff 2009; Klibanoff et al. 2009). The application of this model would be similar to the applications of the Recursive MEU Models above.

### **Case-Based Decision Theory (CBDT)**

An alternative form of updating from Bayesianism has been developed in the CBDT literature. Billot et al. (2005) provide an axiomatic framework for how individuals could use past experiences and observations to update priors based on the frequency of observations and their similarity to the current circumstances. Eichberger and Guerdjikova (2010) extend this framework to include an “optimism” parameter that is analogous to the  $\alpha$ -MEU for decision-making under ambiguity.

### **Other Models**

Hansen and Sargent (2001) developed their Multiplier Preferences model for dynamic applications. Their framework was designed to be used under conditions where past observations made it impossible for a decision-maker to distinguish the correct model from a set of alternative models. In this framework, as time passes, new information would be incorporated into the entropy function, which would alter the set of plausible models the decision-maker cannot distinguish from the correct one, and the decision-maker would update his policies accordingly, always choosing the policy that maximizes utility in the worst plausible model.

Lempert and Collins develop a model driven by applied, rather than theoretical, concerns. Their model is motivated by a precautionary principle that decision strategies should be robust to alternative possibilities given ambiguity. This framework follows a more heuristic approach that attempts to present the modeling information in a way that non-technical decision-makers can better understand to inform the development of policies that are robust to many possible future outcomes, though not necessarily optimal according to subjected expected utility maximization (Lempert and Collins 2007; Lempert et al. 2009).

It is important to note that not all models of climate change find that learning occurs on a sufficiently short time scale. Leach (2007) conducts a simulation where the true impact of greenhouse gases on temperature increases are defined by an unknown parameter or parameters, and there is some variance in annual temperature values. Learning occurs as observed temperature values are used to update estimates of the unknown parameter(s). Leach simulated two types of models, a reduced form and an

Integrated Assessment model. The paper finds that with Bayesian updating alone, it takes hundreds or thousands of years to resolve the uncertainty about the true underlying parameters.

## **6. Discussion and Directions for Future Research**

The field of climate change economics is beset with uncertainties. This survey has attempted to both give a sense of what those uncertainties are, why they are important, and what the main considerations are that economists need to take into account when thinking about them. This review has also surveyed the main theoretical developments in modeling ambiguity in the hopes of pointing the way for future research to incorporate what advances in theory and experiments have told us about ambiguity aversion and decision-making under ambiguity into climate change decision-making.

By including ambiguity aversion into climate change decision-making, the precautionary principle is being incorporated, but with a more rigorous theoretical basis. In doing so, the decision-maker is implicitly making a tradeoff by sacrificing standard expected utility in order to gain greater resilience of their decisions to the multiple possible climate futures. By maximizing *ambiguity sensitive* expected utility, an ambiguity averse decision-maker will choose a strategy that is more robust to alternative climate models.

Additionally, future research could focus on how ambiguity attitude is affected by income. Developing and developed country policy-makers and citizens may have different attitudes toward ambiguity due to their differing wealth levels that could affect their climate change decision-making.

## Appendix A: Formal Treatment of Decision Theories

In the text, decision theories were presented informally so they would be understandable to non-technical readers. This appendix includes the formal presentation of each.

In this section, the notation will be as follows: von-Neumann-Morgenstern utility function  $u$ , a set of priors  $C$  which is the set of all possible probability distributions with  $P \in C$  where  $P$  is a prior distribution in the set  $C$ ,  $L$  is a set of outcomes of a “horse lottery,” or a lottery where each outcome has an uncertain probability as in the probability of any horse winning a race (subjective probabilities),  $Y$  is a set of outcomes of a “roulette lottery,” or a lottery where each outcome has a known probability of occurring, such as the spinning of a roulette wheel (objective probabilities) (Anscombe and Aumann 1963), acts  $f$ ,  $g$ , and  $h$  are outcomes of horse lotteries,  $\alpha, \beta \in [0,1]$ , and  $L_c$  is a lottery with a certain outcome.

Additional notation will be defined in each subsection as it is needed.

### A.1 Choquet Expected Utility (Gilboa & Schmeidler 1989)

Schmeidler's Choquet Expected Utility began the formalization of ambiguity in decision theory. This model also forms the basis of the MEU and  $\alpha$ -MEU models.

