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Human Recognition in Economic Development:
A Theoretical Model

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Abstract: This paper presents a model of human recognition, a concept defined as the acknowledgement provided to an individual that he is of inherent value with intrinsic qualities in common with the recognizer. The model describes provision and receipt of human recognition, its contribution to utility, its effects on health and labor supply, and the role it plays in development programs. The model provides a theoretical basis for understanding human recognition, lays the foundation for empirical study, and offers an example of how non-material components of development can be formally modeled. Key predictions from the model are that human recognition has a positive, causal relationship with utility, health outcomes, and labor supply; that multiple equilibria for human recognition can exist, and groups can be stuck in low-level equilibria; and that only accounting for the instrumental effects recognition has on material outcomes while ignoring its direct effects on utility leads to suboptimal programs.

JEL Codes: I31, O15, I14

Keywords: human recognition, dehumanization, economic development, health, poverty, program model, well-being, dignity, respect

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1. Introduction

Human recognition is defined as the acknowledgement provided to an individual by other individuals, groups, or organizations that he is of inherent value with intrinsic qualities in common with the recognizer (Castleman 2011). It is hypothesized that this concept of recognition as a fellow human being plays a significant role in social and economic development processes and outcomes.

This paper develops a theoretical economic model of human recognition to formally describe human recognition, provide the framework for quantifying recognition transactions, and generate specific hypotheses for empirical testing. Human recognition is a challenging concept to model because there may not be visible costs or benefits associated with human recognition transactions. This initial model of human recognition describes several aspects of human recognition and may also serve as a basis for modeling other non-material dimensions of development such as empowerment, social capital, or social exclusion. An earlier paper (Castleman 2011) reviewed work done by others studying non-material dimensions of development, and where possible this paper builds on that work.

The model in this paper describes the flow and stock of human recognition, determinants of human recognition provision, the contribution of human recognition to utility, the effects human recognition has on health and labor supply, and the role human recognition plays in program design and outcomes. The primary predictions from the theory are: human recognition levels significantly affect utility, health outcomes and labor supply; these relationships are positive; there is complementarity in human recognition provision, which under certain circumstances leads to multiple equilibria with a low-level equilibrium where people mostly provide negative recognition to others and a high-level equilibrium where people mostly provide positive recognition; and full consideration of human recognition in the design of interventions improves program outcomes.

Section II describes determinants of the human recognition that an individual receives and presents a functional relationship between received recognition and total recognition. Section III models the provision of human recognition and examines conditions for multiple equilibria of human recognition. The contribution of human recognition to utility is modeled in Section IV, and a specific case of recognition's contribution to labor supply is modeled in Section V. Section VI incorporates human recognition into a model of development programs, and Section VII concludes.

II. Receipt of Human Recognition

Received recognition

Individuals receive human recognition from other individuals, and in some cases from groups or organizations. Human recognition may be positive or negative. Positive recognition refers to acknowledgment that an individual is of value as a human being, and negative human recognition refers to viewing an individual as lacking inherent value as a human being or not acknowledging this value. A previous paper (Castleman 2011) provides a more in-depth description and analysis of the nature of human recognition.

The total quantity of human recognition an individual receives is a function of the recognition received from each of the individuals, groups, and organizations that provide recognition to her. Based on observed behavior and understanding of the nature of human recognition, it is posited that the expression for the level of human recognition an individual receives should satisfy the four properties described below.

Property 1 – MONOTONICITY: *For an individual i who receives recognition from a vector of individuals (1, 2 …n) where each providing individual provides a quantity of recognition,* q_h, *h = 1…n and* $q \in \mathbb{R}$, *an increase in the quantity of human recognition* q_j *provided by one individual j, holding all other values of* q_h *constant, increases the total recognition received by i,* r_{i_r} .

Property 1 states that holding all else equal, increasing the magnitude of positive recognition (or decreasing the magnitude of negative recognition) that one individual provides to i will increase the total recognition received by i.

A corollary property is given below for the special case in which an individual receives the same quantity of recognition from all individuals who provide recognition to him/her.

Property 1A - MONOTONICITY: *For an individual i who receives a given quantity of human recognition, q, from each of a vector of individuals (1, 2 …n) such that* $q \in \mathbb{R}$, *and q is the same for each individual* ($q_1 = q_2 \ldots = q_n$), *the magnitude of the total human recognition received by i,* r_{i_r} *, is directly proportional to the number of individuals providing recognition to him, i.e. the magnitude of* r_{i_r} *will be larger for larger n.*

For example, receiving five units[1] of positive recognition from 20 people (and no

[1] Practically speaking, measurable units of recognition do not exist in the same sense that measurable units of income or education do, but quantifying units offers a useful structure for the model, and as demonstrated in other papers (Castleman 2011b; Castleman 2011c), human recognition levels can be quantified empirically.

recognition transactions with anyone else) will result in a higher level of total received recognition than receiving five units of positive recognition from only one person (and no recognition transactions with anyone else). Note that the property refers to magnitudes and in the case of negative recognition ($q < 0$), increasing the number of individuals will decrease the level but increase the magnitude of total recognition received.

Property 1 describes the special case in which an individual receives the same quantity of human recognition from each of the individuals providing recognition to him, but the property applies to more general cases as well. Consider a set of individuals (e.g. one neighbor, one colleague, and one relative), each of whom provides a fixed quantity of recognition to individual *i*. Neighbors may provide different quantities of recognition than colleagues, but every neighbor provides the same quantity of human recognition to *i*. If Property 1 holds, then the magnitude of recognition received will be greater the more sets of individuals that *i* interacts with.

Property 2 - DIMINISHING RETURNS TO ADDITIONAL PROVIDERS: *For an individual i who receives a given quantity of human recognition, q, from each of a vector of individuals (1, 2 …n) such that* $q \in \mathbb{R}$ *and q is the same for each individual* ($q_1 = q_2 \ldots = q_n$)*, the effect an additional input of q quantity of recognition by an additional individual, n+1, has on the magnitude of total human recognition received by i,* $|r_{i_r}|$*, is inversely proportional to the number of individuals providing recognition to him, i.e. the magnitude of the change in* r_{i_r} *will be smaller for larger n.*

The intuition behind this corollary is that receiving recognition from a larger number of people insulates an individual, dampening the effect of any single human recognition transaction. Receiving five units of recognition from someone will have a greater effect on an individual if that is the only person from whom she receives recognition than it will if she is already receiving the same or similar quantities of recognition from others.

This property can also be expressed in terms of a given quantity of recognition received.

Property 2A – DIMINISHING RETURNS TO ADDITIONAL PROVIDERS: *For a given quantity, Q, of human recognition received by individual i from a vector of individuals (1, 2 …n) such that* $Q = \sum_{h=1}^{n} q_h$ *where* $q \in \mathrm{R}$ *and where there is no restriction that* $q_1 = q_2 \ldots = q_n$*, the magnitude of the total human recognition received by i,* r_{i_r}*, is inversely proportional to the number of individuals providing recognition to him, i.e. the magnitude of* r_{i_r} *will be smaller for larger n.*

This property means, for example, that receiving one unit of human recognition from each of ten people leads to less total recognition received than receiving ten units of recognition from one person. The intuition behind this property is that receipt of large quantities of positive recognition (e.g. an empathetic sacrifice) or negative recognition (e.g. violence or severe humiliation) has a larger effect on the receiving individual than repeated receipt of small quantities of recognition from various individuals.

Property 3 – EQUIVALENCE OF PROVIDERS NOT REQUIRED: *If individuals j and h each provide the same quantity of recognition, r, to individual i, the effects of j's and h's provision of recognition on the total level of recognition individual i receives,* r_{i_r}*, will not necessarily be equal and will* *depend on the relationship between the providing individual and i.*

The intuition behind this property is clear. For example, the same recognition provided by a spouse or parent may have greater weight than that provided by a neighbor, shopkeeper, or stranger.

Property 4 – INCREASING EFFECTS OF DIFFERENCES IN RECEIVED RECOGNITION: *For an individual i who receives a quantity, Q, of recognition from a vector of individuals (1, 2 …n) such that* $Q = \sum_{h=1}^{n} q_h$ *where* $q \in \mathrm{R}$ *and where there is no restriction that* $q_1 = q_2 \ldots = q_n$, *the magnitude of the effect that recognition provided by an additional individual,* q_{n+1}, *has on total human recognition received by i,* r_{i_r}, *is directly proportional to the magnitude of the difference between the new recognition received,* q_{n+1} *and the net quantity of recognition currently received, Q, for all cases in which* $|q_{n+1}| > |Q|$.

That is, if $|q_{n+1}| > |Q|$, then $|q_{n+1} - Q| \uparrow \Rightarrow |\Delta r_{i_r}| \uparrow$,

where $\Delta r_{i_r} = r_{i_r}(1....n+1) - r_{i_r}(1....n)$.

This property states that in cases where an additional input of recognition is of greater magnitude than the level of recognition otherwise received, the input of recognition has a greater impact on an individual the greater the magnitude of the difference between the new input and the other received recognition inputs[2]. This property is of particular relevance in developing country settings where provision of

[2] The condition $|q_{n+1}| > |Q|$ is necessary because inputs of small quantities of recognition may not have a significant effect on total recognition received even if it is much less than the quantity of recognition otherwise received. For example, if Q=20, and additional input of $q_{n+1} = 30$ is likely to have greater effect than $q_{n+1} = 5$, even though the magnitude of the difference $|q_{n+1} - Q|$ is greater for $q_{n+1} = 5$ than it is for $q_{n+1} = 30$.

positive recognition may have the greatest impact on individuals who otherwise receive significant negative recognition from others. Consider a poor woman who toils under inhumane conditions at a factory and is the subject of regular domestic abuse at home. If someone, say a health care worker, provides her with a large quantity of negative human recognition and treats her in a humiliating manner, the change in the total level of recognition the woman receives may be relatively small. But if the health care worker provides her a high magnitude of positive recognition – asking about and addressing her concerns, providing counseling, etc. – there may be a significant change in the total level of recognition the woman receives.

The following proposition identifies an expression for human recognition that satisfies these properties:

Proposition 1: *The expression,* $r_{i_r} = \frac{1}{\sqrt{n}} \sum_{h=1}^{n} \rho_{hi} r_{hi}$ *, satisfies Properties 1 – 4.*

The term r_{hi} is the recognition that individual *h* provides to individual *i*, and ρ_{hi} is a parameter that represents a provider-specific weight that captures differences in the impact a given level of provided recognition has on individual *i*'s received recognition. In addition to capturing differences between providers (e.g. between a spouse and neighbor), ρ_{hi} also captures differences among receiving individuals in how they convert provision of a given level of provided recognition into received recognition. Individuals may vary in their inherent resilience or "set points" for recognition, which may affect how they absorb recognition provided by others, e.g. some may brush off negative recognition provided by others while others may take it to heart.

During the period of analysis, individual *i* interacts with *n* individuals who provide varying levels of human recognition to her. An individual likely interacts with more than *n* individuals during the period, but only interactions that involve receipt of human recognition are included in *n*. Groups, organizations, or institutions that provide recognition through processes or mechanisms other than one-to-one interactions can also be included in the *n* entities, but for simplicity in the model we refer to the *n* entities as individuals. This model combines human recognition received from all sources into one expression. Another paper (Castleman 2011b) focuses on measurement and distinguishes among recognition received in different domains of one's life.

The *r* terms may be positive or negative, signifying positive or negative human recognition. The expression

$$\frac{1}{\sqrt{n}}\sum_{h=1}^{n}\rho_{hi}r_{hi}$$

represents the weighted sum of human recognition received through interactions with others.

Proof of Proposition 1:

1. MONOTONICITY: To prove monotonicity, we need to show that for the conditions described in Property 1, an increase in q_j leads to an increase in r_{i_r}, or

   $$\frac{\partial r_{i_r}}{\partial q_j} > 0.$$

   Let $Q_0 = \sum_{h=1}^{n}\rho_h r_{hi} - \rho_j r_{ji}$. That is, Q_0 represents all recognition inputs that *i* receives except for *j*'s input.

   $$r_{i_r} = \frac{Q_0 + q_j}{\sqrt{n}}.$$

$\frac{\partial r_{i_r}}{\partial q_j} = \frac{1}{\sqrt{n}} > 0$. The Q_0 term can be treated as a constant in the differentiation because it does not change with q_j. QED.

1A. We need to show that $\frac{\partial | r_{i_r} |}{\partial n} > 0$ for the case described in Property 1A.

