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Sumit Joshi
George Washington University

Ahmed Saber Mahmud
Johns Hopkins University

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Institute for International Economic Policy
1957 E St. NW, Suite 502
Voice: (202) 994-5320
Fax: (202) 994-5477
Email: **iiep@gwu.edu**
Web: **www.gwu.edu/~iiep**

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Unilateral and multilateral sanctions: A network approach☆

Sumit Joshi[a,*], Ahmed Saber Mahmud[b]

[a] Department of Economics, George Washington University, 311 Monroe Hall, 2115 G Street NW, Washington, DC 20052, USA
[b] Advanced Academic Programs, Zanvyl Krieger School of Arts and Sciences, Johns Hopkins University, Suite 104, 1717 Massachusetts Avenue, Washington, DC 20036, USA

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ABSTRACT

The extensive literature on efficacy of sanctions has been mainly focused on a dyadic interaction between sender and target. In contrast, this paper examines sanctions when the sender and target are embedded in a network of linkages to other agents and each agent's utility is a function of the size of the agent's component. Efficacy of sanctions is then a function of two factors: the network structure binding the sender and target, and the concavity/convexity of utility in the component size. We consider both unilateral sanctions and multilateral sanctions. We demonstrate how the network architecture, together with the specification of utility, qualifies and sometimes reverses the main tenets of the dyadic approach. We add to the recent work on identifying network architectures that sustain cooperation via the threat of exclusion by showing that the utility specification matters. Thus the same network can be efficacious for sanctions if utility is convex in component size but not if it is concave.

1. Introduction

1.1. Motivation

Sanctions have a long history and refer to the mechanism through which one agent exercises punitive measures against another for failing to implement some desired action. The sanctioning agent is called the *sender* (S), and the sanctioned agent is called the *target* (T). The recalcitrance of T, measured by a "resistance" parameter $\beta > 0$, is due to the fact that the desired action is costly to implement. Examples include providing a social favor when asked, making requisite transfers to those affected by negative shocks, removing trade barriers, investing in public goods, or sharing information in a cooperative agreement. Sanctions can induce compliance from T only if they impose costs in excess of β. The existing literature has largely considered sanctions within a dyadic or two-way interaction between S and T. It is seldom that S and T are isolated and more often than not will find themselves bound in some *network* of links (social, economic, or political). Our main

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* Corresponding author.
E-mail addresses: sumjos@gwu.edu (S. Joshi), amahmud2@jhu.edu (A.S. Mahmud).

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contribution is to consider sanctions in an environment which explicitly accounts for the network of links that bind *S* and *T*. The underlying rationale is that when *S* and *T* are embedded within a network, then the costs inflicted by sanctions on *T* is critically dependent on the location of *S* and *T* within the network architecture. The effectiveness of sanctions to secure the desired action from *T* is thus best analyzed in a framework that explicitly accounts for the web of linkages connecting *S* and *T* as compared to one in which their relationship is purely dyadic and shorn of the finer details of their connection to others. We examine sanctions in the context of a network model where an agent's utility is a function of the size of the agent's component. We show that two factors play a critical role in determining whether *T* can be forced to comply: the network *architecture* and the *concavity/convexity* of utility in component size. Both factors have important implications when considering either the duration of sanctions (short run versus long run) or its organization (unilateral versus multilateral).

The literature on a dyadic interaction between *S* and *T* is voluminous and spans diverse fields (economics, politics, international relations), treatments (theoretical, empirical) and approaches (interest group models, single-rational actor models).[1] Culling this large literature we can identify some recurring themes that can be said to constitute the main tenets of the dyadic approach.

1. *Asymmetry between sender and target*: Sanctions are successful when the cost to *T* is relatively high and that to *S* is relatively low (Hufbauer et al., 1990; Morgan and Schwebach, 1997; Drury, 1998; Eaton and Engers, 1999). This in turn requires that *S* be significantly "tougher", and *T* correspondingly "weaker", along some payoff-relevant dimension. For example, *T* can be more impatient (Eaton and Engers, 1992), politically or economically distressed (Drury, 1998) or smaller as measured by GNP or market size (Hufbauer et al., 1990; Drury, 1998).[2]

2. *Short run versus long run*: The duration of sanctions is negatively related to their effectiveness (Kaempfer and Lowenberg, 2007 and the references therein, Drury, 1998). Sanctions that are imposed over a longer horizon generally have their effectiveness blunted by allowing *T* a longer reaction time to explore substitution possibilities.

3. *Unilateral versus multilateral sanctions*: Multilateral sanctions – when *S* also draws on other agents to sanction *T* – are generally less effective than unilateral sanctions (Doxey, 1987; Drury, 1998; Kaempfer and Lowenberg, 1988). This is often due to the coordination and implementation problems associated with organizing a sanctioning coalition composed of agents with dissimilar objectives or asymmetric leverage over *T*.

Before outlining the qualifications offered by networks on these tenets of the dyadic approach, we need to specify how an agent's position in the network impacts her (reduced form) utility. Networks are complex constructs and can influence an agent's utility through myriad channels. Therefore no one model can fully capture the impact of a network on the effectiveness of sanctions. In this paper we utilize the well known *components* model in which an agent's utility is a function of those she is connected to directly or indirectly. This model is flexible enough to accommodate a large variety of examples of sanctions. We will call a network *effective* if its architecture induces *T* to comply and switch to the desired action when *S* threatens to sanction. In addition to the network topology, effectiveness of sanctions is also seen to crucially hinge on the *concavity/convexity* of utility in the component size. Henceforth, for brevity, we will only say *concave* (respectively *convex*) utility, taking it as understood that it is with respect to the component size and holds strictly, i.e. the incremental utility of an agent from a unit increase in her component size is *strictly decreasing* (respectively *strictly increasing*). We now offer a heuristic overview of the connection between the network architecture, concavity or convexity of utility, and effectiveness of sanctions.

1.2. The network approach

Let us begin with unilateral sanctions when *S* acts alone to sanction *T* by deleting their link (denoted as *ST*).[3] With regard to asymmetry between *S* and *T*, in networks the dominance of *S* over *T* is manifested through *centrality* in location. Therefore *S* could be disadvantaged along dimensions such as GNP or market size and still exercise significant leverage over *T* simply by serving as a "bridge" who provides singular connectivity to *T* to a large component. Consider Fig. 1. With concave utility, an effective network needs to be highly asymmetric placing *S* in a position of significant advantage over *T*. Thus *S* could be the center of a "star" as in g^5 with the ability to relegate *T* to a singleton via unilateral sanctions. The network g^4 is however more interesting because *S* has the same number of links as *T* but can force *T* to comply through the threat of denying connectivity to a large component. With convex utility, networks such as g^1 and g^0 that post-sanctions place *S* and *T* into fragmented

[1] Single-rational actor theoretical models have been examined among others by Barrett (1997), Eaton and Engers (1992,1999). Theoretical models of interest groups have been examined for example by Gershenson (2002) and Kaempfer and Lowenberg (1988, 1999). For reasons of space we have cited only a few papers for illustration. Please see Kaempfer and Lowenberg (2007) for an excellent survey of the theoretical and empirical literature and an exhaustive reading list.

[2] Hufbauer et al. (1990) find the size effect to be insignificant although the empirical literature on sanctions has been criticized for selection bias (Kaempfer and Lowenberg, 2007).

[3] Note that sanctions also impose costs on *S*. This may call into question *S*'s incentives to carry out sanctions. There are two possible responses to this concern. First, *S* may be guided by broader concerns to impose sanctions even when they are costly for *S*; this is explicitly accounted for when considering long run multilateral sanctions by introducing a "tolerance" level for *S*. Second, in an effective network, the sanctions will never actually be carried out because *T* will have an incentive to comply. This is reminiscent of the punishments embedded into trigger strategies in repeated games that are never executed along the equilibrium path.