Define a finite step function for non-additive probabilities and its Choquet Integral:

$$a = \sum_{i=1}^k \alpha_i E_i^* \text{ where } \alpha_1 > \alpha_2 > \dots > \alpha_k$$
$$\int_C a \, dv = \sum_{i=1}^k (\alpha_i - \alpha_{i+1}) v \left( \bigcup_{j=i}^k E_j \right)$$

### Utility Representation given the Choquet Integral

$$U(f) = \int_C u \cdot f \, dP$$

The utility representation depends on the nonadditive probabilities defined above. The important concept is that nonadditive probabilities allow for the inclusion of information about ambiguity attitude that additive probabilities do not (additive probabilities assume ambiguity neutrality). The difficulty encountered for practical application is there is no clear method for determining probabilities that are not additive.

#### Ambiguity Averse if:

$$U[\alpha f + (1-\alpha)g] \geq \alpha U(f) + (1-\alpha)U(g)$$

#### Ambiguity Loving if:

$$U[\alpha f + (1-\alpha)g] \leq \alpha U(f) + (1-\alpha)U(g)$$

### A.2 Maximin Expected Utility (MEU) (Gilboa and Schmeidler 1989)

#### Axiom Results

This paper defines a set of axioms that yield the following theorem:



$$f \succsim g \Leftrightarrow \min_{P \in C} \int u \cdot f dP \succsim \min_{P \in C} \int u \cdot g dP$$

Therefore, act  $f$  is preferred to  $g$  if and only in the worst possible prior distribution, the utility of  $f$  is greater than that of  $g$  in its worst possible prior distribution.

### Utility Representation

$$U(f) = \min \left\{ \int u \cdot f dP \mid P \in C \right\}$$

### A.3 $\alpha$ -Maxmin Expected Utility ( $\alpha$ -MEU) (Paolo Ghirardato, Maccheroni, and Marinacci 2004)

The Choquet Expected Utility and MEU representations serve as the theoretical basis for the  $\alpha$ -MEU Expected Utility.

Define a function  $a$  which takes any action  $f$  given the uncertainty over the information available so that for all possible  $f$  and information sets,  $a(f) \in [0, 1]$ .

### Utility Representation of $\alpha$ -MEU

$$U(f) = a(f) \min_{P \in C} \int u \cdot f dP + (1 - a(f)) \max_{P \in C} \int u \cdot f dP$$

The  $\alpha$ -MEU includes a characterization with a constant  $\alpha$ . In this case the ambiguity attitude *only* depends on the maximum and minimum expected values from the priors.

$$U(f) = \alpha \min_{P \in C} \int u \cdot f dP + (1 - \alpha) \max_{P \in C} \int u \cdot f dP$$

### A.4 Smooth Ambiguity (Maccheroni et al. 2006)

Define a function  $\phi$  which captures the ambiguity attitude of the decision maker.  $\phi$  can be thought of as an analog of the von-Neumann-Morgenstern utility function, except it is a utility function over ambiguity. As such, a concave  $\phi$  results in ambiguity aversion just as a concave  $u$  results in risk aversion.  $\mu$  is a measure of the probability that the specific  $P$  is the correct probability distribution. This can be thought of as a second order probability distribution.

### Utility Representation of Smooth Ambiguity

$$U(f) = \int_{P \in C} \phi \left( \int_{p \in Y} u \cdot f dP \right) d\mu$$

Another advantage of this model is that it does not require piecewise linear (or kinked) utility curves. The authors characterize this as “second-order ambiguity sensitivity” as opposed to the first-order sensitivity of a kinked indifference curve. The term Smooth Ambiguity reflects the non-kinked nature of the indifference curves.