Using the above expression for r_{i_r} and the conditions given in Property 1A,

$q_h = \rho_h r_{hi}$ and $q_1 = q_2 \ldots = q_n$.

So $r_{i_r} = \frac{1}{\sqrt{n}} nq$.

$$\frac{\partial r_{i_r}}{\partial n} = \frac{\partial(\frac{1}{\sqrt{n}} nq)}{\partial n} = \frac{q}{2\sqrt{n}}.$$

When q > 0, $r_{i_r} > 0$. When q < 0, $r_{i_r} < 0$. For q > 0, $\frac{\partial r_{i_r}}{\partial n} = \frac{q}{2\sqrt{n}} > 0$.

For q < 0, $\frac{\partial r_{i_r}}{\partial n} = \frac{q}{2\sqrt{n}} < 0$. In both cases $\frac{\partial | r_{i_r} |}{\partial n} > 0$. QED.

2. DIMINSHING RETURNS TO ADDITIONAL PROVIDERS: To prove this property, we need to show that for the conditions described in Property 2,

$$\frac{\partial | \Delta r_{i_r} |}{\partial n} < 0.$$

$$\Delta r_{i_r} = \frac{1}{\sqrt{n+1}}(n+1)q - \frac{1}{\sqrt{n}}(n)q$$

$$\frac{\partial \Delta r_{i_r}}{\partial n} = q(\frac{1}{2\sqrt{n+1}} - \frac{1}{2\sqrt{n}}) = \frac{q}{2}(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}).$$

When q > 0, $\Delta r_{i_r} > 0$. When q < 0, $\Delta r_{i_r} < 0$.

$\frac{1}{\sqrt{n+1}}$ will be smaller than $\frac{1}{\sqrt{n}}$. (n is always positive.)

So for q > 0, $\frac{\partial \Delta r_{i_r}}{\partial n} = \frac{q}{2}(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}) < 0$.

And for q < 0, $\frac{\partial \Delta r_{i_r}}{\partial n} = \frac{q}{2}(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}) > 0$.

In both cases $\frac{\partial |\Delta r_{i_r}|}{\partial n} < 0$. QED.

2A. We need to show that $\frac{\partial |r_{i_r}|}{\partial n} < 0$ for the case described in Property 2A.

Using the above expression for r_{i_r} and the conditions given in Property 2A,

Q = $\sum_{h=1}^{n} \rho_h r_{hi}$.

$r_{i_r} = \frac{1}{\sqrt{n}} Q$. $\frac{\partial r_{i_r}}{\partial n} = \frac{\partial(\frac{1}{\sqrt{n}} Q)}{\partial n} = -\frac{Q}{2} n^{-3/2}$.

When Q > 0, r_{i_r} > 0. When Q < 0, r_{i_r} <0. For Q > 0, $\frac{\partial r_{i_r}}{\partial n} = -\frac{Q}{2} n^{-3/2} < 0$.

For Q < 0, $\frac{\partial r_{i_r}}{\partial n} = -\frac{Q}{2} n^{-3/2} > 0$. In both cases $\frac{\partial |r_{i_r}|}{\partial n} < 0$. QED.

3. EQUIVALENCE OF PROVIDERS NOT REQUIRED: The parameter ρ_{hi} is a provider-specific weight that accounts for differences among providers in the impact a given level of provided recognition has on an individual's received recognition. Since these parameters differ for different receiving individuals (i) as well as for different providing individuals (h), the parameters also capture

differences in receiving individuals' conversion of provided recognition into received recognition.

4. INCREASING EFFECTS OF DIFFERENCES IN RECEIVED RECOGNITION: To prove this property, we need to show that for the conditions given in the property, $\frac{\partial |\Delta r_{i_r}|}{\partial |q_{n+1} - Q|} > 0$.

Let d = q_{n+1} - Q. Given the condition that $|q_{n+1}| > |Q|$, there are two possible cases:

$q_{n+1} > Q \Rightarrow q_{n+1} > 0$, d > 0, $\Delta r_{i_r} > 0$.

$q_{n+1} < Q \Rightarrow q_{n+1} < 0$, d < 0, $\Delta r_{i_r} < 0$.

That is, when $q_{n+1} > Q$, the addition of q_{n+1} increases total received recognition so $\Delta r_{i_r} > 0$, and when $q_{n+1} < Q$, the addition of q_{n+1} decreases total received recognition so $\Delta r_{i_r} < 0$. In both cases, d and Δr_{i_r} are the same signs, so for an increase in the magnitude of d to lead to an increase in the magnitude of r_{i_r} as Property 4 states, it means that an increase in d leads to an increase in r_{i_r}. That is, if $|q_{n+1}| > |Q|$, then $\frac{\partial |\Delta r_{i_r}|}{\partial |d|} > 0 \Leftrightarrow \frac{\partial \Delta r_{i_r}}{\partial d} > 0$.

$$\Delta r_{i_r} = \frac{Q + q_{n+1}}{\sqrt{n+1}} - \frac{Q}{\sqrt{n}}.$$

d = $q_{n+1} - Q$.

By the chain rule, $\frac{\partial \Delta r_{i_r}}{\partial d} = \frac{\partial \Delta r_{i_r}}{\partial Q}\frac{\partial Q}{\partial d} + \frac{\partial \Delta r_{i_r}}{\partial q_{n+1}}\frac{\partial q_{n+1}}{\partial d} + \frac{\partial \Delta r_{i_r}}{\partial n}\frac{\partial n}{\partial d}$.

$$\frac{\partial \Delta r_{i_r}}{\partial Q}\frac{\partial Q}{\partial d} = \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}\right)(-1) > 0$$

$$\frac{\partial \Delta r_{i_r}}{\partial q_{n+1}} \frac{\partial q_{n+1}}{\partial d} = \frac{1}{\sqrt{n+1}} > 0$$

$$\frac{\partial \Delta r_{i_r}}{\partial n} \frac{\partial n}{\partial d} = 0$$

$$\therefore \frac{\partial \Delta r_{i_r}}{\partial d} > 0 \Rightarrow \frac{\partial \,|\, \Delta r_{i_r} \,|}{\partial \,|\, d \,|} > 0. \quad \text{QED.}$$

The expression $r_{i_r} = \frac{1}{\sqrt{n}} \sum_{h=1}^{n} \rho_h r_{hi}$ satisfies Properties 1 – 4.

One way in which the above expression may not accurately reflect actual experience is that the model suggests that for individuals receiving high magnitudes of positive (negative) recognition, the impact of receiving modest quantities of positive (negative) recognition from additional providers is to reduce the magnitude of total recognition received. In some cases this may be true, but generally it is unlikely that receipt of modest levels of positive recognition reduces the level of total recognition received for those already receiving high levels of positive recognition. This issue can be addressed to some extent by introduction of time units for analysis.

However, this issue is related to the summing of recognition received from different individuals and does not affect empirical analysis if measurement of recognition relies on indicators of the incidence of specific types of interactions or relies on self-reported levels of recognition received, instead of relying on a summation of recognition received from each provider. As discussed elsewhere (Castleman 2011b), the empirical measurement methods employed use indicators of specific interactions and self-reported recognition and does not sum over individual interactions. Therefore, the issue described above does not pose problems for the empirical applications.

Total recognition

The r_{i_r} term represents the quantity of human recognition an individual receives in a given period of analysis. Related but distinct from this is an individual's total level of human recognition, which is signified by R_i in the model. The total level of human recognition refers to the overall level of recognition an individual has at a given point of time and is determined by 1) the quantity of human recognition received in the time period of analysis, r_{i_r} ; 2) the quantity of recognition received in the past (including during childhood) which may be discounted to account for its diminishing effect on one's current total level of recognition; and 3) how received recognition accumulates and is "stored".

An individual's total level of recognition, R_i, is determined by a function

$$R_i = f(r_{i_r}, \overline{r_i}) = f(r_{i_r} + \overline{r_i})$$

where $\overline{r_i}$ is the base level of recognition individual *i* has at the beginning of the period of analysis and includes the discounted present value of human recognition received in the past. The function f(..) describes how received recognition accumulates and is stored as total recognition.

Based on understanding of how human recognition transactions operate (Castleman 2011), it is posited that the function f(r), for r = $r_{i_r} + \overline{r_i}$, should satisfy the following properties:

Property 1 – NON-DECREASING IN THE ARGUMENT: $f'(r) \geq 0$ *for all r.*

The function does not decrease in its argument because, consistent with intuition, higher (lower) levels of received or base recognition lead to higher (lower) levels of total recognition. (This could be expressed as "increasing in the argument" except for cases where r = 0 in which case f'(r) may equal 0; see proof below.)

Property 2 - DECREASING MARGINAL EFFECTS FOR HIGHER ABSOLUTE VALUE OF THE ARGUMENT: *f''(r) < 0 for r > 0, and f''(r) > 0 for r < 0. An inflection point exists at r = 0.*

The marginal effects of the function decrease in greater positive values of the argument and decrease in greater magnitudes of negative values of the argument. These second order conditions mean that as the absolute value of r increases, the marginal effect that additional received recognition has on total levels of recognition diminishes. That is, a given input of human recognition has less of an effect on individuals who already have very large positive or negative levels of recognition than it does on those with moderate or low magnitudes of recognition. For example, an instance of being humiliated publicly by one's employer has less of an effect on the total recognition level of an individual who is regularly treated poorly by his employer and family than the same incident would have on someone who is treated moderately well by both employer and family. This is again a case of high magnitudes of human recognition insulating an individual from the effect of additional recognition inputs, or viewed another way, high magnitudes diluting the impact of a given input of recognition.

Proposition 2: *The following functional form satisfies Properties 1 and 2 above:*

$$R = f(r) = \begin{cases} r^{\alpha}, r \geq 0 \\ -[(-r)^{\alpha}], r < 0 \end{cases} \quad \text{where } 0 < \alpha < 1$$

Proof of Proposition 2:

1. NON-DECREASING IN THE ARGUMENT:

f'(r) = $\begin{cases} \alpha r^{\alpha-1} \geq 0, r \geq 0 \\ \alpha(-r)^{\alpha-1} > 0, r < 0 \end{cases}$ QED.

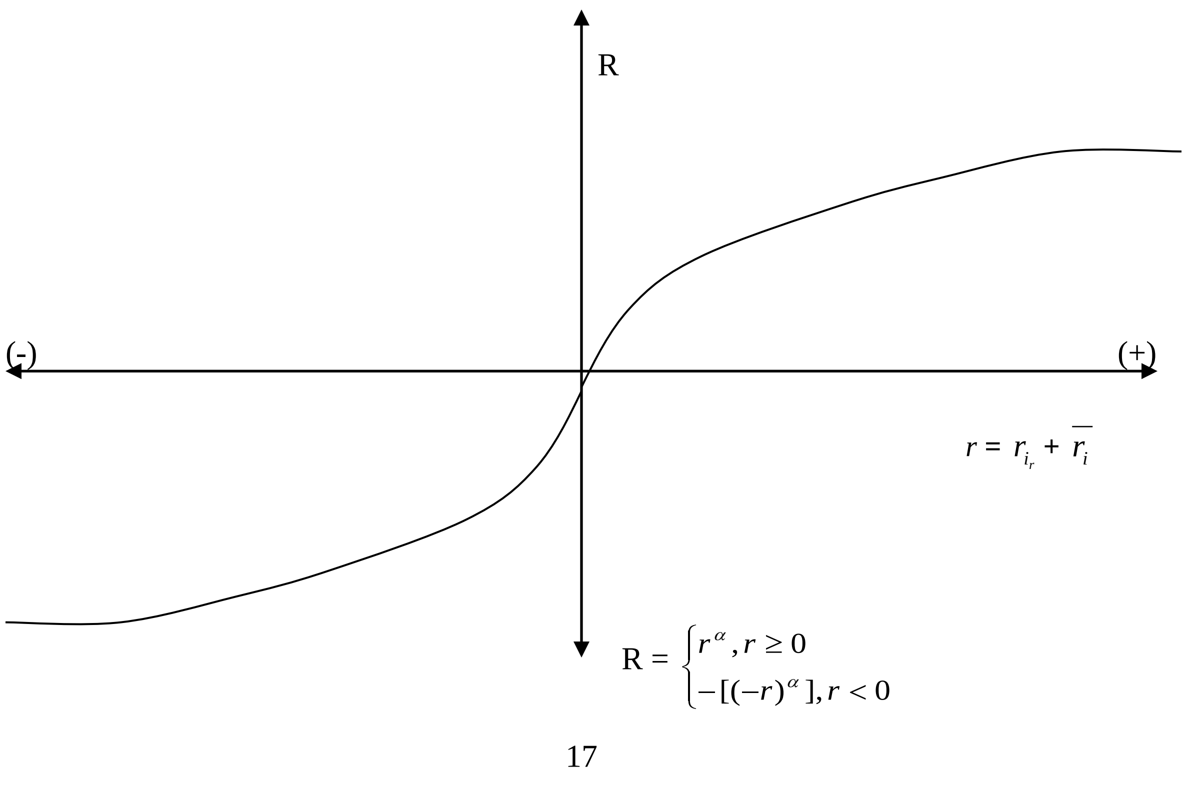
2. DECREASING MARGINAL EFFECTS FOR HIGHER ABSOLUTE VALUE OF THE ARGUMENT:

f''(r) = $\begin{cases} \alpha(\alpha-1)r^{\alpha-2} < 0, r > 0 \\ -\alpha(\alpha-1)(-r)^{\alpha-2} > 0, r < 0 \end{cases}$

For r > 0, as r increases (and |r| increases), f''(r) decreases; for r < 0, as r decreases (and |r| increases), f''(r) decreases, meeting the condition of decreasing marginal effects for higher absolute value of r. QED.