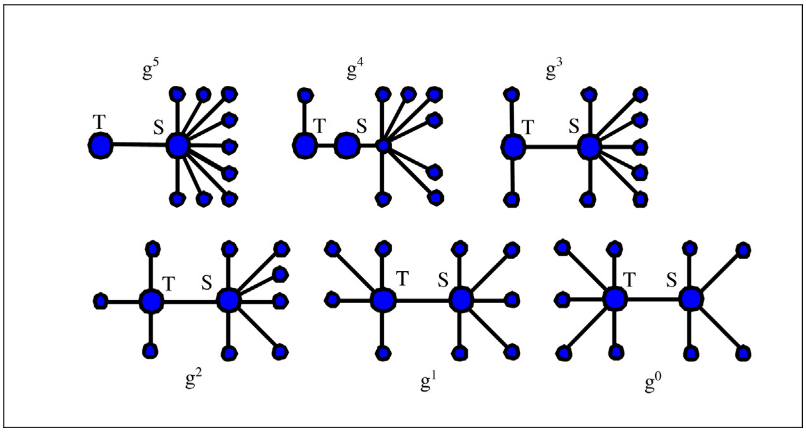

Fig. 1. Networks linking *S* and *T* (Link *ST* is a “Bridge”).

components that are roughly symmetric in size are sufficiently effective to induce compliance. Explicitly recognizing the network architecture and the specification of utility affords a more nuanced view of the effectiveness of sanctions.

With regard to short run versus long run effectiveness, we first need to define these terms in the network context. We let *short run* denote the period of time over which agents can only delete existing links in some historically given network. In contrast, *long run* is the period of time over which agents can also bilaterally form costly new links with others. The rationale underlying this distinction is that deleting a link can be achieved unilaterally and immediately. However forming a new link requires approval of the two agents involved which in turn needs time to build mutual trust. Sanctions imposed on *T* over a longer duration permits *T* the possibility of circumventing these sanctions by finding agents other than *S* to link with. The network architecture explicitly captures the substitution possibilities available to *T* to replace the severed link with *S*. Consider Fig. 1 once again. In the short run g^5 is more effective than g^4 since post-sanctions the component of *S* is larger (and that of *T* smaller) in the former. Over the long run, for low linking costs, *T* has an incentive to reconnect the two components by establishing a link with any agent (say *i*) in *S*'s component, and agent *i* in turn has an incentive to reciprocate since sanctions adversely impact her utility as well. We can therefore compare the relative long run effectiveness of networks in supporting sanctions in terms of the set of linking costs that prevent the post-sanctions fragmented components from reconnecting. With concave utility, a “*topsy-turvy*effective in the short run are in fact less effective in the long run. Therefore g^5 is less effective than g^4 in the long run because, paradoxically, the greater punitive impact of sanctions in g^5 creates a stronger incentive for *T* to link up with agent *i* with the concomitant willingness to accept a larger share of the linking costs. On other hand, if utility is convex, then the incentives of *T* and agent *i* to link up are diluted and g^5 is more effective than g^4 in both the short and the long run. Therefore it is no longer true that the duration of sanctions reduces their effectiveness.

Networks can provide particularly rich insights with regard to multilateral sanctions. In Fig. 1 the link *ST* is a *bridge* that disconnects the network when deleted. However in a large number of networks the link *ST* may not be a bridge.[4] Multilateral sanctions, where *S* assembles a *sanctioning coalition* to delete multiple links, is now the only recourse. In the short run multilateral sanctions can be effective if: (i) there exists a sanctioning coalition that *S* can assemble to restrict *T* to a sufficiently small component, and (ii) *S* can successfully persuade the members of the sanctioning coalition to sever appropriate links. Once again it turns out that these two requirements are more efficaciously implemented under convex utility. Since *T*'s post-sanctions component under convex utility does not have to be reduced to the same extent as under concave, there are networks that are effective under convex but never under concave utility. Further, in networks that are supported for both utility specifications, convex utility require sanctions of lower order in the sense that *S* has to draw into the sanctioning coalition neighbors that are closer in the network. With concave utility *S* has to rely on more distant neighbors as well thereby increasing the size of the group whose incentives need to be aligned to *S*. Convex utility continue to be more efficacious than concave in the long run as well in the sense of permitting *S* to judiciously form new links to convert an ineffective network into an effective one at lower cost.

[4] The network is historically given and it is possible that agents have inherited links that are not bridges. For instance, it is plausible that in the past, when the network was being established, agents had concerns over the durability of links and over-connected with other agents to ensure continued linkage to the overall network. This tendency to over-link could have been supported by negligible linking costs in the initial phases of network formation. However, after the original network has endured for some time, any new links henceforth involve costs. Another possible explanation lies in reinterpreting linking costs. We will see that new links in the long run will be initiated by either *S* or *T* to respectively enforce sanctions or rebuff them. Therefore, we can interpret costs of linking as transfers by *S* and *T* to other agents to draw them to their side.

1.3. Existing literature

We now place our paper in the context of existing work. Our paper builds on Joshi and Mahmud (2016) who considered *multilateral* sanctions in networks and examined the implications of the network architecture for effectiveness of sanctions. The paper did not consider the role of concave/convex utility, nor did it discuss unilateral sanctions. In contrast this paper considers both unilateral and multilateral sanctions and shows that, in addition to the network architecture, the concavity or convexity of utility also matters. Thus the same network can be efficacious for sanctions of one specification of utility but not for another. Other papers examine sanctions in the context of supporting cooperation in informal networks. Ambrus et al. (2014), Bloch et al. (2008), and Jackson et al. (2012) consider networks in which T is an agent who does not make a requisite income transfer or provide a favor when asked. S imposes sanctions either unilaterally or by having all agents to whom she is connected severing their direct links to T. While Bloch et al. (2008) and Jackson et al. (2012) focus on characterizing networks in which informal risk-sharing agreements or reciprocal favors can be supported by punishments involving severance of links, Ambrus et al examine the extent of informal risk-sharing in a given network. Our paper differs from them in that we consider both unilateral and multilateral sanctions and, in the case of unilateral sanctions, both the short run and the long run. Moreover, we examine the role of a concave or convex specification of utility in dictating the efficacy of sanctions.

Our paper is organized as follows. The components model and examples are presented in Section 2. Sections 3 and 4 examine unilateral sanctions in the short run and long run respectively. Sections 5 and 6 consider the case of multilateral sanctions. Since the characterization of effective networks for multilateral sanctions is already available in Joshi and Mahmud (2016), the focus in Sections 5 and 6 is on drawing out the role of concave/convex utility in determining the efficacy of sanctions. Section 7 briefly considers an alternate formulation of the utility function, and Section 8 concludes. All proofs are contained in an appendix.

2. The model

Let $\mathcal{N} = \{1, 2, \ldots, N\}$, $N \geq 3$, denote the set of agents. The starting point for our analysis is a historically given network that we describe next. We then outline the components model and present a number of examples to illustrate the wide range of instances of sanctions that can be fruitfully analyzed using this model.

2.1. Networks

Let $g^{\mathcal{N}}$ denote the collection of all two-agent subsets of $\mathcal{N}$. A bilateral relationship between i and j is denoted by the link ij and the collection of all such links that currently exist is denoted by $g \subset g^{\mathcal{N}}$. A *network* (or *graph*) is the tuple $(\mathcal{N}, g)$, though for brevity we will simply refer to g as the network. $\mathcal{G}$ is the set of all networks. The set $\mathbf{N}_i(g) = \{j \in \mathcal{N} \backslash \{i\} : ij \in g\}$ denotes the *neighbors* of i in g and $n_i(g) = |\mathbf{N}_i(g)|$ its cardinality.