A constant ambiguity aversion representation of  $\phi$  is presented in the paper as:

$$\phi(x) = \begin{cases} \frac{1 - e^{-\alpha x}}{1 - e^{-\alpha}} & \text{if } \alpha > 0 \\ x & \text{if } \alpha = 0 \end{cases}$$

### A.5 Variational Preferences (Hansen & Sargent 2001; Maccheroni et al. 2006)

This preference model is a generalization of models that were developed for use in situations with multiple possible prior literature and robust control literature. For this model, define a function  $b: c \rightarrow [0, \infty)$  and  $x_f$  as the certainty equivalent of action  $f$ . The notion of Variational Preferences states that:

$$f \succsim g \Leftrightarrow \min_{P \in C} \left( \int u \cdot f dP + c(P) \right) \geq \min_{P \in C} \left( \int u \cdot g dP + c(P) \right)$$

Where for each utility function  $u$ , there exists a unique, minimal index of ambiguity aversion  $c^*$ , such that:

$$c^*(P) = \sup \left( u_{x_f} - \int u(f) dP \right)$$

#### Utility Representation

$$U(f) = \min_{P \in C} \left( \int u \cdot f dP + c^*(P) \right)$$

### A.6 Robust Control/Multiplier Preferences (Hansen and Sargent 2001; Maccheroni et al. 2006)

This is a special case of Variational Preferences that developed from the Robust Control to be used in macroeconomic and financial modeling. Multiplier Preferences follows the Variational Preferences model above where  $c^*(P) = R^w(P \parallel q) \forall p \in C^\sigma$ ,  $q$  is the probability distribution the agent has chosen to use,  $C^\sigma$  is the set of all possible subjective priors,  $\theta$  is a positive constant measure of ambiguity aversion (lower  $\theta$  implies greater ambiguity aversion),  $w$  is a nonuniform weighting function, and  $R(P \parallel q)$  is the weighted relative entropy of  $q$  defined by:

$$R^w(P \parallel q) = \begin{cases} \int w(x) \left( \frac{dP}{dq}(x) \log \left[ \frac{dP}{dq}(x) \right] - \frac{dP}{dq}(x) + 1 \right) dq(x) & \text{if } P \in C^\sigma(q) \\ \infty & \text{otherwise} \end{cases}$$

#### Utility Representation

$$U(f) = \min_{P \in C^\sigma} \int \{ u \cdot f dP + \theta R(P \parallel q) \}$$

### A.7 Case-Based Decision Theory (CBDT) (Epstein & Schneider 2003; Epstein & Schneider 2007)

This model uses a separate notation from the ones above. It depends on a discrete set of similar circumstances in the past that the decision-maker uses to compare to the current situation. Define individual problems and a set of decision problems as  $p, q \in P$ , actions  $a, b \in A$ , outcomes  $r \in R$ , and similar past circumstances in the decision-maker's memory  $M$  with  $n$  individual cases. Each case can be identified as a problem-action-outcome triple  $(p, a, r), (q, b, r) \in C = P \times A \times R$ . The decision-maker has a similarity function  $s[(p, a), (q, b)] \rightarrow \mathbb{R}_+$  which identifies the similarity between

circumstance-act combinations.

**Utility Representation**

$$U_s(a) = \sum_{(q,b,r) \in M} s[(p,a), (q,b)] u(r)$$

**A.8 Recursive Multiple Priors (Epstein and Schneider 2003; Epstein and Schneider 2007)**

Using the same notation as before the CBDT, this model includes a time and state (past realization) component. States are represented by  $s$ , where  $s_t$  represents the state in time  $t$  and  $s^t$  represents all past states up to and including  $t$ ,  $s^t = \{s_1, s_2, \dots, s_{t-1}, s_t\}$ . In this case, the prior set of probability distributions  $C$  is depends on the history of past states of the world, so  $C(s^t)$  and is updated each period.

**Utility Representation**

$$U_t(x; s^t) = \min_{P \in C_t(s^t)} E^P [u(x_t) + \beta u_{t+1}(x; s^t, s_{t+1})]$$

This utility representation could be from any expected utility theory.