Figure 1 graphs this function.

Figure 1: Relationship between an Individual's Received and Base Levels (r) and Total Level (R) of Human Recognition



When health and consumption functions are specified in Section IV, the R function is assumed to be asymptotic, with a maximum magnitude R'. For the general model, however, the function is not required to be asymptotic.

One issue of relevance to development settings is that when an individual has a high level of negative recognition (deep in the 4th quadrant) the marginal effect of an input of negative recognition is less than the marginal effect of an equivalent input of positive recognition. The difference in these effects grows larger the greater the magnitude of negative recognition the individual has (the deeper into the 4th quadrant one is located). An example of this is the case described earlier of asymmetry between the effects a health worker's positive and negative recognition have on an otherwise poorly treated woman.

In the model, this asymmetry between positive and negative inputs stems from two sources. First, according to Property 4 of the expression for received recognition, the greater the difference between a new input of recognition and the existing level of received recognition, the greater the new input's impact will be (for cases where the magnitude of the new input is greater than the existing level of inputs). So when an individual who has received large quantities of negative recognition receives an additional large input of negative recognition, its effect on the level of recognition received is significantly less than the effect of an additional input of positive recognition of the same magnitude. The level of received recognition is then translated into an individual's total level of recognition by the f(r) function, which is an increasing function. Therefore, at negative (or positive) magnitudes of total recognition there is asymmetry between the effects an input of negative recognition and an input of positive recognition of the same magnitude will have on total recognition level. At negative recognition levels positive inputs will have a greater effect than negative inputs of the same magnitude, and at positive recognition levels negative inputs will have a greater effect.[3]

The second source of asymmetry is the second order condition of the f(r) function. Because f'(r) decreases as the magnitude of r increases, for an individual receiving net positive recognition (quadrant 1 of the graph) the marginal effect of an additional input of positive recognition is less than the marginal effect of the same quantity of negative recognition; and vice versa for an individual receiving net negative recognition (quadrant 4).

This model of receipt of recognition lays the foundation for methods to measure receipt of human recognition, and the model can be built on to develop a method for empirically measuring receipt of human recognition (Castleman 2011b).

III. Provision of Human Recognition

In addition to describing receipt of human recognition, a model of human recognition transactions must also describe provision of human recognition. In this model individuals choose how much recognition to provide to others by balancing the marginal benefits and marginal costs of recognition provision. There are also likely to be

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[3] A simple numerical example helps to illustrate these points. Suppose an individual receives -5 units of human recognition from 8 individuals all of whom have weights of $\rho = 1$. Applying the expression for received recognition, the individual's level of received recognition is r_{i_r} = -14.1. Assuming a base level of recognition of zero and $\alpha = 0.5$, his total level of recognition is $R_i = -3.76$. If someone then provides him with -8 units of human recognition, his new r_{i_r} = -16 and his new $R_i = -4$. If instead of -8 someone provides +8 units of human recognition, the individual's new r_{i_r} = -10.67 and his new $R_i = -3.27$. The positive recognition input leads to a change in total recognition level (+0.49) that is twice the magnitude of the change in total recognition (-0.24) from the negative recognition input.

exogenous determinants of human recognition provision that are independent of benefits and costs, such as an individual's personality, cultural factors, or conditioning by role models in the individual's past or present environment. These exogenous determinants may be quite important and affect the total quantity of recognition provided and can be influenced by interventions such as counseling, other psychosocial interventions, or deliberate introduction of role models. However, these determinants are not included in the model, which focuses on individual choices of recognition provision based on benefits and costs in order to enable more meaningful analysis of human recognition transactions.

The amount of human recognition individual *i* provides to others is represented by r_{ij}, for j = 1….m, where *i* interacts with *m* individuals to whom she has the opportunity to provide human recognition. r_{ij} can be positive or negative. Note that *m* does not necessarily equal *n* from the previous section because the people from whom one receives recognition do not necessarily coincide with those to whom one provides recognition. Individuals often have the greatest opportunity and choice about provision of recognition to those with less power than oneself (Castleman 2011) so the individuals one receives recognition from may differ from those one provides recognition to. The model allows an individual to provide different levels of human recognition to different individuals, so values of r_{ij} can vary for j = 1…m. The value of r_{ij} is determined by individual *i*'s maximization process. For each individual to whom she provides recognition, *i* chooses the level of human recognition to provide based on her own benefits and costs.

The benefits to individual *i* of providing a given level of human recognition, r_{ij}, to individual *j* are given by:

$$B(r_{ij}) = \mu(r_{ij}) + \Psi(R_i, r_{ij}).$$

The $\mu(r_{ij})$ function represents the material benefits to person *i* of providing human recognition r_{ij} to person *j*. μ' can be positive or negative, depending on the situation. μ' will be negative in cases where provision of negative recognition enables exploitation that materially benefits the individual providing recognition, e.g. inhumane workplace conditions that generate greater profits, or domestic abuse that extracts higher dowry payments. μ' will be positive in cases where provision of positive recognition materially benefits the individual providing it, e.g. a teacher who can improve enrollment or attendance – and consequently increase his salary – by providing students with greater human recognition.

There are two types of cases in which provision of human recognition generates material benefits. The examples given above of using inhumane conditions or abuse to extract greater financial reward are examples of how provision of human recognition (negative in this case) can lead to changes in production or terms of exchange based on factors outside of market mechanisms. In the case of inhumane workplace conditions, provision of negative recognition enables employers to reduce costs of production by, for instance, requiring long hours and offering poor benefits. In the case of dowry extraction, provision of negative recognition enables coercion of additional payment without offering additional commensurate goods or services – other than the implicit offer to reduce or terminate abuse upon receipt of payment.

This type of case requires significant differences in power between the provider and receiver of recognition in order for the former to be able to extract additional material benefits or rent from the latter. And the provision of negative recognition can further reinforce this power differential. If employees have sufficient power or alternative employment options, they may demand higher wages to compensate for the poor conditions, which leads to the second type of case described below. If the wife or her family has sufficient power or alternative options and has cultural and social latitude, she may leave the household rather than tolerate the abuse or pay the additional dowry.

The second type of case is a hedonic market, in which provision of recognition serves as a compensating differential to which prices (including wages), supply, and demand adjust. The labor model presented in Section V is an example of this type of case. Workers choose to provide greater supply of labor if greater levels of human recognition are provided at the workplace, for a given wage and non-labor income. The model predicts that an employer can increase labor inputs by increasing the levels of human recognition provided to employees.

As referred to above, a distinguishing condition that influences which type of case occurs is the power differential between the provider and receiver of recognition. Greater power differentials increase the possibilities for human recognition transactions to generate non-market factors that affect production or terms of exchange, and smaller power differentials increase the likelihood of hedonic markets.

The two types of cases are not mutually exclusive. Situations exist that can be reasonably interpreted as either type of case. The example given above of the teacher whose provision of positive human recognition improves enrollment and attendance, thereby increasing his salary, could be interpreted as a case of recognition being a non-market factor that increases enrollment and teacher compensation. Alternatively, one could interpret the recognition as a compensating differential leading to greater demand for education on the part of students and their families.

The $\Psi(R_i, r_{ij})$ function in the expression of benefits represents the psychic benefits to person *i* of providing human recognition r_{ij} to person *j*. U.S. Supreme Court Justice Thurgood Marshall said, "In recognizing the humanity of our fellow beings, we pay ourselves the highest tribute", which eloquently describes the positive psychic utility that can be gained by providing positive human recognition to others. R_i is in the argument because one's own level of recognition can affect the psychic benefits gained from providing human recognition to others, and the properties of the function Ψ determine the direction and extent of this effect. The sign of the cross-partial $\frac{\partial^2 \Psi}{\partial r_{ij} R_i}$ determines whether there is complementarity in provision of human recognition among individuals, and the magnitude of $\frac{\partial^2 \Psi}{\partial r_{ij} R_i}$ determines the strength of the relationship between one's own recognition level and the recognition one provides to others. The model presented below restricts the value of the cross-partial to $\frac{\partial^2 \Psi}{\partial r_{ij} R_i} > 0$, indicating a positive relationship between one's own level of recognition and the psychic benefits of recognition provided to others. This is based on documented experiences about how the treatment people receive affects their treatment of others. For example, it has been widely documented that those abused as children are more likely to abuse their own children (Oliver 1993), and there is evidence that domestic violence offenders who are treated respectfully by arresting officers are less likely to become repeat offenders (Lind and Tyler 1988). Note that even with this assumption of a positive cross-partial and complementarity, an individual with positive levels of recognition may still choose to provide negative recognition to others (or vice versa) if the material benefits and costs dominate the psychic benefits.

The costs to individual *i* of providing a given level of human recognition, r_{ij}, to person *j* are given by:

$$C(r_{ij}) = \alpha(r_{ij}^2) + \beta(r_{ij}).$$

There are some positive costs, $\alpha(r_{ij}^2)$, associated with providing either positive or negative human recognition to others; these costs may involve time, effort, or other inputs. For these costs, the marginal costs are increasing, as the polynomial term indicates; this reflects situations in which providing small amounts of recognition incurs very little cost but providing larger quantities requires larger investments of inputs. There are also some costs, $\beta(r_{ij})$, that are positive for provision of positive human recognition and negative for provision of negative human recognition. In some situations providing negative recognition incurs fewer short-term costs than providing positive recognition does because treating people well requires greater investment of time or other inputs than treating them poorly does.

An individual determines how much recognition to provide someone by maximizing her own net benefits or payoff. The following simple linear functional forms,

$$\mu(r_{ij}) = \mu r_{ij};\ \Psi(r_i, r_{ij}) = r_{ij}\psi(R_i);\ \alpha(r_{ij}^2) = \alpha r_{ij}^2;\ \beta(r_{ij}) = \beta r_{ij},\ \text{for } \mu, \alpha, \beta > 0,$$

yield the following payoff function:

$$\pi_i(r_{ij}) = \mu r_{ij} + r_{ij}\psi(R_i) - \alpha r_{ij}^2 - \beta r_{ij}.$$

Note that only r_{ij} is a choice variable, not R_i since what the individual has control over is the level of recognition provided to others, not one's own level of recognition. So the individual solves

$$\max_{\{r_{ij}\}} \pi_i(r_{ij}) = \mu r_{ij} + r_{ij}\psi(R_i) - \alpha r_{ij}^2 - \beta r_{ij}.$$

The marginal payoff from provision of human recognition is

$$\frac{\partial \pi_i(r_{ij})}{\partial r_{ij}} = \mu + \psi(R_i) - 2\alpha r_{ij} - \beta.$$

At the optimum, the marginal payoff equals zero by the first-order condition, and

$$r_{ij}^* = (\mu + \psi(R_i) - \beta)/2\alpha.$$

The first order condition indicates that individuals choose how much recognition to provide by balancing the marginal material benefit, the marginal psychic benefit, and the marginal cost of human recognition provision. The model assumes ψ' is positive, so the marginal payoff of providing positive human recognition increases in one's own level of recognition:

$$\frac{\partial \pi_i^2(r_{ij})}{\partial r_{ij} R_i} = \frac{\partial^2 \Psi}{\partial r_{ij} R_i} = \psi'(R_i) > 0.$$

Furthermore, the *magnitude* of the human recognition one provides to others depends on one's own level of human recognition. Higher levels of positive recognition lead one to provide less negative recognition (for situations and parameters with positive R_i and negative r_{ij}) or greater positive recognition (for situations with positive R_i and positive r_{ij}). Greater magnitudes of negative recognition lead one to provide greater amounts of negative recognition (for situations and parameters with negative R_i and negative r_{ij}) or less positive recognition (for situations with negative R_i and positive r_{ij}).