A *path* in g connecting i and j is the sequence of distinct links $ii_1, i_1 i_2, \ldots, i_{n-1} i_n, i_n j \in g$. The number of links on the shortest path in g connecting i to j is the *distance*, $d_{ij}(g)$, between these agents in g. A network is *connected* if there exists a path between any pair $i, j \in \mathcal{N}$; otherwise the network is *unconnected*. A sub-network, $C(g) \equiv (\mathcal{N}', g')$, $\mathcal{N}' \subset \mathcal{N}$, $g' \subset g$, is a *component* of the network $(\mathcal{N}, g)$ if it is connected and if $ij \in g$ for $i \in \mathcal{N}'$, $j \in \mathcal{N}$, implies $j \in \mathcal{N}'$ and $ij \in g'$. With some abuse of notation, we let $|C(g)|$ denote the number of agents in the component $C(g)$. We will denote that agent i belongs to a component $C(g)$ in a network g by writing it as $C_i(g)$. The historically given network g will be assumed to be connected.[5] Consider a network g such that $ij \notin g$; then $g + ij$ will denote the network derived from g in which the link between i and j has been added. Similarly, if $ij \in g$, then $g - ij$ will denote the network derived from g in which the link ij has been deleted. We will say that a link ij forms a *bridge* or a 1-*link cut* in a network g if $g - ij$ has more components than g.

2.2. The components model

In the components model each agent benefits equally from both direct and indirect connections to other agents. Recalling that $C_i(g)$ is the component to which agent i belongs in a network g, the utility of agent i is given by a continuous function:

$$u_i(g) = u(|C_i(g)|), \; u(h+1) > u(h) \tag{1}$$

This formulation assumes that all agents are ex-ante symmetric in the sense of having identical utility functions, u, but may get different utilities in a network based on the size of their respective components. The symmetry assumption ensures that effectiveness of sanctions stems from the network characteristics of agents rather than differences in ex-ante utility. We will let utility be linear, strictly concave, or strictly convex in the size of the component. We will denote these utility

[5] This assumption is without loss of generality in the short run. Our analysis of course requires S and T to be in the same component before sanctions. Players in other components do not have any role in dictating the efficacy of sanctions in the short run and thus can be ignored. In the long run, we simply have to impose the condition that T cannot circumvent sanctions by forming jointly profitable links with players in either S's component or in other components.

functions as u^0, u^-, and u^+ respectively. To meaningfully compare the three cases we will impose the following boundary restrictions:

$$u^0(1) = u^-(1) = u^+(1),\ u^0(N) = u^-(N) = u^+(N) \tag{2}$$

We now present examples of sanctions in networks that can be considered within the components framework. The first two examples illustrate the case of linear utility.

Example 2.1. *The Connections Model*

This model is adapted from Jackson and Wolinsky (1996) and is useful in considering situations where sanctions take the form of social excommunication of the target. There are N agents connected via a social network g. Each agent has social "worth" V. Agents derive utility directly from the social worth of their neighbors as well as indirectly from the social worth of neighbors of their neighbors and so on with utility falling with distance. Letting $0 < \delta < 1$, the value that agent i gets from agent j in the component $C_i(g)$ is equal to $\delta^{d_{ij}(g)}V$ where $d_{ij}(g)$ is the shortest distance between the two agents in the component. The utility of agent i in the network g is equal to:

$$u_i(g) = V + V \sum_{j \in C_i(g)} \delta^{d_{ij}(g)} \tag{3}$$

When $\delta = 1$, social worth of indirect connections provide the same utility as direct connections and therefore:

$$u_i(g) = u^0(|C_i(g)|) = V|C_i(g)| \tag{4}$$

Therefore utility is linear in the size of the component. □

Example 2.2. *Exchange of Favors*

This model is adapted from Jackson et al. (2012). Agents are linked via social ties in a network g and can receive favors from or provide favors to those in their component, $C(g)$. Failure to provide a favor when asked results in sanctions that take the form of dissolving social ties. In any period t an agent may need a favor (which gives utility V) or be asked to grant a favor (which costs c) with probability $p \in (0, 1)$, where $V > c > 0$. Letting $\delta \in (0, 1)$ denote the agent's discount factor, and noting that the agent may have to grant favors to or receive favors from $|C(g)| - 1$ other members in the component, the agent's expected utility is:

$$u_i(g) = u^0(|C_i(g)|) = \frac{p(V - c)(|C(g)| - 1)}{1 - \delta} \tag{5}$$

Therefore expected utility is linear in the size of the component.□

The next three examples illustrate the case of strictly concave utility.

Example 2.3. *Trade Networks*

This model is based on Basu, 1986; Basu, 2000. It is useful in modeling situations where agents are connected via a trade network and sanctions take the form of restricting the target's access to trade. An alternative application following Barrett (1997) is when the failure to take a desired action in one sphere (reducing greenhouse emissions, forswearing nuclear weapons, respecting human rights) is punished through sanctions in another sphere (severing trading links). There are N distinct goods and agent i has an endowment of ω units of good i and zero units of the $N - 1$ other goods. Letting x_i^j denote agent i's consumption of good j, the utility function of agent i is given by:

$$u_i(\mathbf{x}_i) = \sum_{j=1}^{N} \sqrt{x_i^j} \tag{6}$$

where $\mathbf{x}_i = (x_i^1, x_i^2, \ldots, x_i^N)$. Let $\mathbf{e}^i$ denote the row vector with 1 in the ith coordinate and 0's elsewhere. Under autarky, agent i's utility is:

$$u_i(\omega \mathbf{e}^i) = \sqrt{\omega} \tag{7}$$

An agent can only trade with those whom it knows directly or indirectly. If agent i belongs to the component $C_i(g)$, then the utility of agent i is:

$$u_i(g) = u^-(|C_i(g)|) = \sqrt{\omega |C_i(g)|} \tag{8}$$

Therefore utility is strictly concave in the size of the component. □

Example 2.4. *Risk-Sharing Networks*

This model is based on Bramoullé and Kranton (2007b). It is useful in analyzing situations involving group lending and joint liability where sanctioning of the target due to non-provision of the requisite transfers takes the form of social exclusion from a risk-sharing mechanism.[6] There are N risk-averse individuals, each with utility function v over income, who

[6] See for instance Besley and Coate (1995).

are connected to each other via a risk-sharing network. Each individual's income is subject to iid shocks. Formal insurance mechanisms are not available and therefore individuals reduce risk through income-sharing. After the income shocks are realized, risk sharing proceeds sequentially with each agent being randomly matched to one of her neighbors at least once and the linked pair sharing their monetary holdings equally. Bramoullé and Kranton show that the expected utility of agent i in component $C_i(g)$ is:

$$u_i(g) = u^-(|C_i(g)|) = Ev\left(\frac{\sum_{j\in C_i(g)} y_j}{|C(g)|}\right) \quad (9)$$

where the expectation, Ev, is with respect to all income realizations, y_j, for all individuals j in $C_i(g)$. Bramoullé and Kranton consider parametrizations under which utility is strictly increasing and strictly concave in the size of the agent's component.[7]□

Example 2.5. *The Public Goods Network*

This model is adapted from Bramoullé and Kranton (2007a) and is useful in examining situations where sanctions via social exclusion is triggered when the target either fails to contribute to a public good or contributes less than the required amount. The N agents are connected in a network g, and each agent i exerts effort $e_i \in [0, \infty)$ at constant marginal cost $c > 0$ that is a perfect substitute for the effort expended by other agents in $C_i(g)$ and non-excludable from them.[8] Agent i's utility in the network g is given by:

$$u_i(\mathbf{e}; g) = b\left(e_i + \sum_{j\in C_i(g)} e_j\right) - ce_i \quad (10)$$

where $b(0) = 0$, $b' > 0$ and $b'' < 0$. Agents play a Nash game in efforts, and in a Nash profile the aggregate effort exerted in each component is e^* where e^* satisfies $b'(e^*) = c$. If all agents in a component exert equal effort,[9] then agent i's utility becomes:

$$u_i(g) = u^-(|C_i(g)|) = b(e^*) - \frac{ce^*}{|C_i(g)|} \quad (11)$$

The utility function is therefore strictly increasing and strictly concave in the size of the component. □

The final two examples consider the case of strictly convex utility.