## References

- Ahn, David, S. Choi, D. Gale, and S. Kariv. 2009. Estimating ambiguity aversion in a portfolio choice experiment.
- Aldy, J.E., Alan Krupnick, R.G. Newell, I.W.H. Parry, and WA Pizer. 2009. Designing Climate Mitigation Policy. *papers.ssrn.com*. Washington, DC, May.
- Anscombe, Fj, and Rj Aumann. 1963. A definition of subjective probability. *Annals of mathematical statistics* 34, no. 1: 199–205.
- Arrow, K.J., and AC Fisher. 1974. Environmental preservation, uncertainty, and irreversibility. *The Quarterly Journal of Economics* 88, no. 2 (May): 312–319. doi:10.2307/1883074.
- Backus, Dk, Br Routledge, and Se Zin. 2004. Exotic preferences for macroeconomists. *NBER Macroeconomics Annual* 19, no. 2004: 319-390.
- Baker, E. 2005. Uncertainty and learning in a strategic environment: global climate change. *Resource and Energy Economics* 27, no. 1 (January): 19-40. doi:10.1016/j.reseneeco.2004.05.002.
- Baroang, Kye Mesa, Molly Hellmuth, and Paul Block. 2009. Identifying uncertainty and defining risk in the context of the WWDR-4. Perugia, Italy.
- Basili, M, a Chateauneuf, and F Fontini. 2008. Precautionary principle as a rule of choice with optimism on windfall gains and pessimism on catastrophic losses. *Ecological Economics* 67, no. 3 (October): 485-491. doi:10.1016/j.ecolecon.2007.12.030.
- Billot, Antoine, Itzhak Gilboa, Dov Samet, and David Schmeidler. 2005. Probabilities as similarity-weighted frequencies. *Econometrica* 73, no. 4 (July): 1125-1136. doi:10.1111/j.1468-0262.2005.00611.x.
- Bossaerts, P., P. Ghirardato, S. Guarnaschelli, and W.R. Zame. 2010. Ambiguity in asset markets: theory and experiment. *Review of Financial Studies* 23, no. 4: 1325-1359. doi:10.1013/rfs/hhp106.
- Budescu, D.V., and Sara Templin. 2008. Valuation of vague prospects with mixed outcomes. *Decision Modeling and Behavior in Complex and Uncertain Environments*: 253–275.
- Cabantous, Laure. 2007. Ambiguity Aversion in the Field of Insurance: Insurers' Attitude to Imprecise and Conflicting Probability Estimates. *Theory and Decision* 62, no. 3 (January): 219-240. doi:10.1007/s11238-006-9015-1.
- Camerer, Colin, and Martin Weber. 1992. Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty* 5, no. 4 (October): 325-370. doi:10.1007/BF00122575.
- Chakravarty, Sujoy, and Jaideep Roy. 2008. Recursive expected utility and the separation of attitudes towards risk and ambiguity: an experimental study. *Theory and Decision* 66, no. 3 (June): 199-228. doi:10.1007/s11238-008-9112-4.
- Choi, Syngjoo, Raymond Fisman, Douglas Gale, and Shachar Kariv. 2007. Consistency and heterogeneity of individual behavior under uncertainty. *The American economic review* 97, no. 5: 1921–1938.
- Claussen, Martin, Claudia Kubatzki, Victor Brovkin, Andrey Ganopolski, Philipp Hoelzmann, and Hans-Joachim Pachur. 1999. Simulation of an abrupt change in Saharan vegetation in the Mid-