We know from the determinants of R_i,

$$R_i = f(r_{i_r}, \overline{r_i}) = f(r_{i_r} + \overline{r_i}), \text{ where } r_{i_r} = \frac{1}{\sqrt{n}}\sum_{h=1}^{n}\rho_{hi}r_{hi},$$

that an individual's own level of recognition, R_i, depends on the levels of human recognition others provide to her, r_{hi}. Combined with the result from the above comparative static, this suggests there is complementarity in provision of human recognition. Individual *i*'s decision about how much human recognition to provide to *j*, r_{ij}, directly affects *j*'s level of recognition, R_j, which in turn directly and positively affects the level of recognition *j* chooses to provide to others, r_{jk}, r_{jl}, etc. That is,

$$\frac{\partial \pi_j^{\,2}(r_{jk})}{\partial r_{jk} r_{ij}} > 0.$$

This complementarity is the result of an externality in recognition provision: when *i* provides recognition to *j*, it affects the recognition *j* will provide to *k* and *l*, but *i* does not include this result of his action when determining how much human recognition to provide to others. The complementarity occurs in both positive and negative directions. Greater positive magnitudes of r_{ij} increase the benefits to *j* of providing greater positive magnitudes (or smaller negative magnitudes) of human recognition to others, r_{jk}; greater negative magnitudes of r_{ij} increase the benefits to *j* of providing greater negative magnitudes (or smaller positive magnitudes) of recognition to others, $r_{jk.}$

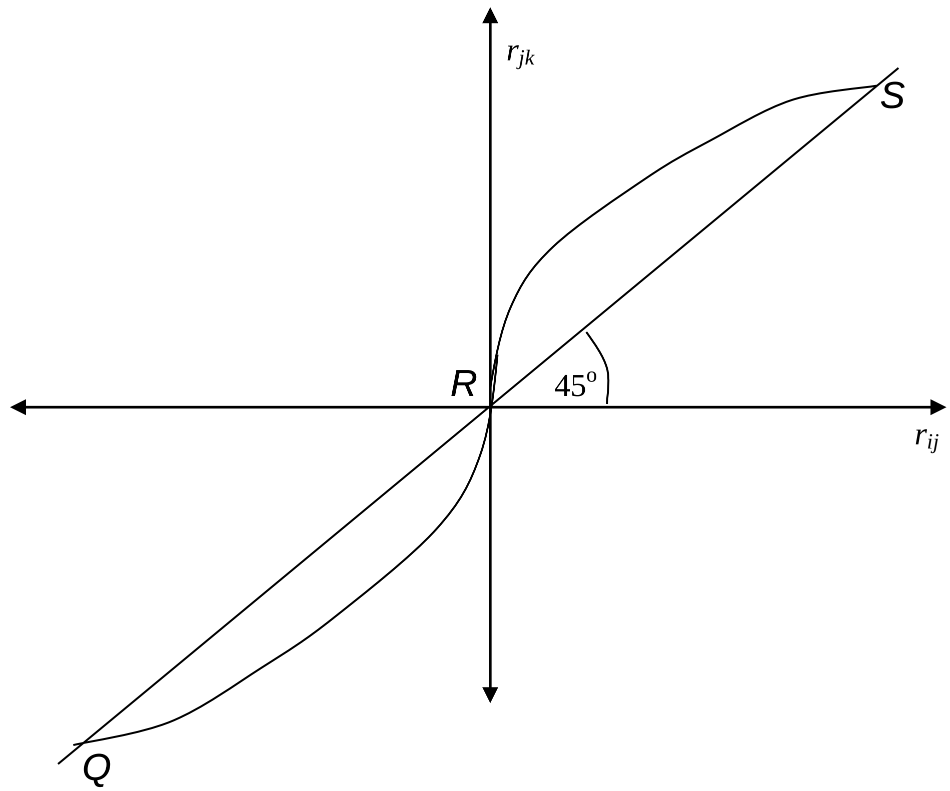
Depending on the properties of the reaction curves between r_{ij} and r_{jk}, this complementarity may result in multiple equilibria. That is, there can be situations with both a stable low-level equilibrium in which members of a group of interacting individuals mostly provide negative recognition to others, and a stable high-level equilibrium in which people mostly provide positive recognition to others. An equilibrium with low magnitudes of recognition provision (i.e. neutral) may also exist.

In a case of continuous actions such as human recognition provision, if we assume identical agents[4], multiple equilibria will occur if the reaction curve for human recognition provision cuts the 45^o line at an interior point with a slope greater than one (Hoff 2001). While the parameters μ, α, β and the function $\psi(R_i)$ vary across individuals and interactions, multiple equilibria can exist for a given set of parameters corresponding to a set of individuals and interactions. Figure 2 depicts such a case of multiple equilibria with a graph of an individual's (j's) reaction curve for provision of human recognition. Higher magnitudes of positive (negative) human recognition provided by *i* lead *j* to provide higher magnitudes of positive (negative) human recognition.

Three equilibria exist in Figure 2 (*Q*, *R*, and *S*). *Q* is a stable equilibrium in which people provide negative recognition to each other, *S* is a stable equilibrium in which people provide positive recognition to each other, and *R* is an unstable equilibrium in which people provide neutral recognition to each other.

[4] The assumption of identical agents means differences among individuals' reactions to others' recognition provision based on one's place in the social or economic heirarchy or on other characteristics are not explicitly considered. One way to interpret this assumption is to view individuals *i* and *j* as "representative agents" for the population.

Figure 2: Reaction Curve for Human Recognition Provision



The graph depicts diminishing marginal reaction to the magnitude of recognition received; that is, individual *j*'s choice of human recognition provision is more sensitive to the quantity of recognition provided to her the closer recognition provision is to zero. At higher magnitudes of recognition provision, changes in others' behavior have less of an effect on one's own recognition provision behavior. For the model of recognition provision described above, diminishing marginal reaction would occur when the function $\psi(R_i)$ is of a similar form to the f(r) function discussed in the previous section. Diminishing marginal reaction is a necessary condition for the type of multiple equilibria depicted in Figure 2. This is a reasonable property for this function to have because recognition provision decisions are likely to be more affected by others' behavior at moderate or neutral levels of recognition than at extreme values, when one's behavior may be more entrenched. For example, consider a 70-year-old prisoner who has been dehumanized and provided high magnitudes of negative recognition for 40 years, and consider his 20-year-old grandson who is not a prisoner and receives a combination of positive and negative recognition from those surrounding him. It is expected that a given input of recognition received in the current time period would be less likely to change the 70-year-old prisoner's own recognition provision behavior than it would be to change his grandson's.

Complementarity in the model is based on psychic components of utility, $\frac{\partial^2 \Psi}{\partial r_{ij} R_i}$. Those with greater positive (negative) recognition obtain greater psychic utility from providing positive (negative) recognition to others. However, material factors can reinforce this complementarity: once low (high) recognition provision becomes the norm it may be materially costly to provide high (low) levels of recognition to others. For example, if all employers apply inhumane employee conditions in factories, an employer who tries to provide better conditions and benefits may increase costs and lose his competitive edge. Conversely, once high recognition provision becomes the norm, an employer may lose employees if he provides negative recognition to employees through poorer conditions, a case that Section V explores more formally.

Existence of multiple equilibria can contribute to understanding the persistence of inefficiencies in human recognition and resulting behaviors. Based on a study of domestic violence in households in southern India and its relationship to intra-household resource allocation, Rao concludes:

> Clearly everyone in a violent household would be better off with the same allocation and without the violence. Why then does violence exist? No theoretical model of intra-household behavior, that I am aware of, allows for inefficient equilibria. (Rao 1998)

Multiple equilibria in human recognition may explain this situation. The households Rao studied are stuck at equilibrium Q in Figure 2 in a sort of low recognition trap. It would be more beneficial for them to be at equilibrium S with positive recognition and no violence, but they are stuck at Q. Since recognition behavior affects other behaviors and outcomes – in Rao's case intrahousehold resource allocation, as well as possibly health outcomes related to violence – remaining at equilibrium Q has implications for these outcomes as well.

In cases where equilibria *Q*, *R*, and *S* are Pareto ranked, a coordination failure will exist. Coordination failure in this context could involve a household in which disrespect and abuse are the norm, or a community in which people provide low levels of human recognition to each other in schools, health care facilities, and places of employment. Complementarity keeps everyone at equilibrium *Q* through a combination of psychic components of utility ($\frac{\partial^2 \Psi}{\partial r_{ij} R_i}$), conditioning and role modeling, and human recognition's effect on material components of utility as mentioned above and formally modeled in Section IV. Everyone would be better off at equilibrium *S*, but an individual's utility declines if she provides greater positive recognition (i.e. moves toward *S*) while everyone else is providing negative recognition (at *Q*). If someone starts moving up the reaction curve from Q, she reacts to the behavior of others by providing greater magnitudes of negative recognition and slides back to Q. Therefore, everyone remains at the low equilibrium.

However, if some individuals – such as those at the top of the power structure – benefit from being at equilibrium *Q* and would lose utility if everyone shifted to equilibrium *S*, then strictly speaking this is not a case of coordination failure. In this case the equilibria are not Pareto ranked, and it may not be a failure to coordinate that is keeping the population at the low equilibria, but rather deliberate efforts or resistance to change on the part of those who would lose utility from such a shift. For example, in the household Rao describes, his premise is that "everyone in a violent household would be better off with the same allocation and without the violence". But if the provision of negative human recognition and the violence influence the allocation of consumption goods in the household – for example, through extraction of greater dowry or control of household income – then some individuals – perhaps an abused woman's husband and his parents – materially gain from the violence and negative recognition. In this case it is not a coordination failure. Removing the violence is not a Pareto improvement because some members will lose goods, and it is not lack of coordination that is preventing removal of the violence but rather active efforts on the part of those at the top of the household's power structure.

In cases of coordination failure (and possibly in cases of multiple equilibria without coordination failure such as the household situation described above), external interventions can help move the population from the low equilibrium to the high equilibrium. Depending on the context, interventions may include: 1) issuing and enforcing specific laws (e.g. laws regarding labor conditions, domestic violence, minimum standards of privacy and care at health care facilities); and/or 2) implementation of programs that provide substantial positive recognition to a significant proportion of the population, thereby raising recognition levels high enough that it becomes beneficial for people to provide positive recognition to others, i.e. moving the population past point *R* and into quadrant 1 of Figure 2. Incorporation of human recognition components into the design of programs and policies is formally modeled in section VI below. In addition to external interventions, opportunities may also exist for internal mechanisms – such as mothers groups or worker solidarity movements – to help move populations from a low equilibrium to a high one.

Note that despite the complementarity in human recognition provision, the analysis of an individual's choice of how much human recognition to provide to others ignores the "feedback" psychic effect on one's own receipt of human recognition that provision of recognition to others will have. That is, r_{ij} affects R_j, R_j affects r_{ji}, which in turn affects R_i. But this effect is not included in the expression for the optimal level of recognition one provides to others, which is derived from the first order condition: $r_{ij}^* = (\mu + \psi(R_i) - \beta)/2\alpha$.

There are two reasons for ignoring this feedback effect in modeling the determinants of human recognition provision. First, it is assumed that an individual interacts with large enough populations *m* and *n* that the effect of such feedback on any given decision an individual makes about providing human recognition to another individual is likely to be relatively small. Second, in many contexts individuals have the greatest influence on human recognition levels of those less powerful than themselves. As discussed elsewhere (Castleman 2011), recognition tends to move down the pecking order, from the more powerful to the less powerful. Therefore, the people whose recognition an individual most influences may not have a strong influence on her own recognition levels; when *i* determines how much recognition to provide to *j*, then *j* may be able to influence *k* and *l* who are lower in the social or power hierarchy, but *j* may not be able to significantly affect *i*'s recognition level in response.