Example 2.6. *Collusive Networks*

This example is based on Joshi and Vora (2013).[10] It can be utilized to examine scenarios where sanctions take the form of exclusion from a cooperative agreement such as a cartel or a joint venture following a deviation by the target. There are N symmetric risk-neutral bidders in a second-price private value auction. Each bidder's valuation for the good is private information and is drawn independently from [0, 1] according to a distribution function F with associated density function f. These facts are common knowledge among the bidders. A bidding ring is a subset of bidders who belong to the same component. Graham and Marshall (Theorem 1) describe a preauction knockout mechanism which is individually rational and incentive compatible. If z is the random variable denoting the highest valuation inside the ring, and y the highest valuation outside the ring, then the ring makes positive profits if $z > y$. The utility to a ring member i who belongs to a component $C(g)$ is given by:

$$u_i(g) = u^+(|C_i(g)|) = \int_0^1\int_0^z F(y)^{N-|C_i(g)|}F(z)^{|C_i(g)|-1}f(z)dydz,\ z > y \quad (12)$$

Utility is strictly increasing and strictly convex in the size of the component. □

Example 2.7. *R&D Collaboration Networks*

This example is adapted from Goyal and Joshi (2003). Consider N local monopolies, where monopoly i has inverse demand function $P_i = a - Q_i$. Each monopoly produces with constant marginal cost of production. Each monopoly can be connected to others through a network of R&D collaboration links. The links can be interpreted as an arrangement to share R&D costs and cost-reducing innovations among monopolies that are connected directly and indirectly. Therefore monopoly i's marginal cost is given by $MC(g) = c_0 - c|C_i(g)|$, where $a > c_0$ and $c > 0$, and its profits are equal to:

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[7] Bloch et al. (2008) consider a more general formulation in which agents can informally insure each other through transfers determined by prior bilateral norms.

[8] In Bramoullé and Kranton (2007a), each agent's effort is non-excludable only along the direct links of that agent. Therefore each agent i gains only from those agents who belong to $\mathbf{N}_i(g)$.

[9] Bramoullé and Kranton (2007a) refer to this case as a distributed Nash equilibrium. Another possibility is a specialized Nash equilibrium in which only one agent specializes and exerts e^* while the others agents in the component free ride by exerting zero effort. If the specialist is randomly picked from the component, then we get the same utility as in (11) below.

[10] The model is a network adaptation of bidding rings in second-price auctions analyzed by Graham and Marshall (1987).

$$u_i(g) = u^+(|C_i(g)|) = \frac{1}{4}(a - c_0 + c|C_i(g)|)^2 \tag{13}$$

Profits are strictly increasing and strictly convex in the size of the component. If a member in a component reneges on sharing R&D costs or cost-reducing breakthroughs, then sanctions take the form of severing collaborative links. □

3. Unilateral sanctions in the short run

In the short run agents can only sever links but cannot form new ones. We justify this on the grounds that forming a new link requiring approval of both agents needs time and resources to build the necessary level of trust. Unilateral sanctions occur when $ST \in g$ and this link is severed when T does not take the desired action. T will comply if:

$$u_T(g) - u_T(g - ST) \geq \beta \tag{14}$$

i.e. the reduction in utility from noncompliance and the ensuing sanctions exceeds T's resistance (assuming compliance in the case of indifference).[11] We will say that the network g is *effective in the short run with respect to unilateral sanctions*, or simply effective when the context is clear, if and only if (14) holds. Note that in an effective network the sanctions are not actually implemented because T complies. It is immediate that unilateral sanctions are effective only if the link ST exists in the network and constitutes a bridge. Therefore in this section we will focus on the subset $\mathcal{G}^{ST} \subset \mathcal{G}$ of networks that contain the link ST and in which ST is a bridge.

In order to relate efficacy of sanctions to the structure of the underlying network, we will now introduce the notion of *essentiality* which measures how important one agent is to the connectivity of the other. Accordingly for $i, j \in C(g)$, the essentiality of agent j to agent i in g is defined as:

$$\mathcal{E}_{ij}(g) = |C(g)| - |C_i(g - ij)| \tag{15}$$

It measures the extent to which agent i's component breaks up when agent j severs their link. Since g is assumed to be connected, deleting the link ST from g partitions the set $\mathcal{N}$ into two sets comprising of agents connected (directly or indirectly) respectively to S and T with cardinalities $\mathcal{E}_{TS}(g) = |C_S(g - ST)|$ and $\mathcal{E}_{ST}(g) = |C_T(g - ST)|$ such that $\mathcal{E}_{ST}(g) + \mathcal{E}_{TS}(g) = N$.

To avoid the trivial case where sanctions are never effective we will maintain the following parametric restriction throughout for all utility specifications:

$$0 < \beta < u(N) - u(1) \tag{16}$$

Since u is continuous, the following is well-defined:

$$h^T(\beta) = \max\{h = 1, 2, \ldots, N-1 : u(N) - u(h) \geq \beta\},\ h^S(\beta) = N - h^T(\beta) \tag{17}$$

$(h_0^T(\beta), h_0^S(\beta))$, $(h_-^T(\beta), h_-^S(\beta))$ and $(h_+^T(\beta), h_+^S(\beta))$ are correspondingly defined for utility functions u^0, u^-, and u^+ respectively. There is no loss of generality in supposing that for linear utility, $u(N) - u(h_0^T(\beta)) = \beta$. The following proposition now follows directly from the definitions.

Proposition 1. *A network g is effective in the short run with regard to unilateral sanctions if and only if* $\mathcal{E}_{TS}(g) \geq h^S(\beta)$.

A network is effective if and only if S is sufficiently essential in the sense of relegating T to a small enough component after sanctions. Of course sanctions will also exact costs on S. We would therefore like to order effective networks in terms of the costs imposed on S. Accordingly we first rank networks in $\mathcal{G}^{ST}$ by defining a binary order $\succsim$ on $\mathcal{G}^{ST}$ as follows:

$$g' \succsim g \Leftrightarrow \mathcal{E}_{TS}(g') \geq \mathcal{E}_{TS}(g),\ g, g' \in \mathcal{G}^{ST}$$

S retains a larger component post-sanctions in g' relative to g and S's utility loss is correspondingly smaller in g' relative to g. This ordering is complete on $\mathcal{G}^{ST}$. The orderings $\sim$ and $\succ$ can be induced from $\succsim$ in the standard way. Since $\mathcal{G}^{ST}$ is finite, we can reindex and order the elements of this set as:

$$g_M \succsim g_{M-1} \succsim \cdots \succsim g_3 \succsim g_2 \succsim g_1 \tag{18}$$

In Fig. 1 for example, $g^5 \succ g^4 \succ g^3 \succ g^2 \succ g^1 \succ g^0$. Note that the ranking depends only on the size of the components after the link ST is deleted and not on the configuration of links within each component.

We now address *relative* effectiveness. If g and g' are effective, and $\mathcal{E}_{TS}(g') \geq \mathcal{E}_{TS}(g)$, then we say that g' is relatively more effective than g. Consider a linear utility and pick any network g in $\mathcal{G}^{ST}$ for which $|C_T(g - ST)| = h_0^T(\beta)$ and let its index be n_0. (If there is more than one network satisfying this property, then arbitrarily pick any one as n_0). Recalling (18), assign succeeding indices to networks in the order of increasing relative effectiveness. The indices n_- and n_+ can be defined similarly for concave and convex utilities for the network with the lowest index for which the post-sanctions component of T is less than or equal to respectively $h_-^T(\beta)$ and $h_+^T(\beta)$. We can now prove:

[11] Note that no additional links will be deleted by agents following the dissolution of ST. In the components model, deleting any additional link will either decrease utility or leave it unchanged depending respectively on whether the link is a bridge or not in the network.

Proposition 2. *Suppose utility functions* $\left(u^0, u^-, u^+\right)$ *satisfy* (2).