- Holocene. *Geophysical Research Letters* 26, no. 14: 2037. doi:10.1029/1999GL900494.
- Cline, W.R. 2007. *Global warming and agriculture: Impact estimates by country*. Public Policy. Peterson Institute.
- Dessai, S., and Jp Van Der Sluijs. 2007. Uncertainty and Climate Change Adaptation-a Scoping Study. *Copernicus Institute, Utrecht University*: 1-95.
- Dessai, Suraje, and Mike Hulme. 2004. Does climate adaptation policy need probabilities. *Climate Policy* 4, no. 2: 107–128.
- Di Mauro, Carmela, and Anna Maffioletti. 2004. Attitudes to risk and attitudes to uncertainty: experimental evidence. *Applied Economics* 36, no. 4 (March): 357-372. doi:10.1080/00036840410001674286.
- Easterling, D. R. 2000. Climate Extremes: Observations, Modeling, and Impacts. *Science* 289, no. 5487 (September): 2068-2074. doi:10.1126/science.289.5487.2068.
- Edenhofer, O., K. Lessmann, C. Kemfert, M. Grubb, and J. Köhler. 2006. Induced technological change: Exploring its implications for the economics of atmospheric stabilization: Synthesis report from the innovation modeling comparison project. *Energy Journal* 27, no. Special Issue: Endogenous Technological Change and the Economics of Atmospheric Stabilisation (August): 57-106.
- Eichberger, J, and A Guerdjikova. 2010. Ambiguity, Data and Preferences for Information—A Case-Based Approach. *arts.cornell.edu*: 1-57.
- Ellsberg, Daniel. 1961. Risk, Ambiguity, and the Savage Axioms. *The Quarterly Journal of Economics* 75, no. 4 (November): 643. doi:10.2307/1884324.
- Epstein, L.G., and M Schneider. 2003. Recursive multiple-priors. *Journal of Economic Theory* 113, no. 1: 1–31.
- Epstein, L.G., and Martin Schneider. 2007. Learning under ambiguity. *Review of Economic Studies* 74, no. 4: 1275–1303.
- . 2008. Ambiguity, information quality, and asset pricing. *The Journal of Finance* 63, no. 1: 197.
- . 2010. Ambiguity and asset markets. *Annual Review of Financial Economics* 2, no. 1 (January): 1325-1359. doi:10.1093/rfs/hhp106.
- Epstein, Larry G. 2010. Three Paradoxes for the “Smooth Ambiguity” Model of Preferences.
- Fischer, Carolyn, and R.D. Morgenstern. 2006. Carbon Abatement Costs: Why the Wide Range of Estimates? *The Energy Journal* 27, no. 2: 73.
- Fischer, Carolyn, and Richard G. Newell. 2008. Environmental and technology policies for climate mitigation. *Journal of Environmental Economics and Management* 55, no. 2 (March): 142-162. doi:10.1016/j.jeem.2007.11.001.
- Fisher, AC, and U. Narain. 2003. Global warming, endogenous risk, and irreversibility. *Environmental and Resource Economics* 25, no. 4: 395–416. doi:10.1007/BF00324689.
- Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci. 2004. Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory* 118, no. 2: 133–173.
- Gilboa, I. 1987. Expected utility with purely subjective non-additive probabilities. *Journal of*

- Mathematical Economics* 16, no. 1: 65-88. doi:10.1016/0304-4068(87)90022-X.
- Gilboa, I, D. Schmeidler, and P.P. Wakker. 2002. Utility in Case-Based Decision Theory. *Journal of Economic Theory* 105, no. 2 (August): 483–502. doi:10.1006/jeth.2001.2858.
- Gilboa, Itzhak, Offer Lieberman, and David Schmeidler. 2006. Empirical Similarity. *Review of Economics and Statistics* 88, no. 3 (August): 433-444. doi:10.1162/rest.88.3.433.
- Gilboa, Itzhak, and David Schmeidler. 1989. Maxmin Expected Utility with Non-Unique Prior. *Journal of Mathematical Economics* 18, no. 2: 141-153. doi:10.1016/0304-4068(89)90018-9.
- . 1995. Case-based decision theory. *The Quarterly Journal of Economics* 110, no. 3 (August): 605–639. doi:10.2307/2946694.
- Gollier, Christian, and N. Treich. 2003. Decision-making under scientific uncertainty: The economics of the precautionary principle. *Journal of Risk and Uncertainty* 27, no. 1: 77–103.
- Guerdjikova, A. 2008. Case-based learning with different similarity functions. *Games and Economic Behavior* 63, no. 1 (May): 107-132. doi:10.1016/j.geb.2007.10.004.
- Halevy, Yoram. 2007. Ellsberg Revisited: An Experimental Study. *Econometrica* 75, no. 2 (March): 503-536. doi:10.1111/j.1468-0262.2006.00755.x.
- Hall, Jim, Guangtao Fu, and Jonathan Lawry. 2007. Imprecise probabilities of climate change: aggregation of fuzzy scenarios and model uncertainties. *Climatic Change* 81, no. 3-4 (January): 265-281. doi:10.1007/s10584-006-9175-6.
- Halsnæs, K., P. Shukla, D. Ahuja, G. Akumu, R. Beale, J. Edmonds, C. Gollier, et al. 2007. Framing Issues. In *Climate Change 2007: Mitigation*, ed. B. Metz, O. R. Davidson, P. R. Bosch, R. Dave, and L. A. Meyer, 15:261-71. Cambridge: Cambridge University Press, December. doi:10.1007/s10728-007-0063-7.
- Hanany, Eran, and Peter Klibanoff. 2009. Updating ambiguity averse preferences. *The B.E. Journal of Theoretical Economics* 9, no. 1: 1-51.
- Hansen, L.P., and T.J. Sargent. 2001. Robust control and model uncertainty. *American Economic Review* 91, no. 2: 60–66.
- Harding, Ronnie, and E Fisher, eds. 1999. *Perspectives on the precautionary principle. Human and Ecological Risk Assessment: An International Journal*. Vol. 6. Sydney: The Federation Press.
- Hayashi, Takashi, and Ryoko Wada. 2008. Choice with imprecise information: an experimental approach. *Theory and Decision* (September). doi:10.1007/s11238-008-9119-x.
- Heal, Geoffrey, and Bengt Kriström. 2002. Uncertainty and climate change. *Environmental and Resource Economics* 22, no. 1 (June): 3–39. doi:10.1023/A:1015556632097.
- Held, Hermann, Elmar Kriegler, Kai Lessmann, and Ottmar Edenhofer. 2009. Efficient climate policies under technology and climate uncertainty. *Energy Economics* 31: S50-S61. doi:10.1016/j.eneco.2008.12.012.
- Hennlock, Magnus. 2009. Robust Control in Global Warming Management: An Analytical Dynamic Integrated Assessment. *papers.ssrn.com*. Washington, DC.
- Ingham, Alan, Jie Ma, and Alistair Ulph. 2007. Climate change, mitigation and adaptation with uncertainty and learning. *Energy Policy* 35, no. 11 (November): 5354-5369.