This dynamic clearly depends on the context and on cultural and other factors. There may be changes in power structures due to upward or downward mobility, though depending on the timeframe for these changes, individuals may not change their recognition provision behavior in anticipation of an altered power structure. In some cases, human recognition provided among peers is a primary source of recognition, and the feedback effect may apply to small groups of peers such as spouses with relatively equal power, small cooperatives, or women's groups. Furthermore, in some situations strong positive peer-to-peer human recognition transactions can be a catalyst for development such as solidarity movements or women's self-help groups. Such cases can be important opportunities and targets for programs and policies. While the model assumes a more traditional, hierarchical power structure and does not explicitly account for this type of feedback effect, the model could be extended to incorporate such an effect.

IV. Contribution of Human Recognition to Utility

A variety of types of utility function can be used to model human recognition's contribution to utility. This section uses a simple model in which an individual's utility is determined by consumption, health, human recognition level, and human recognition provided to others. The next section incorporates human recognition into a simple labor supply model, illustrating how human recognition's effect on specific economic behaviors can be modeled.

Consider a utility function given by

$U_i = U(h_i, c_i, R_i, R_{i_p})$

where h_i is individual *i*'s health status, c_i is her consumption, R_i is her level of human recognition, and R_{i_p} is the level of human recognition she provides to others, i.e. the net total of all the r_{ij}'s, $R_{i_p} = \sum_{j=1}^{m} r_{ij}$.

For simplicity and since there are not individual-specific interactions in this part of the model, the i subscripts are dropped, and the utility function becomes

$U = U(h, c, R, R_p)$

where R is an individual's level of human recognition and R_p is the level of recognition she provides to others.

Health status and consumption are determined by the functions

$h = h(H, R)$ and $c = c(C, R, R_p)$

where H and C are factors and inputs other than human recognition that determine health status and consumption respectively, e.g. household income, age, proximity to health facilities, etc. Note that there may be interactions between H and R because one's level of human recognition can affect access to health services or nutritious food or affect adherence to behaviors that determine health status, and conversely some of these factors can affect recognition levels. Similarly, there may be interactions between C and R and between C and R_p. The R_p term is included as part of the argument in the *c*(..) function because the level of human recognition provided to others can affect one's own consumption level. For example, Bloch and Rao find that systematic domestic violence in southern India – a manifestation of a husband's negative human recognition of his wife – is used to extract dowry payments from the wife's family, which increases the husband's wealth and consumption (Bloch and Rao 2002). The R_p term is not included in the h(…) function because the direct impact recognition provision has on one's own health is expected to be minimal, though in some cases it could affect mental health.

R_p represents the total recognition an individual provides to all others, 1…m, and its effect on the individual's utility is determined directly from the payoff function derived earlier. The individual payoff function, $\pi_i(r_{ij}) = \mu r_{ij} + r_{ij}\psi(R_i) - \alpha r_{ij}^2 - \beta r_{ij}$, can be converted to a payoff function for total recognition provided, $\pi(R_p) = \mu R_p + R_p\psi(R) - \alpha R_p^2 - \beta R_p$. For simplicity, the parameters, μ, ψ, α, and β, in this function are taken to be the same for all *m* individuals, or alternatively can be interpreted as the sum or the average of parameters across all *m* individuals.

The payoff function, $\pi(R_p) = \mu R_p + R_p\psi(R) - \alpha R_p^2 - \beta R_p$, is a subutility function. Although one's own recognition level R is a variable in the subutility function, since R is not a choice variable in an individual's utility maximization and can be treated as exogenous, two-stage maximization can be used whereby an individual chooses how much recognition to provide others and then chooses his other choice variables related to consumption and health (Varian 1992). Optimizing this subutility function as described in the previous section yields $R_p^* = (\mu + \psi(R) - \beta)/2\alpha$.

Incorporating the determinants of each component and the optimal level of human recognition provision, the utility function becomes:

$$U = U[h(H, R), c(C, R, \{\mu + \psi(R)- \beta\}/2\alpha), R, (\mu + \psi(R) - \beta)/2\alpha].$$ [5]

[5] An individual's level of recognition, R, could also be expressed as a function of c, h, and other factors d, *R*(c, h, d), to indicate that consumption and health determine the level of recognition an individual receives.

Resource constraints could be added to generate a constrained maximization problem for the individual, such as an expenditure constraint on H and C. Since the focus of this section is on human recognition's overall contribution to utility, constrained utility maximization problems are not included. In Section VI, we add resource constraints associated with a development program to examine a program's maximization problem. Because individual maximization problems are not explored here, there is no need to incorporate the R = f(r) function into this part of the model.

A simple linear function, $u(R) = \phi R$, is used for the direct psychic effect one's human recognition level has on one's utility, with $\phi > 0$. Using a simple additive utility function, the utility function becomes:

$$U = U[h(H, R), c(C, R, R_p), R, R_p]$$

$$= u_h[h(H, R)] + u_c[c(C, R, R_p)] + \phi R + \pi(R_p)$$

where u_h and u_c are utility functions for health and consumption.

A simple linear function, $\psi(R) = \psi R$, is used for the marginal psychic benefit of human recognition provision, with $\psi > 0$. Recalling that $R_p^* = \{\mu + \psi(R) - \beta\}/2\alpha$, the subutility of human recognition provision is now given by the expression, $\pi(R_p) = \mu R_p + R_p \psi R - \alpha R_p^2 - \beta R_p$.

The utility function becomes:

$$U = u_h[h(H, R)] + u_c[c(C, R, \{\mu + \psi R - \beta\}/2\alpha)] + \phi R + \mu\{\mu + \psi R - \beta\}/2\alpha +$$

$$\psi R\{\mu + \psi R - \beta\}/2\alpha - \alpha(\{\mu + \psi R - \beta\}/2\alpha)^2 - \beta\{\mu + \psi R - \beta\}/2\alpha.$$

This is hypothesized to be a simultaneous relationship, with recognition determining health and consumption as well. However, since the result of interest here is the effect that changes in recognition have on utility, $\frac{\partial U}{\partial R}$, including such a functional relationship in this model does not enrich the results.

The expression for the total marginal utility of one's own level of human recognition is:

$$\frac{\partial U}{\partial R} = \frac{\partial U}{\partial h}\frac{\partial h}{\partial R} + \frac{\partial U}{\partial c}\frac{\partial c}{\partial R} + \phi + \frac{\psi}{2\alpha}(\mu - \beta + \psi R).$$

Derivation of the last term of this expression is given in Appendix 1.

This expression indicates that one's level of human recognition affects well-being in three ways: a) through its effect on health and consumption, $\frac{\partial h}{\partial R}$ and $\frac{\partial c}{\partial R}$, which in turn affect utility, $\frac{\partial U}{\partial h}$ and $\frac{\partial U}{\partial c}$ (*material effects*); b) through its direct effect on well-being, ϕ (*psychic effects*); and c) through its effect on the recognition one provides to others, which affects one's own well-being, $\frac{\psi}{2\alpha}(\mu - B + \psi R)$ (*psychic and material effects*).

Three results emerge from the model for empirical testing:

1) The significance and signs of $\frac{\partial h}{\partial R}$ and $\frac{\partial c}{\partial R}$, the effects of human recognition on health and consumption. Both are hypothesized to be positive.
2) The significance and sign of ϕ, the direct psychic effect of human recognition on utility. It is hypothesized to be positive.
3) The significance and sign of $\frac{\partial U}{\partial R}$, the total effect of human recognition on utility. It is hypothesized to be positive.

If hypotheses 1) and 2) above hold, then hypothesis 3) will hold unless the last term, $\frac{\psi}{2\alpha}(\mu - \beta + \psi R)$, in the marginal utility expression is negative and of greater magnitude than the other terms combined. This last term represents the effect on an individual's utility of the change in his provision of recognition to others that is caused by a change in his own level of recognition. Using notation, this effect can be expressed as:

$$\Delta R \longrightarrow \Delta R_p \longrightarrow \Delta U$$

This term will be negative if and only if R_p is negative. This can be seen by observing that the sign of this term, $\frac{\psi}{2\alpha}(\mu - \beta + \psi R)$, is the same as the sign of the optimal level of recognition provided to others, $R_p^* = \{\mu + \psi R - \beta\}/2\alpha$.[6]

When this term is negative, there are two cases to consider. The first case is an individual who is providing negative recognition, R_p, and whose own recognition level, R, *increases*. Because of complementarity, the increase in R leads the individual to decrease the magnitude of negative recognition he is providing others. This may decrease the (psychic and/or material) utility the person obtains from providing negative recognition to others. So the rise in the individual's recognition level decreases the utility obtained from provision of recognition and hence the final term in the marginal utility expression is negative.

However, in terms of the individual's total utility, in most of these cases the magnitude of the increase in utility from the increased recognition level (due to improved health, increased consumption, and psychic benefits) will be larger than the magnitude of

[6] The result can also be seen on an individual basis from the earlier first order condition, $\frac{\partial \pi_i(r_{ij})}{\partial r_{ij}} = \mu + \psi R_i - 2\alpha r_{ij} - \beta = 0$. Given this condition, if $\mu - \beta + \psi R_i < 0$, then $r_{ij} < 0$. And in order for the sign of the last term in the marginal utility expression, $\frac{\psi}{2\alpha}(\mu - \beta + \psi R)$, to be negative, $\mu - \beta + \psi R_i$ must be negative.

the decrease in utility due to the change in recognition provision. That is, even if $\frac{\psi}{2\alpha}$ $(\mu - \beta + \psi R) < 0$, it is very likely that $\frac{\partial U}{\partial h}\frac{\partial h}{\partial R} + \frac{\partial U}{\partial c}\frac{\partial c}{\partial R} + \phi > |\frac{\psi}{2\alpha}(\mu - \beta + \psi R)|$.

Furthermore, in terms of total social welfare, while providing less negative recognition may reduce the individual provider's utility, it will increase the utility of the individuals who receive the lower quantities of negative recognition. So while there may be a private loss of utility to the provider due to the reduced magnitude of negative recognition provision, there is likely a net social gain. This has implications for the benefits of interventions aimed at increasing provision of positive recognition and decreasing provision of negative recognition, though of course this is distinct from whether individual utility increases or decreases.

The second case is similar and involves an individual who is providing negative recognition to others, R_p, and whose own recognition level, R, *decreases*. The decrease in R leads the individual to provide a higher magnitude of negative recognition to others, which may increase the (psychic and/or material) utility the individual obtains from providing recognition to others. So the decrease in the individual's recognition level increases the utility obtained from provision of negative recognition and hence the final term in the marginal utility expression is negative.

Again, in terms of the individual's total utility, it is unlikely that the magnitude of this increase in utility will be equal to or greater than the magnitude of the decrease in utility caused by the decline in the individual's own level of recognition. That is, it is very likely that

$$|\frac{\partial U}{\partial h}\frac{\partial h}{\partial R} + \frac{\partial U}{\partial c}\frac{\partial c}{\partial R} + \phi| > \frac{\psi}{2\alpha}(\mu - \beta + \psi R).$$

And again, in terms of social welfare, while the increase in utility caused by greater negative provision of recognition would be a private gain, it would likely represent a social loss given the decreased utility of those on the receiving end of the negative recognition.

Therefore, for all practical purposes, we assume that if hypotheses 1) and 2) both hold, then hypothesis 3) will hold also. However, with suitable measurement techniques and data, this assumption can be tested by empirically testing hypothesis 3), a process that is begun in another paper (Castleman 2011c).

Specifying the model further using simple, explicit functional forms for the u_h and u_c functions yields:

$$U = \eta h + \kappa c + \phi R + \pi(R_p), \text{ where } \eta, \kappa > 0.$$

The following explicit functional forms are then used for the *h* and *c* functions:

$$h = h(H, R) = H + \frac{HR}{\lambda} + \sigma R, \text{ where } H \geq 0, \sigma > 0 \text{ and } \lambda > R'.$$

$$c = c(H, R, R_p) = C + \frac{CR}{\gamma} + \delta R + \theta R_p - \tau R_p, \text{ where } C \geq 0, \gamma > R', \delta > 0, \theta \geq 0, \tau < 1.$$

The σR and δR terms represent the direct effects one's recognition level has on health and consumption. For example, recognition levels may affect mental and emotional health status.

The θR_p and τR_p terms represent the positive and negative effects, respectively, that provision of human recognition has on one's own consumption. These parameters are related to μ, α, and β, the parameters for the material benefits and costs of human recognition provision, but they are not identical to those parameters because recognition provision may generate other material benefits and costs in addition to those related to consumption (e.g. political power within the community, or changes in health).

R' is the upper bound of the scale used to measure R and the inverse of the lower bound, which requires the function f(r) shown in Figure 1 to be asymptotic.