(a) *All networks* g_n*,* $n > n_-$*, are effective with concave utility and are arranged in decreasing order of effectiveness as* $g_M \succsim g_{M-1} \succsim \cdots \succsim g_{n_-}$*. Moreover,* $n_- \geq n_0$*.*
(b) *All networks* g_n*,* $n > n_+$*, are effective with convex utility, and are arranged in decreasing order of effectiveness as* $g_M \succsim g_{M-1} \succsim \cdots \succsim g_{n_+}$*. Moreover,* $n_+ \leq n_0$*.*

According to Proposition 2, concave utililty reduces the number of effective networks relative to the linear case. The intuition is simple. With concave utility, the severance of a given set of links in a connected network has a smaller marginal impact on the utility of T than with linear utility; therefore, T needs be to relegated to a post-sanctions component of much smaller size for the sanctions to be effective. By the same logic, convex utility increases the number of effective networks relative to the linear case. In this sense, convex utility is more conducive to the implementation of sanctions. The following example illustrates this proposition.

Example 3.1. Let $u^0(h) = h$. For both concave and convex utility, let $\Delta u(h) = u(h) - u(h-1)$. Now consider the following:

Case	h									
	2	3	4	5	6	7	8	9	10	11
$\Delta u^-(h)$	3	1.75	1.5	1.25	1	0.5	0.4	0.3	0.2	0.1
$\Delta u^+(h)$	0.26	0.5	0.92	0.93	0.94	0.95	1	1.25	1.5	1.75

Consider the networks in Fig. 1. Let $\beta = 7$. With linear utility, $h_0^T(\beta) = 4$, $n_0 = 2$, and all four networks, $g^5 \succ g^4 \succ g^3 \succ g^2$, are effective. Consider concave utility and note that $h_-^T(\beta) = 2$ and $n_- = 4$. Therefore both networks, $g^5 \succ g^4$, are effective. With convex utility, $h_+^T(\beta) = 5$ and $n_+ = 1$. Therefore all five networks, $g^5 \succ g^4 \succ g^3 \succ g^2 \succ g^1$, are effective. □

4. Unilateral sanctions in the long run

In the long run agents can form new links. Forming links is costly since it entails a commitment of resources. Let $f > 0$ denote the cost to any agent of forming a link with another. In an effective network in the short run, T complies because otherwise the network is disconnected. However if T can form links to reconnect the fragmented components then it can circumvent the unilateral sanctions imposed by S. In fact it is possible to envisage situations where T would be willing to cover the entire cost of another agent's link in order to induce that agent to reciprocate. Our definition of effectiveness in the long run should accordingly allow agents to not only form links, but also possibly subsidize the linking costs of potential partners. Borrowing from Bloch and Jackson (2006), we will require in the long run that any two unconnected agents in $g - ST$ should not find it *jointly* profitable to form a link. To be precise, given linking cost $f > 0$, for any $i, j \in \mathcal{N}$ with $ij \notin g - ST$:

$$[u_i(g - ST + ij) - u_i(g - ST)] + \left[u_j(g - ST + ij) - u_j(g - ST)\right] \leq 2f \tag{19}$$

A network g will be *effective in the long run with respect to unilateral sanctions* given linking cost $f > 0$ (or effective for brevity when there is no ambiguity) if along with (14) we have (19). In other words, (i) unilateral sanctions should relegate T to a sufficiently small post-sanctions component and (ii) linking costs should be sufficiently high to prevent T from mustering the necessary investment needed to reconnect the fractured components by inducing an agent from S's component to reciprocate the link.[12]

We explore two issues in the long run. The first is to keep the specification of utility fixed and compare the effectiveness of networks across the short run and long run. The second is to restrict ourselves to the long run, and compare the effectiveness of networks across the three types of utilities. With regard to the first issue, the general consensus in the received dyadic literature is that sanctions are harder to maintain in the long run than in the short run. Our result qualifies this view by showing that short run versus long run effectiveness depends on the utility specification. Let $f(g - ST; u)$ denote the linking cost for which (19) is satisfied as an equality for network g and utility function u. If g satisfies (14), then it is long run effective for all linking costs $f \geq f(g - ST; u)$.

Proposition 3. *Suppose utility functions* $\left(u^0, u^-, u^+\right)$ *satisfy* (2)*. Then:*
(*a*)$f(g' - ST; u^0) = f(g - ST; u^0)$ *for all* $g' \succsim g \succsim g_{n_0}$.
(*b*)$f(g' - ST; u^-) > f(g - ST; u^-)$ *for all* $g' \succ g \succsim g_{n_-}$.
(*c*)$f(g' - ST; u^+) < f(g - ST; u^+)$ *for all* $g' \succ g \succsim g_{n_+}$.

With linear utility, all networks that are effective in the short run continue to be effective in the long run as well for the same set of linking costs. Since incremental utility is independent of the size of an agent's component, once the set of linking

[12] Agents in a component are identically placed in terms of incentives. If T cannot form a jointly profitable link with an agent from S's component, then no pair of agents from the two components can form a jointly profitable link.

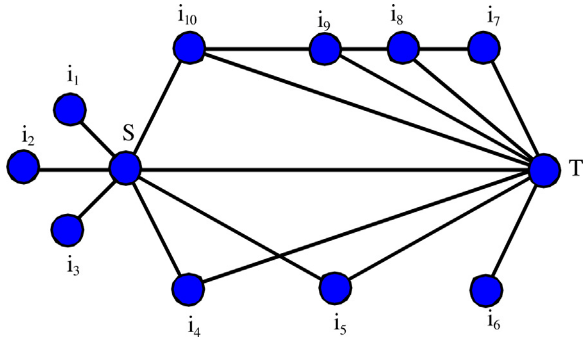

Fig. 2. The case for multilateral sanctions.

costs that support the short run effective network g_{n_0}, $(f(g_{n_0} - ST; u^0), \infty)$, has been determined, then this set of linking costs continues to support all networks $g \succsim g_{n_0}$.

In the case of concave utility on the other hand we observe an interesting "*topsy-turvy principle*than another, g, at supporting sanctions in the short run is less effective at doing so in the long run. In particular, there is a non-empty set of linking costs for which unilateral sanctions will be effective in the long run in g but ineffective in g'. Putting it differently, long run compliance from T can be secured in the network g for a strictly larger set of cost of link formation than in g'.

An argument exactly opposite to the topsy-turvy principle applies in the case of convex utility. Therefore convex utility overturns the conventional point of view that sanctions are less effective in the long run relative to the short run. If g' is more effective than g in the short run, then it continues to be more effective than g in the long run as well in the sense of being supported by a larger set of link formation costs. In this sense, convex utility is more conducive to the effective implementation of sanctions as compared to concave.

Next we restrict attention to the long run and compare effectiveness of a network across the three specifications of utility in terms of the link formation costs.

Proposition 4. *Suppose utility functions* $\left(u^0, u^-, u^+\right)$ *satisfy* (2). *In the long run*:

$$f(g - ST; u^+) > f(g - ST; u^0) > f(g - ST; u^-).$$

Here we see an interesting caveat to the result that a larger set of networks are effective under convex utility. For the *same* network that is effective under all utility specifications, convex utility requires linking costs to be much greater in order to support sanctions. It confers much larger incremental utility gains as compared to concave utility and therefore linking costs need to be correspondingly greater to circumvent the re-linking of post-sanctions components.

Example 4.1. Let us consider the utility functions of Example 3.1. We can compute the following:

Utility function	$f(g^2 - ST;u)$	$f(g^3 - ST;u)$	$f(g^4 - ST;u)$	$f(g^5 - ST;u)$
Linear	5.5	5.5	5.5	5.5
Strictly concave	•	•	3.65	5.05
Strictly convex	6.91	6.87	6.5	5.9

Reading along the rows illustrates Proposition 3. For instance, the row for concave utility highlights the topsy-turvy principle: g^5 is more effective than g^4 in the short run, but is less effective in the long run since, for all $f \in (3.65, 5.05)$, T will successfully reconnect to S's component in g^5. Reading down each column illustrates Proposition 4. □

5. Multilateral sanctions in the short run

Unilateral sanctions require that the link $ST \in g$ is a bridge. However, if this is not the case, then S will have to act in conjunction with other agents to sanction T with multi-link cuts. S will need to identify the precise links to be deleted to disconnect T and determine the resulting size of the post-sanctions component of T. These considerations will influence S's choice of a *sanctioning coalition* to exert pressure on T. Formalizing these ideas will require the graph-theoretic notions of spanning trees and cutsets. Consider network g in Fig. 2.