doi:10.1016/j.enpol.2006.01.031.

- IPCC. 2007a. *Climate Change 2007: The Physical Science Basis*. Ed. S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis, K.B. Averyt, M. Tignor, and H.L. Miller, Jr. Cambridge: Cambridge University Press.
- . 2007b. *Climate Change 2007: Impacts, Adaptation and Vulnerability*. Ed. ML Parry, O.F. Canziani, J.P. Palutikof, P.J. Van Der Linden, and C.E. Hanson. Cambridge: Cambridge University Press.
- Jaffe, Ab, R Newell, and Rn Stavins. 2003. Technological change and the environment. *Handbook of environmental economics* 43, no. 2-3 (December): 105-126. doi:10.1016/S0921-8009(02)00209-4.
- . 2005. A tale of two market failures: Technology and environmental policy. *Ecological Economics*, no. October.
- Keller, K, B Bolker, and D Bradford. 2004. Uncertain climate thresholds and optimal economic growth. *Journal of Environmental Economics and Management* 48, no. 1 (July): 723-741. doi:10.1016/j.jeem.2003.10.003.
- Kelly, D L, C D Kolstad, and G T Mitchell. 2005. Adjustment costs from environmental change. *Journal of Environmental Economics and Management* 50: 468-495.
- Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji. 2005. A smooth model of decision making under ambiguity. *Econometrica* 73, no. 6 (November): 1849–1892. doi:10.1111/j.1468-0262.2005.00640.x.
- . 2009a. On the smooth ambiguity model: a reply. *Series Working Papers*.
- . 2009b. Recursive smooth ambiguity preferences. *Journal of Economic Theory* 144, no. 3 (May): 930-976. doi:10.1016/j.jet.2008.10.007.
- Lange, Andreas, and Nicolas Treich. 2008. Uncertainty, learning and ambiguity in economic models on climate policy: some classical results and new directions. *Climatic Change*: 7-21. doi:10.1007/s10584-008-9401-5.
- Leach, Andrew. 2007. The climate change learning curve. *Journal of Economic Dynamics and Control* 31, no. 5 (May): 1728-1752. doi:10.1016/j.jedc.2006.06.001.
- Leippold, Markus, Fabio Trojani, and Paolo Vanini. 2008. Learning and Asset Prices Under Ambiguous Information. *Review of Financial Studies* 21, no. 6 (October): 2565-2597. doi:10.1093/rfs/hhm035.
- Lempert, Robert, and Myles T Collins. 2007. Managing the risk of uncertain threshold responses: comparison of robust, optimum, and precautionary approaches. *Risk analysis* 27, no. 4 (August): 1009-26. doi:10.1111/j.1539-6924.2007.00940.x.
- Lempert, Robert, J. Scheffran, and D.F. Sprinz. 2009. Methods for Long-Term Environmental Policy Challenges. *Global Environmental Politics* 9, no. 3: 106–133.
- Löschel, A. 2002. Technological change in economic models of environmental policy: a survey. *Ecological Economics* 43, no. 2-3 (December): 105–126. doi:10.1016/S0921-8009(02)00209-4.
- Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini. 2006. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica* 74, no. 6: 1447–1498.
- Margulis, Sergio, and Urvashi Narain. 2010. The Costs to Developing Countries of Adapting to Climate