The $\frac{HR}{\lambda}$ and $\frac{CR}{\gamma}$ terms represent the effects that one's human recognition level has on the "productivity" of other factors in producing health and consumption respectively. For example, if one component of H is proximity to health care facilities, an individual with a higher level of human recognition may obtain greater health benefits from living a given distance from health facilities than an individual with a lower level of recognition. Greater recognition from family members may enable an individual to visit the facility more freely; greater recognition from health care providers at the facility may encourage more frequent attendance and better adherence to treatment provided and practices recommended. The restriction on the parameters λ and γ that they are greater than the maximum level of R and the inverse of the minimum level of R means that the effect these interactions between recognition and other factors have on health and consumption will always be smaller in magnitude than the direct effect that non-recognition factors, H and C, have on health and consumption. The expressions for health and consumption can be rewritten as

$$h = H(1 + \frac{R}{\lambda}) + \sigma R$$

$$c = C(1 + \frac{R}{\lambda}) + \delta R + \theta R_p - \tau R_{p.}$$

Since $|\frac{R}{\lambda}| < 1$, the effect of the interactive terms is to enhance or diminish the impacts H and C have on health and consumption, not to supercede or eliminate these impacts.

Incorporating these explicit functions, the additive utility function becomes:

$$U = \eta(H + \frac{HR}{\lambda} + \sigma R) + \kappa(C + \frac{CR}{\gamma} + \delta R + \theta R_p - \tau R_p) + \phi R + \pi(R_p).$$

Substituting the optimized level of recognition provision and the subutility function yields:

$$U = \eta(H + \frac{HR}{\lambda} + \sigma R) + \kappa(C + \frac{CR}{\gamma} + \delta R + \frac{(\theta - \tau)(\mu + \psi R - \beta)}{2\alpha}) + \phi R +$$

$$\mu\{\mu + \psi R - \beta\}/2\alpha + \psi R\,\{\mu + \psi R - \beta\}/2\alpha - \alpha(\{\mu + \psi R - \beta\}/2\alpha)^2 - \beta\{\mu + \psi R -$$

$$\beta\}/2\alpha$$

The marginal utility of one's level of human recognition is given by:

$$\frac{\partial U}{\partial R} = \eta(\frac{H}{\lambda} + \sigma) + \kappa(\frac{C}{\gamma} + \delta + \frac{\psi(\theta - \tau)}{2\alpha}) + \phi + \frac{\psi}{2\alpha}(\mu - \beta + \psi R).$$

The same three results emerge from the model, now with specific functional forms that are all positive given the assigned ranges of parameter values[7]:

1) $\frac{\partial h}{\partial R} = \eta(\frac{H}{\lambda} + \sigma) \geq 0.$

$$\frac{\partial c}{\partial R} = \kappa(\frac{C}{\gamma} + \delta + \frac{\psi(\theta - \tau)}{2\alpha}) > 0.$$

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[7] It is theoretically possible, but practically unlikely, that the expression for $\frac{\partial c}{\partial R} > 0$. This would require that $\tau > \frac{2\alpha}{\psi}(\frac{C}{\gamma} + \delta) + \theta$. That is, it requires that the negative marginal effect that provision of recognition has on one's own consumption, τ, be significantly greater than the positive marginal effect, θ, and that the difference in these effects, $\frac{\psi(\tau - \theta)}{2\alpha}$ be of greater magnitude than the sum of the marginal effect one's own level of recognition has on one's consumption and the marginal effect one's own human recognition level has on the productivity of other consumption factors, $\frac{C}{\gamma}$. This can be empirically tested.

2) The direct psychic effect of recognition is $\phi > 0$.

3) $$\frac{\partial U}{\partial R} = \eta(\frac{H}{\lambda} + \sigma) + \kappa(\frac{C}{\gamma} + \delta + \frac{\psi(\theta - \tau)}{2\alpha}) + \phi + \frac{\psi}{2\alpha}(\mu - \beta + \psi R) > 0.$$

The same factors apply for signing the last term of the expression in (3) as were discussed above for the general function.

As in the general case, these three results suggest that increases in one's level of human recognition improve utility through both direct (psychic) and indirect (material) channels and improve health and consumption outcomes. With suitable measurement methods and data (Castleman 2011b; Castleman 2011c), these three results can be empirically tested.

V. Role of Human Recognition in Labor Supply

Human recognition is relevant to the workplace, especially in developing country contexts where wide ranges of working conditions and employee treatment norms exist. Much of the research on determinants of women's labor supply in developing countries has focused on factors such as women's education levels, household income and wealth, husbands' education levels, cultural mores, and opportunity costs in terms of household and own-farm production, child care and other household duties (Cameron et al. 2001; Fafchamps and Quisumbing 1998; Fafchamps and Quisumbing 1999; Khandker 1988; Mammen and Paxson 2000). These factors are external to the nature of the employment itself. Yet job characteristics can also affect labor supply, the most obvious characteristic being wage levels. Other, non-wage characteristics of a job that affect labor supply decisions are often combined into the concept of "job satisfaction" and include components such as enjoyment gained from work tasks themselves, working environment, social interactions at the workplace, the job's effect on social status within one's community, and indirect opportunities to gain social or economic benefits such as increasing the chances that other household members obtain employment.

It is posited that human recognition transactions at the workplace are a relevant characteristic that contributes to job satisfaction. Employees receive positive or negative human recognition through working conditions and employer treatment of employees, such as the physical working environment, availability of basic facilities, policies and norms for sick or hurt employees, disciplinary policies and actions, and gifts or bonuses that help to meet employees' basic needs. Since these factors affect employees' well-being (through both material and psychic channels), it is hypothesized that they also affect labor supply decisions. For example, a woman who is generally treated disrespectfully or even abusively in her household but who is treated with respect and dignity at her workplace may choose to spend more time on the job (for a given wage and income level) than if she were treated poorly on the job.

A simple static labor supply model is adapted to incorporate human recognition as a factor that influences labor supply decisions. The adapted model generates a testable hypothesis about the relationship between the level of human recognition an individual receives at the workplace and the individual's labor supply. The model can be used, for example, to describe labor supply decisions of women in developing country contexts.

Because this is a labor supply model, it requires a somewhat different parameterization than the earlier general utility model. Utility is given by the function

$$U_i = U(c_i, L_i, R_i)$$

where L_i is time individual *i* spends not working in the labor market (i.e. time not spent working for remuneration from employers outside of one's own household), and c_i and R_i are consumption and total level of recognition respectively as in the earlier model. The utility function is increasing in all three arguments. Since interactions among specific individuals are not part of the model, the subscripts are dropped for simplicity to become U = *U*(c, L, R).

L satisfies the constraint K = H + L, where K is the total time available and H is time spent in market work. R is determined by a function R = $R(\overline{R}$, K, L), where $\overline{R}$ is the level of recognition one receives from sources other than the workplace.

In this model the *R* function is specified as R = $\overline{R}$ + (K-L)r, where r is the quantity of human recognition an individual receives at work per time unit of labor. That the level of human recognition is proportional to the hours one works makes intuitive sense. For example, a given workplace practice that affects one's level of human recognition is likely to have a greater effect on an employee who spends 40 hours per week at the workplace than one who spends only 10 hours per week there.

Workers face the following problem:

max *U*(c, L, R) subject to
1) $W(K-L) + N \geq Pc$
2) $R = r(K-L) + \overline{R}$, such that $R \in [0, 2)$
3) $K = H + L$

where W is the wage, N is the individual's non-labor income, and P is the price of consumption.

An explicit functional form for utility is used to solve the optimization problem and generate comparative statics with meaningful interpretations.

$$U = \alpha\log(c) + \beta\log(L) + \gamma\log(R), \quad \alpha, \beta, \gamma > 0, R \in [0,2).$$

In this model an individual's level of human recognition is bounded by 0 and 2.[8] Human recognition levels between 1 and 2 mean an individual has positive net human recognition, from which she derives positive utility. Human recognition levels between 0 and 1 mean an individual has negative net human recognition, from which she derives disutility. A human recognition value of 1 represents "neutral" net human recognition from which an individual derives neither positive nor negative utility. This could be viewed as shifting Figure 1 up so that a lower asymptote is at R = 0, an upper asymptote is at R = 2, and the inflection point is at R = 1. However, for simplicity in the labor supply model, the R = f(r) function is not used; instead total recognition is equal to recognition received on the job, r(K-L), and off the job, $\overline{R}$, but is bounded between 0 and 2.

The worker's utility maximization problem becomes:

$$\max_{(c,\,L,\,R)} U = \alpha\log(c) + \beta\log(L) + \gamma\log(R) \quad \text{s.t.} \quad \begin{aligned} &W(K-L) + N \geq Pc \\ &R = r(K-L) + \overline{R} \\ &K = H + L \end{aligned}$$

Substituting K – H for L yields the following Lagrangean:

$$\max_{(c,\,H,\,R)} \mathcal{L} = \alpha\log(c) + \beta\log(K-H) + \gamma\log(R) + \lambda(WH + N - Pc) + \mu(rH + \overline{R} - R)$$

The price of consumption goods is normalized to 1, and the first order conditions become:

1. c: $\frac{\alpha}{c} - \lambda = 0$
2. H: $-\frac{\beta}{K-H} + \lambda W + \mu r = 0$

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[8] This implies that while r can be positive, negative, or zero, $\overline{R}$ is also bound between 0 and 2.

3. R: $\frac{\gamma}{R} - \mu = 0$
4. λ: $WH + N = c$
5. μ: $rH + \overline{R} = \text{R}$

Combining the five first order conditions from a worker's utility maximization yields:

$$\frac{\beta}{K-H} = \frac{\alpha W}{WH+N} + \frac{\gamma r}{rH+\overline{R}}.$$

This condition is derived in Appendix 2. From this equation, the effect on labor supply of changes in r can be predicted:

$$\frac{\partial H}{\partial r} = \frac{\gamma \overline{R}(K-H)^2(WH+N)^2}{\beta(WH+N)^2(dH+\overline{R})^2 + \alpha W^2(K-H)^2(dH+\overline{R})^2 + \gamma r^2(K-H)^2(WH+N)^2} > 0.$$

Derivation of this comparative static is given in Appendix 2.

This result suggests that increases in human recognition provided at the workplace lead to increases in labor supply. This is an example of human recognition affecting economic behavior through a hedonic market mechanism (Castleman 2011). This can be tested empirically if data are available on both measurable variations in the levels of recognition provided at workplaces and variations in labor supply at these workplaces.

VI. Programs and Policies

The model can be extended to describe development programs and policies. Programs and policies influence human recognition levels, which in turn can affect the utility of the target population through recognition's direct psychic effects, recognition's effects on materials outcomes that are part of the program's objectives, and recognition's effects on other material outcomes. Whether and how human recognition is considered in program design and allocation of resources can influence program outcomes for targeted beneficiaries. Furthermore, complementarity in human recognition suggests that programs can also affect the human recognition levels of populations that are not direct program beneficiaries.

The model in this section examines how consideration of human recognition factors in the design of a health program affects resource allocation and outcomes. Similar models could be applied to programs in other sectors, e.g. microcredit or education. The model focuses on a program, but could be adapted to apply either more broadly to policies or more narrowly to specific services.

In the model a program maximizes a welfare function for a targeted population[9]:

$$\max_{H,R} W = \sum_{i}^{q} w_i \text{ , } w_i = U_i(h(H_0, H, R_0, R), r(R_0, R))$$

$$= \eta[H_0 + H + \frac{(H_0 + H)(R_0 + R)}{\lambda} + \sigma(R_0 + R)] + \phi(R_0 + R)$$

subject to $Hp_H + Rp_R \leq M$,

where there are q members of the targeted population; w_i is the welfare of individual i; H_0 are pre-existing health-related factors that are not the result of program interventions; H are program interventions aimed at improving health; R_0 are pre-existing recognition-related factors that are not the result of program interventions[10]; R are program interventions that directly target recognition; p_H and p_R are the costs of health and

[9] Depending on the program, the target population may be either direct program beneficiaries or the entire population in the program catchment area.
[10] These non-program factors are known or estimated by the program designers or managers in that they are considered in resource allocation choices.

recognition interventions respectively; and M is the level of resources available to the program. Parameters μ, τ, σ, and ϕ are interpreted as described earlier with the same restrictions on their values.