A *spanning tree* in a (connected) network g is a connected acyclic sub-network. Formally, for a given g a subset of links $\tau(g)$ is called a *spanning tree* if the network $(\mathcal{N}, \tau(g))$ is connected and acyclic. We will consider spanning trees that include the link ST although it is not central to the analysis that follows. It simply reflects the fact that, in most instances, sanctions commence with the rupture of ST. Given $\tau(g)$, deleting the link ST defines a partition of $\mathcal{N}$ into two subsets, $\mathcal{N}^S_{\tau(g)}$ and $\mathcal{N}^T_{\tau(g)}$. All links in g of the form $\{kl : k \in \mathcal{N}^S_{\tau(g)}, l \in \mathcal{N}^T_{\tau(g)}\}$ comprise the *cutset* of $\tau(g)$ with respect to ST. We represent this cutset as $D_{\tau(g)}(ST) \equiv (\mathcal{N}^S_{\tau(g)}, \mathcal{N}^T_{\tau(g)})$. The links in $D_{\tau(g)}(ST)$ are called *multi-link cuts* and deleting them disconnects g into two

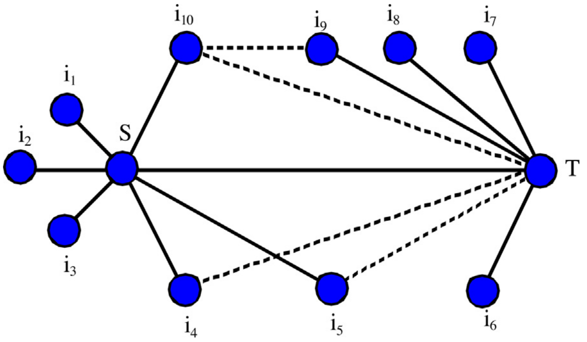

Fig. 3. Spanning tree and cutset of multi-link cuts.

connected components. The set $\mathcal{N}^S_{\tau(g)}$ is called the sanctioning coalition. Fig. 3 illustrates with unbroken lines a spanning tree corresponding to the network of Fig. 2, and the broken lines show the links in the cutset with respect to *ST*.

The sanctioning coalition in Fig. 3 is $\mathcal{N}^S_{\tau(g)} = \{S, i_1, i_2, i_3, i_4, i_5, i_{10}\}$ and is made up of agents who are at distance 1 from *S*, i.e. the neighbors of *S*. We refer to the underlying spanning tree as an *order*-1 spanning tree. Let us postpone for the moment the issue of why agents in the sanctioning coalition will delete their links in the cutset and assume that indeed they do. We will refer to the multilateral sanctions as *order*-1 sanctions. Then *T* will comply if the post-sanctions component of *T* is sufficiently small to overcome *T*'s resistance, i.e. $|\mathcal{N}^T_{\tau(g)}| \leq h^T(\beta)$. Of course it is possible that order-1 sanctions are not restrictive enough. It is evident from Fig. 2 that *S* will then have to expand the size of the sanctioning coalition to include agents who are at distance 2 or greater from *S*. We will say that a spanning tree is of *order-l*, and represent it as $\tau_l(g)$, if the agent in the sanctioning coalition who is furthest from *S* is at a distance of *l*. Therefore the order of the spanning tree indicates the distance to which the influence of *S* extends. The corresponding multilateral sanctions are called *order-l* sanctions. Note that severity of sanctions is increasing in the order of the spanning tree since the post-sanctions component of *T* becomes smaller as *S* incorporates more distant agents into the sanctioning coalition. We will let $\Gamma_l(g)$ denote the set of all order-*l* spanning trees and let:

$$\Gamma^e_l(g) = \left\{ \tau_l(g) \in \Gamma_l(g) : D_{\tau_l(g)}(ST) \equiv (\mathcal{N}^S_{\tau_l(g)}, \mathcal{N}^T_{\tau_l(g)}),\ |\mathcal{N}^T_{\tau_l(g)}| \leq h^T(\beta) \right\} \tag{20}$$

A network *g* is *effective in the short run with respect to order-l multilateral sanctions* (or simply order-*l* effective) if and only if $\Gamma^e_l(g) \neq \emptyset$, and for some $\tau_l(g) \in \Gamma^e_l(g)$ the sanctioning coalition $\mathcal{N}^S_{\tau_l(g)}$ on $\tau_l(g)$ have an incentive to delete links in the cutset when *S* sanctions *T*. A network is *effective* if and only if it is order-*l* effective for some $l \geq 1$.

Suppose that *S* has recourse to an appropriate order-*l* spanning tree in $\Gamma^e_l(g)$, with associated sanctioning coalition $\mathcal{N}^S_{\tau_l(g)}$. In other words, agents up to a distance of *l* are in the ambit of *S*'s radius of influence. Let us now tackle the issue of whether agents in the sanctioning coalition, who are called upon to delete their links in the cutset, will in fact do so. An agent in $\mathcal{N}^S_{\tau_l(g)}$ is said to be *noncompliant* if she refuses to delete a link when called to do so by *S*, and is *compliant* otherwise. In order to effectively implement multilateral sanctions, *S* has to ensure that all agents in $\mathcal{N}^S_{\tau_l(g)}$ are compliant. Joshi and Mahmud (2016, Proposition 1) show that to ensure compliance from *all* agents in $\mathcal{N}^S_{\tau_l(g)}$ it suffices to check the incentives to remain compliant of *S*'s *immediate neighbors* in $\mathcal{N}^S_{\tau_l(g)}$ who lie on paths connecting *S* to *T*. We say that *S* is *l-essential* if each neighbor of *S* on the path to *T*, under the belief that compliant agents in the sanctioning coalition $\mathcal{N}^S_{\tau_l(g)}$ will delete their links with noncompliant agents, has an incentive to be compliant. Otherwise, *S* is *l-inessential*. If *S* is *l*-inessential for all $l \geq 1$, then *S* is *inessential*. Note that if *S* is *l*-essential, then *S* is also $(l+1)$-essential but may not be $(l-1)$-essential. Joshi and Mahmud (2016, Proposition 1) establish that:

Proposition 5. *A network g is effective if and only if S is l-essential for some $l \geq 1$ and $\Gamma^e_{l'}(g) \neq \emptyset$ for some $l' \geq l$.*

Having characterized effective networks, we now examine effectiveness as a function of the utility specification. Recalling (20), let $\Gamma^e_{l,0}(g)$, $\Gamma^e_{l,-}(g)$ and $\Gamma^e_{l,+}(g)$ denote the set of order-*l* spanning trees under linear, concave, and convex utility respectively for network *g* that suitably restrict the post-sanctions component of *T*. Similarly, let $l^0(g)$, $l^-(g)$ and $l^+(g)$ denote the smallest value of $l \geq 1$, for which order-*l* multilateral sanctions are effective under linear, concave, and convex utility respectively in *g*.

Proposition 6. *For any given network g, $\Gamma^e_{l,-}(g) \subseteq \Gamma^e_{l,0}(g) \subseteq \Gamma^e_{l,+}(g)$. Further, $l^+(g) \leq l^0(g) \leq l^-(g)$.*

Proposition 6 echoes our results from the unilateral case. First, convex utility supports the largest set of networks as effective as compared to linear and concave. Thus for example it is possible that for a network *g*, $\Gamma^e_{l,-}(g) = \Gamma^e_{l,0}(g) = \emptyset$ while $\Gamma^e_{l,+}(g) \neq \emptyset$. Therefore no sanctioning coalition can be assembled under linear or concave utility for *g* but it is possible under convex. Second, for a network that is effective under all three forms of utility, it is possible to garner T's compliance through lower order multilateral sanctions under convex utility as compared to linear and concave. Equivalently, l-essentiality of S of a lower order suffices for a network to be effective under convex utility. The following example substantiates these results.