Change. *aedd.fr*.

- Mukerji, S. 1997. Understanding the nonadditive probability decision model. *Economic Theory* 46 (June): 23-46. doi:10.1073/pnas.1002194107.
- Nordhaus, Wd. 2002. Modeling induced innovation in climate-change policy. In *Technological change and the environment*, ed. Arnulf Grübler, Nebojsa Nakicenovic, and WD Nordhaus. Resources for the Future.
- Nordhaus, W. 2009. An analysis of the dismal theorem. *papers.ssrn.com*.
- Oppenheimer, Michael, Brian C. O'Neill, Mort Webster, and Shardul Agrawala. 2007. The limits of consensus. *Science* 317, no. 5844 (September): 1505-1506. doi:10.1126/science.1144831.
- O'Riordan, Timothy, and Andrew Jordan. 1995. The precautionary principle in contemporary environmental politics. *Environmental Values* 4, no. 3 (August): 191-212. doi:10.3197/096327195776679475.
- Pindyck, R. S. 2007. Uncertainty in Environmental Economics. *Review of Environmental Economics and Policy* 1, no. 1 (January): 45-65. doi:10.1093/reep/rem002.
- Pizer, W a, and David Popp. 2008. Endogenizing technological change: Matching empirical evidence to modeling needs. *Energy Economics* 30, no. 6 (November): 2754-2770. doi:10.1016/j.eneco.2008.02.006.
- Pratt, John W. 1964. Risk Aversion in the Small and in the Large. *Econometrica* 32, no. 1/2 (January): 122. doi:10.2307/1913738.
- Quiggin, John. 2005. The precautionary principle in environmental policy and the theory of choice under uncertainty. *Political Science*.
- Refsgaard, J, J Vandersluijs, A Hojberg, and P Vanrolleghem. 2007. Uncertainty in the environmental modelling process – A framework and guidance. *Environmental Modelling & Software* 22, no. 11 (November): 1543-1556. doi:10.1016/j.envsoft.2007.02.004.
- Schmeidler, David. 1989. Subjective probability and expected utility without additivity. *Econometrica: Journal of the Econometric Society* 57, no. 3 (May): 571-587. doi:10.2307/1911053.
- Shaw, W. Douglass, and Richard T. Woodward. 2008. Why environmental and resource economists should care about non-expected utility models. *Resource and Energy Economics* 30, no. 1 (January): 66-89. doi:10.1016/j.reseneeco.2007.05.001.
- Skees, JR, BJ Barnett, and Benjamin Collier. 2008. Agricultural Insurance: Background and Context for Climate Adaptation Discussions. In *Prepared for the OECD Expert Workshop on "Economic Aspects of Adaptation."*
- Skees, J, B Barnett, and Jason Hartell. 2005. Innovations in government responses to catastrophic risk sharing for agriculture in developing countries.
- Stern, Nicholas, ed. 2007. *Stern Review: The economics of climate change*. Cambridge: Cambridge University Press.
- Sullivan, R., and William Blyth. 2006. *Climate change policy uncertainty and the electricity industry: implications and unintended consequences*. EEDP BP. Vol. 6.
- Ulph, A., and D Ulph. 1997. Global warming, irreversibility and learning. *The Economic Journal* 107,



no. 442: 636–650.

Weitzman, Martin L. 2009a. On modeling and interpreting the economics of catastrophic climate change. *The Review of Economics and Statistics* 91, no. 1: 1–19.

---. 2009b. Additive damages, fat-tailed climate dynamics, and uncertain discounting. *Economics Discussion Papers*: 1-24.