The model assumes a homogenous target population, and H_0, H, R_0, and R represent levels for the entire population[11]. The utility function uses the $r(R_0, R) = (R_0 + R)$ to express an individual's level of human recognition under the simplifying assumption that recognition outcomes are directly proportional to pre-existing factors and program investments targeting recognition. Similarly, $(H_0 + H)$ is used to express non-recognition factors affecting health. In this model, H_0, H, R_0, and R are all confined to be non-negative because the model examines program inputs to improve health and human recognition, and because allowing negative values poses problems for the budget constraint, i.e., p_H and p_R refer to the prices for interventions aimed at improving health and recognition respectively, not explicit savings or profits from reducing health or recognition levels. Therefore, the lowest levels of health, recognition, or utility in the model is 0.

The objective function is a utilitarian welfare function: utility is determined by the sum of beneficiaries' utilities. Alternatively, a Rawlsian welfare function (minmax of utilities) or another type of welfare function could be used. The main conclusions about treatment of human recognition in programs would still hold, though the expressions and comparative statics would differ. Other components may be added to either the objective function or the constraints, such as implementing organizations' institutional priorities or donor requirements, but the model assumes the program's primary objective is

[11] Alternatively, the variables could represent averages for a heterogeneous population.

improvement of beneficiary utility and the primary constraint is a resource constraint.

Note that the program's objectives are limited to particular components of utility, specifically health and recognition levels. The contribution to utility of consumption is not included in the utility function, as it is assumed that health programs focus on improved well-being of beneficiaries brought about by changes in health, not consumption. There is evidence of a strong positive relationship between consumption and health (see Feachem et al., 1992), and some health programs do consider consumption implications in program interventions, but this model is confined to the program's specific health and recognition objectives. There have been recent efforts to identify multidimensional measures of poverty and deprivation (Alkire and Foster 2011) and to design multisectoral program approaches (Sanchez et al. 2007). If broader utility and welfare functions that include consumption and other components were used in the model, it would make the solutions more complicated but is not expected to change the main conclusions about human recognition.

Beneficiaries' human recognition levels are included as an argument in the program's objective function because health programs can influence recognition levels, which in turn affect utility through changes in health outcomes, as well as directly through psychic components of utility. Applying the terminology Sen uses to describe freedom's role in development (Sen 1999), human recognition impacts utility through both instrumental effects as it influences health and constitutive effects as it influences well-being directly. Provision of recognition, R_p, is not included in the utility function because an individual's provision of recognition is not likely to significantly affect one's own health. Beneficiaries' provision of recognition is relevant to health programs insofar as it affects other beneficiaries' recognition levels, but this is captured in the $R_0 + R$ term, the expression for beneficiary recognition levels.

Because of the complementarity in recognition provision discussed earlier, program investment in improving recognition levels may lead to changes in recognition for a wider population than those directly receiving program interventions. Programs can enhance these "spread effects" with outreach efforts, such as by supporting program beneficiaries to lead farmers groups, mothers groups, health education sessions, or microfinance groups, which may help extend both the material and human recognition benefits of the program. In some contexts strengthening peer-to-peer human recognition can contribute to development outcomes, such as through solidarity movements or women's groups that facilitate improved human recognition transactions, empower members, and these opportunities can help spread program benefits beyond direct beneficiaries.

The model includes the use of distinct resources (Rp_R) to improve beneficiaries' human recognition levels. While in some cases, this may involve specific additional interventions such as pycho-social counseling or interventions to reduce domestic violence, in many cases these efforts will not involve separate interventions, but rather spending additional resources on existing health interventions to ensure they produce positive or non-negative recognition outcomes. Examples of such efforts include refining the content and methods of staff training, norms for the amount of time service providers spend with clients, supervision content and methods, or infrastructure such as beneficiary seating. In addition to these interventions, there may also be relatively costless ways that programs can enhance human recognition simply by adjusting how health interventions are implemented, such as changes in interpersonal behavior among service providers and other program staff. These adjustments may not require significant resources, though training and structured supervision and support systems may ensure higher quality and sustained changes. While some such "win-win" opportunities do exist to improve recognition and health simultaneously without significantly affecting the resources allocated to either, the model focuses on cases that invovle choices in resource allocation.

Case 1: Direct and instrumental effects of human recognition considered (*optimal program*)

The optimal program faces the following constrained optimization problem:

$$\max_{H,R} W = \sum_i^q w_i \text{ , } w_i = U_i(h(H_0, H, R_0, R), r(R_0, R))$$

$$= \eta[H_0 + H + \frac{(H_0 + H)(R_0 + R)}{\lambda} + \sigma(R_0 + R)] + \phi(R_0 + R)$$

subject to $Hp_H + Rp_R \leq M$

A program that solves this problem, accounting for recognition's direct effects on utility and its indirect effects through health outcomes, solves the following Lagrangian:

$$\max_{H,R} \mathcal{L} = \eta[H_0 + H + \frac{(H_0 + H)(R_0 + R)}{\lambda} + \sigma(R_0 + R)] + \phi(R_0 + R) + \Lambda(M - Hp_H - Rp_R).$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial H} = \eta + \frac{\eta(R_0 + R)}{\lambda} - \Lambda p_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial R} = \frac{\eta(H_0 + H)}{\lambda} + \eta\sigma + \phi - \Lambda p_R = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H - Rp_R = 0$$

Solving these conditions yields the following optimal levels of investment in health and recognition (the derivation of this solution is given in Appendix 3):

$$H^* = \frac{1}{2}\left[\frac{M + (\lambda + R_0)P_R}{P_H} - H_0 - \lambda(\sigma + \frac{\phi}{\eta})\right]$$

$$R^* = \frac{1}{2}\left[\frac{M}{P_R} - R_0 - \lambda + \frac{P_H}{P_R}[H_0 + \lambda(\sigma + \frac{\phi}{\eta})]\right]$$

Comparative statics indicate that for the program, health and recognition interventions are both ordinary goods, $\frac{\partial H}{\partial P_H} = -\frac{M + (\lambda + R_0)P_R}{2P_H^2} < 0$, $\frac{\partial R}{\partial P_R} = -\frac{M + P_H[H_0 + \lambda(\sigma + \frac{\phi}{\eta})]}{2P_R^2} < 0$, and that health and recognition interventions act as substitutes, $\frac{\partial H}{\partial P_R} = \frac{\lambda + R_0}{P_H} > 0, \frac{\partial R}{\partial P_H} = \frac{H_0 + \lambda(\sigma + \frac{\phi}{\eta})}{2P_R} > 0.$

Comparative statics also indicate that better initial health factors and status (higher H_0) leads to relatively lower investments in the direct health aspects of interventions and relatively higher investments in recognition; and better initial recognition status (higher R_0) leads to relatively lower investments in recognition aspects of interventions and relatively higher investments in health. That is,

$\frac{\partial H}{\partial H_0} = -\frac{1}{2} < 0, \frac{\partial R}{\partial H_0} = \frac{P_H}{2P_R} > 0$ and $\frac{\partial R}{\partial R_0} = -\frac{1}{2} < 0, \frac{\partial H}{\partial R_0} = \frac{P_R}{2P_H} > 0$. This is because the marginal utility of health inputs and the marginal utility of recognition are both constant, $\frac{\partial^2 U}{\partial (H_0 + H)^2} = 0, \frac{\partial^2 U}{\partial (R_0 + R)^2} = 0$, but the cross marginal utilities are increasing,

$\frac{\partial^2 U}{\partial(H_0 + H)(R_0 + R)} = \frac{1}{\lambda} > 0$. Hence, while H and R are substitutes in the standard economic sense, they are also complementary products that enhance each other's contribution to utility. Because of the positive interaction between recognition and health inputs in the health component of the utility function, higher levels of health (recognition) increase the marginal utility from improvements in recognition (health). This implies that programs should aim to balance their investments across health and recognition, taking into account the pre-program status.

For the welfare and utility functions given, these outcomes represent the optimal situation in which program design accounts for human recognition both as a direct component of utility and as a contributing factor to health outcomes, which leads to the maximum utility and welfare levels. We now look at outcomes for programs that either do not consider human recognition or only partially account for it in program design.

Case 2: Human recognition not considered

A program that does not consider human recognition at all in program design solves a variation of the above optimization problem in which no R terms are included. The program solves the following Lagrangian:

$$\max_{H} \mathcal{L} = \eta(H_0 + H) + \Lambda(M - Hp_H)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial H} = \eta - \Lambda p_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H = 0$$

The solution is H** = $\frac{M}{p_H}$. Since recognition is not considered in the design, there are no investments made directly in recognition, and R** = 0. Note this does not mean the program will not have an effect on human recognition levels; most programs influence recognition levels of the targeted population, even if they are "unintended consequences" such as Sen describes (Sen 1999). What it does mean is that the program does not deliberately allocate any resources to specifically improve recognition. Recall the R terms do not refer only to separate interventions for recognition, but also to efforts to ensure that health interventions are implemented in a manner that enhances recognition levels. In fact, in some cases programs that focus exclusively on health objectives and not at all on human recognition objectives may reduce recognition levels among program beneficiaries because what is perceived as the most efficient methods for providing health services may involve provision of negative recognition, e.g. coercive family planning methods or very brief doctor visits.

For the welfare and utility functions used in this model, this outcome is a suboptimal resource allocation, and the program is over-investing in the direct health aspects of interventions and under-investing in the aspects of interventions aimed at improving human recognition. That is, H** > H* and R** < R*, except for the unusual case when R* = 0. According to the model, beneficiaries of a program providing H** and R** will have lower welfare than beneficiaries of a program providing H* and R*.

Case 3: Human recognition considered but no resources allocated

A variation similar to Case 2 is a program that does recognize and consider human recognition's role, but chooses *a priori* not to devote any resources specifically to addressing recognition. Such a program solves a special case of the optimization problem; there is no R term in the expenditure constraint and there is an additional constraint that R = 0. The program solves the following Lagrangian:

$$\max_{H} \mathcal{L} = \eta[H_0 + H + \frac{(H_0 + H)(R_0 + R)}{\lambda} + \sigma(R_0 + R)] + \phi(R_0 + R) + \Lambda(M - Hp_H) + \Gamma(R - 0)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial H} = \eta + \frac{\eta(R_0 + R)}{\lambda} - \Lambda p_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Gamma} = R = 0$$

The solution is the same as above: $H^{**} = \frac{M}{p_H}$ and $R^{**} = 0$, with the same implications for the welfare of targeted beneficiaries.

<u>Case 4: Only instrumental effects of human recognition considered</u>

Many programs do consider human recognition and make allowances for it in program design, though they may not use the term, human recognition. For example, health programs may realize that participatory and respectful approaches lead to greater attendance at services and better adherence to prescribed treatments and recommended behaviors. The design of such a program may account for the instrumental effect recognition has on utility through its effect on health outcomes, but not account for recognition's direct effect on utility.

Such a program solves a special case of the optimization problem in which there is no term for the psychic utility of recognition, $\phi(R_0 + R)$, or alternatively in which $\phi = 0$. The program solves the following Lagrangian:

$$\max_{H,R} \mathcal{L} = \eta[H_0 + H + \frac{(H_0 + H)(R_0 + R)}{\lambda} + \sigma(R_0 + R)] + \Lambda(M - Hp_H - Rp_R)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial H} = \eta + \frac{\eta(R_0 + R)}{\lambda} - \Lambda p_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial R} = \frac{\eta(H_0 + H)}{\lambda} + \eta\sigma - \Lambda p_R = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H - Rp_R = 0$$

Solving these conditions yields the following levels of investment in health and recognition:

$$H^{***} = \frac{1}{2}\left[\frac{M + (\lambda + R_0)P_R}{P_H} - H_0 - \lambda\sigma\right]$$

$$R^{***} = \frac{1}{2}\left[\frac{M}{P_R} - R_0 - \lambda + \frac{P_H}{P_R}(H_0 + \lambda\sigma)\right]$$

The comparative statics have the same signs as in the optimal case, but this is a suboptimal allocation of resources, with overinvestment in the direct health aspects of interventions and underinvestment in the recognition aspects of interventions. With recognition's direct effect on utility not considered in program design, the $\frac{\phi}{\eta}$ terms are no longer in the solutions, and comparison with the the optimal solutions for Case 1 shows that H*** > H* and R*** < R*. Again, this will lead to lower beneficiary welfare than the optimal case.

Case 5: Only direct effects of human recognition considered

The last variation involves a program that accounts for human recognition's direct psychic effect on utility, but does not account for its indirect effect through health outcomes. This is a less likely case than Case 4, but may occur in a program that is attuned to psychological and emotional aspects of well-being for their own sake but does not see a link between these factors and the types of health services the program supports.