Example 5.2. Consider the network of Fig. 2 and the utility specification of Example 3.1. When $\beta = 7$, then we know that $(h_0^T(\beta), h_-^T(\beta), h_+^T(\beta)) = (4, 2, 5)$. Accordingly we have the following:

Utility function	Lowest order of sanctions	Sanctioning coalition
Linear	order-2	$\{i_1, i_2, i_3, i_4, i_5, i_9, i_{10}, S\}$
Concave	order-4	$\{i_1, i_2, i_3, i_4, i_5, i_7, i_8, i_9, i_{10}, S\}$
Convex	order-1	$\{i_1, i_2, i_3, i_4, i_5, i_{10}, S\}$

Utility function	Lowest order of sanctions	Sanctioning coalition
Linear	order-3	$\{i_1, i_2, i_3, i_4, i_5, i_8, i_9, i_{10}, S\}$
Concave	Ineffective	$\emptyset$
Convex	order-2	$\{i_1, i_2, i_3, i_4, i_5, i_9, i_{10}, S\}$

A greater resistance from T requires higher order sanctions under all specifications of utilities. But note that it is no longer possible to sanction T if utility is concave. Since S cannot force i_6 to delete the link with T, there is no way to put together a sanctioning coalition that will reduce T to a singleton. □

6. Multilateral sanctions in the long run

Let $2f$, $f > 0$, denote the total cost of forming a link as in Section 4. It is reasonable to assume that S typically will operate under some budget constraint that will limit the number of additional links that it can form in the long run. Accordingly let $\alpha > 0$ denote the "tolerance" of S to the costs that it can incur to force T to comply. The level of tolerance is specified exogenously (presumably by political, social or economic concerns). Since each new link costs $2f$, the number of additional links that S can form is bounded from above by $\alpha/(2f)$. We will say that a network g is *effective in the long run with respect to multilateral sanctions* (or simply *effective*) if and only if S can derive network g' from g by adding links at a total cost not exceeding the threshold α, and g' is short run effective with respect to multilateral sanctions.

Note first of all that, in contrast to the unilateral case, networks that were effective with respect to multilateral sanctions in the short run will continue to be effective in the long run. The reason is that agents in the sanctioning coalition who had an incentive to delete links in the cutset cannot be persuaded by T to reestablish those links. Let us therefore consider ineffective networks in the short run. From Proposition 5, a network is ineffective with regard to multilateral sanctions if S is inessential (so that no sanctioning coalition can be found that has the incentive to delete links in the cutset) and/or it is impossible to limit T to a sufficiently small post-sanctions component. If the former is true, i.e. S is inessential in the network, then Joshi and Mahmud (2016, Proposition 4) establish that the network cannot be effective in the long run. The reason is that when S is inessential, then T is l-essential for some $l \geq 1$ (Joshi and Mahmud, 2016, Lemma 3) and can block any sanctions. Therefore, the only possible case in which a short run ineffective network g can be effective in the long run is when S is l-essential for some $l \geq 1$ and the only impediment to short run effectiveness is the inability to relegate T to a sufficiently small component. However, this is where the ability to add links to g in the long run offers S some reprieve. It now becomes possible for S to craft a spanning tree with its attendant sanctioning coalition in a manner that the modified network with new links retains S's essentiality and is also capable of suitably restricting T's post-sanctions component. If this modified network can be engineered while remaining below the tolerance threshold of α, then S can effectively sanction T in the long run.

To illustrate this point, consider the network of Fig. 2 and the utility specification of Example 3.1 in the case of concave utility. When $\beta = 8$, then we know that $h_-^T(\beta) = 1$. We have already seen in Example 5.2 that S, even though 1-essential, lacks the capability in the short run to confine T to a singleton component. Now suppose that S offers to form a link with i_6 accepting the full cost of link formation.[13] To keep the analysis tractable and to avoid timing issues, we will make two simplifying assumptions: (i) all offers to form new links are announced simultaneously and publicly by S and (ii) each agent who is offered a link by S believes that all other agents who are also offered links by S will reciprocate. Since the offer to agent i_6 to form a link with S is costless for i_6 (with S bearing the full cost of link formation), there is no incentive for i_6 to reject S's offer. In the network g of Fig. 2, i_6 will accept the link Si_6 leading to the network $g' = g + Si_6$. Given the order-1 spanning tree corresponding to this network g', deleting the link i_6T in the cutset when sanctions are implemented places i_6 in a component of size 8. By rejecting the link Si_6, and in the event that sanctions are enforced, it will be in a component of size 5. Therefore i_6 has an incentive to execute sanctions when required in g'.

We now present long run effective sanctions more formally. Let $\mathcal{G}_{ims} \subset \mathcal{G}$ denote the set of networks that contain the link ST and are ineffective with respect to multilateral sanctions in the short run but in which S is not inessential. Given $g \in \mathcal{G}_{ims}$,

[13] In the components model, agent i_6 will outright reject forming a link with S if she had to bear any cost of link formation. The link Si_6 does not increase the size of the component and therefore will strictly reduce the net utility of i_6.

let $g_{ems} = g + \sum_{l=1}^{m} Sj_l$ denote the network derived from g such that $\Gamma_l^e(g_{ems}) \neq \emptyset$ for some $l \geq 1$. Joshi and Mahmud (2016, Lemma 2) show that since S is l-essential for some $l \geq 1$ in $g \in \mathcal{G}_{ims}$, then S is also l-essential in $g_{ems} = g + \sum_{l=1}^{m} Sj_l$. It follows that there exists $\tau_l(g_{ems}) \in \Gamma_l^e(g_{ems})$ for which the sanctioning coalition $\mathcal{N}^S_{\tau_l(g_{ems})}$ on $\tau_l(g_{ems})$ is compliant. Therefore g_{ems} is the network derived from g so that it is effective with respect to short run multilateral sanctions. Let $c(g_{ems})$ denote the cost to S of the additional links needed to make the network g effective with regard to short run multilateral sanctions. Now let:

$$\tilde{g} = arg\min \left\{ c(g_{ems}) : g_{ems} = g + \sum_{l=1}^{m} Sj_l \right\}, \; g \in \mathcal{G}_{ims}$$

$\tilde{g}$ is an effective network with respect to short run multilateral sanctions that can be derived from g by S incurring the lowest cost of link formation. If there is more than one modified network that keeps link formation cost at a minimum, then any one can be picked arbitrarily. For a given tolerance level $\alpha > 0$, a network $g \in \mathcal{G}_{ims}$ is effective in the long run if and only if $c(\tilde{g}) \leq \alpha$. Let us now define the order $\succsim_{ms}$ on $\mathcal{G}_{ims}$ as follows:

$$g' \succsim_{ms} g \Leftrightarrow c(\tilde{g}') \leq c(\tilde{g}), \; g, g' \in \mathcal{G}_{ims}$$

All networks in $\mathcal{G}_{ims}$ can be reindexed and ordered according to $\succsim_{ms}$. Let n_0, n_-, n_+ be respectively the lowest index for which a network is effective with respect to multilateral sanctions in the long run under linear, concave, and convex utility respectively. Following the already established line of argument we have the following result that is given without proof:

Proposition 7. *Suppose utility functions* $\left(u^0, u^-, u^+\right)$ *satisfy* (2). *Then* $n_+ \leq n_0 \leq n_-$.