Such a program solves a special case of the above optimization problem in which there are no R terms in the expression for beneficiaries' health. The program solves the following Lagrangian:

$$\max_{H,R} \mathcal{L} = \eta(H_0 + H) + \phi(R_0 + R) + \Lambda(M - Hp_H - Rp_R)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial H} = \eta - \Lambda p_H = 0$$

$$\frac{\partial \mathcal{L}}{\partial R} = \phi - \Lambda p_R = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H - Rp_R = 0$$

The solution depends on the ratio of the parameters and the ratio of the prices. If $\frac{\eta}{\varphi} > \frac{p_H}{p_R}$, then H**** = M and R**** = 0. If $\frac{\eta}{\varphi} < \frac{p_H}{p_R}$, then H**** = 0 and R**** = M.

If $\frac{\eta}{\varphi} = \frac{p_H}{p_R}$, then there are an infinite number of solutions that meet the budget constraint.

Intuitively, this makes sense because with a linear utility function and linear cost structure, if the benefit-cost ratio of health $(\frac{\eta}{p_H})$ is greater than the benefit-cost ratio of recognition $(\frac{\phi}{p_R})$, all program investment will go to health, and vice versa if the benefit-cost ratio for recognition is greater. If the two ratios are equal, then there are infinite solutions because any allocation that meets the budget constraint will provide the same utility. Neither of the definite solutions is optimal; one entails over-investing in direct health aspects and under-investing in recognition, and the other entails over-investing in recognition and under-investing in health.

Development practitioners often report that *how* a project is implemented is as great a factor in its success as the technical content of the activity. This is one reason for recent focus on quality assurance and quality improvement processes in health and other programs in developing countries (e.g., USAID Health Care Improvement Project 2008). This model suggests that part of that *how* is whether and how program design accounts for human recognition. Optimal improvements in the utility of the targeted population occur when both the direct and instrumental effects of human recognition are considered in program design and resource allocation. When programs do not consider human recognition or when they account for only a portion of its effects, suboptimal outcomes result.

VII. Conclusions and Areas for Further Study

One of the challenges of incorporating non-material components of economic development into research and practice is the difficulty of modeling the roles these components play in development processes and outcomes. The model presented here offers an approach to understanding and predicting the determinants, contribution to utility, and role in programs of one such non-material component - human recognition. In addition to the foundation the model lays for further work on human recognition, this approach may also be useful for modeling other non-material components of development, such as empowerment, social capital, or social exclusion.

The model predicts a number of hypotheses that can be empirically tested if data that validly measure recognition are available. In particular, the model predicts that increases in one's level of human recognition increase utility through both direct, psychic effects and indirect, material effects such as improved health and consumption outcomes. Development of an approach for measuring human recognition will enable empirical study to better understand several of the issues that emerge from the model, including: determinants of human recognition levels, determinants of recognition provision, extent to which recognition is a determinant of material development outcomes such as health, income and labor supply, extent to which recognition is a determinant of well-being, and the impact of specific program interventions on recognition.

On the theoretical side, there is a need for further study of multiple equilibria of human recognition and the role these play in poverty traps. There is also scope for more comprehensive modeling of human recognition transactions among individuals, perhaps applying models of social and economic networks (see for instance Jackson and Wolinsky 1996).

One notable feature of the model is that the function that maps received recognition to an individual's total level of recognition increases in its argument but has marginal effects that decline for higher absolute values of the argument. This means that a given input of human recognition has less of an effect on individuals who already have very large positive or negative levels of recognition than it does on those with moderate or low levels of recognition. This "insulation" effect also occurs in the model of human recognition provision, in which a given input of recognition one receives from others has less effect on one's own recognition provision behavior when one already receives large magnitudes of positive or negative recognition. While these dynamics require greater study and empirical validation, they may have implications for policies and programs. For example, it may be that while the greatest need for increased receipt of human recognition lies with those who receive large magnitudes of low recognition, and the greatest scope for improving provision of recognition lies with "middle" groups who do not receive large magnitudes of high or low recognition.

Efforts to improve human recognition levels need to be based on a sound understanding of the dynamics of human recognition provision. The model of human recognition provision suggests that in some circumstances, multiple equilibria can exist. A community or group of interacting individuals can be stuck at a stable low-level equilibrium in which people for the most part provide negative recognition to each other, or can be at a stable high-level equilibrium in which people for the most part provide positive recognition to each other. These multiple equilibria may help to explain certain situations at household and community levels, and given the hypothesized interactions between recognition and material outcomes, groups stuck at a low-level equilibrium could be in a sort of low recognition poverty trap. Policy and program interventions that address human recognition may be able to assist such groups escape the traps by helping them move to a high-level equilibrium.

One conclusion that emerges from the model that is of relevance to the design of program interventions is that an exclusively instrumental approach to human recognition issues is incomplete. Only considering the impacts that changes in recognition have on individuals' material outcomes (health, consumption) undervalues recognition's impact by failing to capture its direct impact on well-being. The model predicts that when programs only account for recognition's role in achieving better health, education, or income outcomes and do not consider its direct role in well-being, the resulting program design and resource allocations will be suboptimal. This implies the need to broaden the view of human recognition and related non-material components of development from being only a means of achieving better material outcomes to also being an objective of the same order – though not necessarily of the same priority – as material objectives. Such a broadening of perspective suggests that development at its optimum is a process of simultaneously improving material and non-material outcomes.

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Appendix 1: Derivation of Marginal Utility of Recognition Level

The general utility function is:

$$U = u_h[h(H, R)] + u_c[c(C, R, R_p)] + \phi R + \pi(R_p)$$

Substituting for $R_p{}^* = \{\mu + \psi R - \beta\}/2\alpha$, and $\pi(R_p) = \mu R_p + R_p \psi R_i - \alpha R_p{}^2 - \beta R_p$, the utility function becomes:

$$U = u_h[h(H, R)] + u_c[c(C, R, \{\mu + \psi R - \beta\}/2\alpha)] + \phi R + \mu\{\mu + \psi R - \beta\}/2\alpha + \psi R\{\mu + \psi R - \beta\}/2\alpha - \alpha(\{\mu + \psi R - \beta\}/2\alpha)^2 - \beta\{\mu + \psi R - \beta\}/2\alpha$$

To derive the expression for marginal utility of one's own level of human recognition, the above expression for utility is differentiated with respect to R.

Differentiating the first three terms, $u_h[h(H, R)] + u_c[c(C, R, \{\mu + \psi R - \beta\}/2\alpha)] + \phi R$, yields:

$$\frac{\partial U}{\partial h}\frac{\partial U}{\partial h_i}\frac{\partial h}{\partial R} + \frac{\partial U}{\partial c}\frac{\partial c}{\partial R} + \phi$$

Obtaining the final term of the marginal utility requires differentiating the expression, $\mu\{\mu + \psi R - \beta\}/2\alpha + \psi R\{\mu + \psi R - \beta\}/2\alpha - \alpha(\{\mu + \psi R - \beta\}/2\alpha)^2 - \beta\{\mu + \psi R - \beta\}/2\alpha$, with respect to R.

$$\frac{\partial \pi(R)}{\partial R} = \mu\psi/2\alpha + \mu\psi/2\alpha - \beta\psi/2\alpha + 2\psi^2 R/2\alpha - \psi(\mu + \psi R - \beta)/2\alpha - \beta\psi/2\alpha$$

$$= \mu\psi/2\alpha - \beta\psi/2\alpha + \psi^2 R/2\alpha$$

$$= \frac{\psi}{2\alpha}(\mu - \beta + \psi R)$$

So the expression for total marginal utility of one's own recognition level is

$$\frac{\partial U}{\partial R} = \frac{\partial U}{\partial h}\frac{\partial h}{\partial R} + \frac{\partial U}{\partial c}\frac{\partial c}{\partial R} + \phi + \frac{\psi}{2\alpha}(\mu - \beta + \psi R).$$

Appendix 2: Derivation of First Order Conditions and Comparative Statics in Labor Model

$$\max_{(c,\,H,\,R)} \quad \mathcal{L} = \alpha\log(c) + \beta\log(K\text{-}H) + \gamma\log(R) + \lambda(WH + N - Pc) + \mu(rH + \overline{R} - R)$$

First order conditions:

1. c: $\frac{\alpha}{c} - \lambda = 0$
2. H: $-\frac{\beta}{K-H} + \lambda W + \mu r = 0$
3. R: $\frac{\gamma}{R} - \mu = 0$
4. λ: $WH + N = c$
5. μ: $rH + \overline{R} = R$

Combining these five conditions yields:

$$\frac{\beta}{K-H} = \lambda W + \mu r . \quad \lambda = \frac{\alpha}{c}. \quad c = WH + N.$$

$$\frac{\beta}{K-H} = \frac{\alpha W}{WH+N} + \mu r . \quad \mu = \frac{\gamma}{R}. \quad R = rH + \overline{R} .$$

$$\frac{\beta}{K-H} = \frac{\alpha W}{WH+N} + \frac{\gamma r}{rH+\overline{R}} .$$

Differentiating this condition with respect to r yields:

$$\frac{\beta}{(K-H)^2}\frac{\partial H}{\partial r} = -\frac{\alpha W^2}{(WH+N)^2}\frac{\partial H}{\partial r} + \frac{(rH+\overline{R})\gamma - \gamma r(H + r\frac{\partial H}{\partial r})}{(rH+\overline{R})^2}$$

$$\frac{\partial H}{\partial r}\left[\frac{\beta}{(K-H)^2} + \frac{\alpha W^2}{(WH+N)^2} + \frac{\gamma r^2}{(rH+\overline{R})^2}\right] = \frac{\gamma\overline{R}}{(rH+\overline{R})^2}$$

$$\frac{\partial H}{\partial r}\left[\frac{\beta(WH+N)^2(rH+\overline{R})^2 + \alpha W^2(K-H)^2(rH+\overline{R})^2 + \gamma r^2(K-H)^2(WH+N)^2}{(K-H)^2(WH+N)^2(rH+\overline{R})^2}\right] = \frac{\gamma\overline{R}}{(rH+\overline{R})^2}$$

$$\frac{\partial H}{\partial r} = \frac{\gamma \overline{R}(K - H)^2 (WH + N)^2}{\beta (WH + N)^2 (rH + \overline{R})^2 + \alpha W^2 (K - H)^2 (rH + \overline{R})^2 + \gamma r^2 (K - H)^2 (WH + N)^2} > 0$$

because $\overline{R} > 0$ and $\alpha, \gamma > 0$.

Appendix 3: Derivation of Optimal Program Health and Recognition Investments

(1) $\frac{\partial \mathcal{L}}{\partial H} = \eta + \frac{\eta(R_0 + R)}{\lambda} - \Lambda p_H = 0$

(2) $\frac{\partial \mathcal{L}}{\partial R} = \frac{\eta(H_0 + H)}{\lambda} + \eta\sigma + \phi - \Lambda p_R = 0$

(3) $\frac{\partial \mathcal{L}}{\partial \Lambda} = M - Hp_H - Rp_R = 0$ ⟶ (4) $H = \frac{M - Rp_R}{p_H}$

(1) & (2) ⟶ $\frac{\lambda\eta + \eta(R + R_0)}{\lambda p_H} = \frac{\eta(H + H_0) + \eta\sigma\lambda + \phi\lambda}{\lambda p_R}$

(5) $R = \frac{p_H}{p_R}\left[H + H_0 + \sigma\lambda + \frac{\phi\lambda}{\eta}\right] - \lambda - R_0$

(4) & (5) ⟶ $H = \frac{M}{p_H} + \frac{(\lambda + R_0)P_R}{p_H} - (H + H_0 + \sigma\lambda + \frac{\phi\lambda}{\eta})$

(6) $\mathbf{H^*} = \frac{1}{2}\left[\frac{M + (\lambda + R_0)P_R}{P_H} - H_0 - \lambda(\sigma + \frac{\phi}{\eta})\right]$

(3) ⟶ (7) $R = \frac{M - Hp_H}{p_R}$

(6) & (7) ⟶ $R = \frac{M}{P_R} - \frac{M}{2P_R} - \frac{1}{2}(R_0 + \lambda) + \frac{P_H H_0}{2P_R} + \frac{P_H}{2P_R}\lambda(\sigma + \frac{\phi}{\eta})$

(8) $\mathbf{R^*} = \frac{1}{2}\left[\frac{M}{P_R} - R_0 - \lambda + \frac{P_H}{P_R}[H_0 + \lambda(\sigma + \frac{\phi}{\eta})]\right]$