Consider for instance the comparison between concave and convex utility. If a network g_n is effective in the long run under concave utility, then drawing on Proposition 5 we know that S is essential and that the post-sanctions component of T is sufficiently small. Since convex utility permits a larger post-sanctions component for T, it follows that g_n will then be effective in the long run under convex utility as well. Thus $n_+ \leq n_-$ and we have the familiar result that convex utility supports a larger set of effective networks than concave.

7. Discussion

In this paper we utilized the components model to examine effectiveness of sanctions in a network context. Some of the insights of the paper, such as the importance of the centrality of S in enforcing sanctions or convex utility supporting a larger set of effective networks, generalize to other formulations of the utility function as well. We offer a brief illustration for the case of unilateral sanctions using the connections model outlined in Example 2.1 due to Jackson and Wolinsky (1996). Let us write the utility function in (3) more generally as:

$$u_i(g) = u\left(V + V \sum_{j \in C_i(g)} \delta^{d_{ij}(g)}\right), \; u' > 0$$

Consider network g in Fig. 2. Even though the link ST is not a bridge, severing the link ST will impose costs on T (and S). Note that S lies on the shortest path connecting T to agents i_1, i_2, and i_3. If S unilaterally sanctions T, then the path lengths from T to these three agents increase with a corresponding reduction in utility. Letting $\bar{C}_T(g) = C_T(g) \backslash \{i_1, i_2, i_3\}$, it follows that:

$$u_T(g) - u_T(g - ST) = u\left(V + V \sum_{j \in \bar{C}_T(g)} \delta^{d_{Tj}(g)} + 3V\delta^2\right) - u\left(V + V \sum_{j \in \bar{C}_T(g)} \delta^{d_{Tj}(g)} + 3V\delta^3\right)$$

It is immediately clear that the larger the number of shortest paths connecting T to $\mathcal{N}\backslash\{S, T\}$ that run through S, the greater is T's utility loss from unilateral sanctions. Therefore, a more central S can impose sanctions more effectively. A similar consideration applies to the utility formulations in Ballester et al. (2006) and Goyal and Vega-Redondo (2007).

To elaborate on the role of concave/convex utility, let $\bar{V}_T = V + V\sum_{j \in C_T(g)} \delta^{d_{Tj}(g)}$, and replace the boundary conditions in (2) by:

$$u^0(1) = u^-(1) = u^+(1), \; u^0\left(\bar{V}_T\right) = u^-\left(\bar{V}_T\right) = u^+\left(\bar{V}_T\right)$$

where u^0, u^-, and u^+ continue to denote linear, concave, and convex utility respectively. It then follows that:

$$u_T^-(g) - u_T^-(g - ST) < u_T^0(g) - u_T^0(g - ST) < u_T^+(g) - u_T^+(g - ST)$$

suggesting that convex utility is more conducive for imposing sanctions than concave or linear.

8. Conclusion

In this paper we examined the role of networks and the specification of utility in dictating the efficiency of sanctions imposed by S on T. Much of the existing literature addresses this issue in the context of a dyadic interaction between S and T. In contrast, our paper explicitly accounted for the network within which S and T are embedded and the role of the network architecture and the nature of the utility function in sharpening or dulling the impact of sanctions. There are many further avenues of research to pursue in this context including the case where an agent is in fact a collection of interest groups with diverging objectives (Kaempfer and Lowenberg, 1988; Kaempfer and Lowenberg, 2007; Gershenson, 2002) and S can exploit these differences in enforcing sanctions.

Appendix A.

Proof of Proposition 1. Note that $u_T(g) - u_T(g - ST) = u(N) - u(|C_T(g - ST)|) \geq \beta$ if and only if $|C_T(g - ST)| \leq h^T(\beta)$ if and only if $\mathcal{E}_{TS}(g) = |C_S(g - ST)| = N - |C_T(g - ST)| \geq N - h^T(\beta) = h^S(\beta)$.□

Proof of Proposition 2. We prove this for (a). The first part is clear. For the second, note that using (2), $u^-(N) - u^-(|C_T(g_{n_0})|) \leq u_0(N) - u_0(|C_T(g_{n_0})|) = \beta \leq u^-(N) - u^-(|C_T(g_{n_-})|)$ yielding $u^-(|C_T(g_{n_-})|) \leq u^-(|C_T(g_{n_0})|)$. Since u^- is increasing, $|C_T(g_{n_-})| \leq |C_T(g_{n_0})|$, and thus $\mathcal{E}_{TS}(g_{n_-}) \geq \mathcal{E}_{TS}(g_{n_0})$ since the initial network is connected. Therefore $g_{n_-} \succsim g_{n_0}$, i.e. $n_- \geq n_0$. □

Proof of Proposition 3. Suppose $g' \succ g$. Then $|C_T(g'_t)| < |C_T(g_t)| < |C_S(g_t)| < |C_S(g'_t)|$ where $g_t = g - ST$ and $g'_t = g' - ST$. Consider agents $i \in C_S(g_t)$ and $i' \in C_S(g'_t)$ (they may be the same or different agents). The joint utility gain to T and agent i from forming a link iT in g_t which reconnects the components $C_T(g_t)$ and $C_S(g_t)$ is:

$$\begin{aligned}\Delta u(g) &\equiv [u_T(g) - u_T(g_t)] + [u_i(g) - u_i(g_t)] \\ &= [u(N) - u(|C_T(g_t)|)] + [u(N) - u(|C_S(g_t)|)]\end{aligned}$$

$$\Delta u(g') - \Delta u(g) = \left[u(|C_T(g_t)|) - u\left(|C_T(g'_t)|\right)\right] - \left[u\left(|C_S(g'_t)|\right) - u(|C_S(g_t)|)\right]$$

Proof of Proposition 4. For any connected network g, the smallest value of f for which agents in $C_S(g_t)$ and $C_T(g_t)$ have no incentive to form a link is given by $\tilde{f} = \{[u(N) - u(|C_T(g_t)|)] + [u(N) - u(|C_S(g_t)|)]\}/2$. Therefore $f(g_t; u) = \tilde{f}$. The result follows by noting that under our assumptions, $u^-(N) - u^-(h) < u^0(N) - u^0(h) < u^+(N) - u^+(h)$. □

Proof of Proposition 5. Since $u^-(N) - u^-(h) < u^0(N) - u^0(h) < u^+(N) - u^+(h)$, it follows that $h^T_+(\beta) \geq h^T_0(\beta) \geq h^T_-(\beta)$ and therefore $\Gamma^e_{l,-}(g) \subseteq \Gamma^e_{l,0}(g) \subseteq \Gamma^e_{l,+}(g)$. We now establish $l^+(g) \leq l^0(g)$ and note that the proof of $l^0(g) \leq l^-(g)$ is identical. Suppose g is order-$l^+(g)$ effective under convex utility with respect to the spanning tree $\tau_{l^+(g)}(g)$. Note that this lowest order spanning tree simply needs to satisfy $|\mathcal{N}^T_{\tau_{l^+(g)}(g)}| = h^T_+(\beta)$ in (20). We have two cases to consider. (i) If $h^T_+(\beta) = h^T_0(\beta)$, then g is order-$l^+(g)$ effective under linear utility with respect to the spanning tree $\tau_{l^+(g)}(g)$ yielding $l^0(g) \leq l^+(g)$. If $l^0(g) < l^+(g)$ then, given that $h^T_+(\beta) = h^T_0(\beta)$, the spanning tree that effectively implements order-$l^0(g)$ sanctions for linear utility would work for convex utility as well contradicting the definition of $l^+(g)$. Therefore $l^0(g) = l^+(g)$. (ii) Suppose $h^T_+(\beta) > h^T_0(\beta)$. Since $|\mathcal{N}^T_{\tau_{l^+(g)}(g)}| = h^T_+(\beta) > h^T_0(\beta)$, we can extend $\tau_{l^+(g)}(g)$ by at least one link using the procedure outlined in Joshi and Mahmud (2016, Lemma 1) in order to reduce the size of the sanctioned coalition for linear utility. But then $l^+(g) < l^+(g) + 1 \leq l^0(g)$. □

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