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THE ENERGY IMPLICATIONS OF CITY SIZE AND DENSITY

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Abstract

This paper develops a new urban simulation model with endogenous population, housing supply and demand, and highway use and congestion. These features allow the model to simulate cities of different sizes with a single parameterization and hence to study the partial effect of city size differences on economic activity. The model is applied to the important problem of the energy implications of city size and density. Energy consumption in housing and commuting is calculated based on the structure type and size of housing units, consumption of a numeraire good, and commuting distances and velocities on congested roadways. The surprising conclusion is that per capita energy consumption does not vary as city size increases. Households in larger cities consume less housing, commute longer (and slower), and consume more of the numeraire good. The energy use implications of these effects are offsetting for a laissez-faire city. However, common land use policies, specifically density limits and greenbelts, can positively or negatively affect both city welfare and the elasticity of energy use with respect to city size.

JEL Codes: Q40, R14

Keywords: urban simulation, congestion, commuting, gasoline, greenbelt

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1 Introduction

Since its introduction by Muth (1975), various versions of the urban simulation model have been used to understand the spatial structure of cities. Virtually all of these applications have involved closed cities with exogenous population, a single type of structure, and exogenous urban transportation costs. None of these efforts has attempted to simulate the effects of variation in city size. The model formulated and solved here is, along with Rappaport (2014), the first urban simulation model of an open city with endogenous population, housing supply and demand, and highway use and congestion.[1] This model is calibrated with respect to the characteristics of a city with one million people, and is quite successful in accounting for the effects of city size on city characteristics.

The model is then used to determine the effect of city size and density on energy use, an important policy question that has been the object of recent empirical research. This empirical research begins with the stylized fact that the rise in house prices with city size causes increases in residential density.[2] As the logic goes, the energy efficiency of multifamily dwellings results in reduced energy consumption in larger, denser cities. Offsetting some of these energy savings are longer and more congested commuting trips. Despite the ambiguity in the magnitudes of these countervailing effects, shown in Figure 1, the prevailing view appears to be that there are net savings in per capita energy use associated with city size.

There are a number of problems with using empirical measurements from current cities to make inferences about the effects of size and density on per capita energy use. First, there is substantial heterogeneity in the population and economic structure of cities, because the process that selects individuals and firms into cities is a function of city size and house prices. Second, the current spatial form, housing density, transportation network, and energy use in cities are functions of the historical path of development. Modern housing and transportation systems are technically superior and measures of energy use in actual cities confound technological obsolescence with energy inefficiency. Third, climate and topography have a major effect on energy use in cities and the largest cities often have locational advantages based on topographical features. Fourth, fragmented political systems and arbitrary plan-

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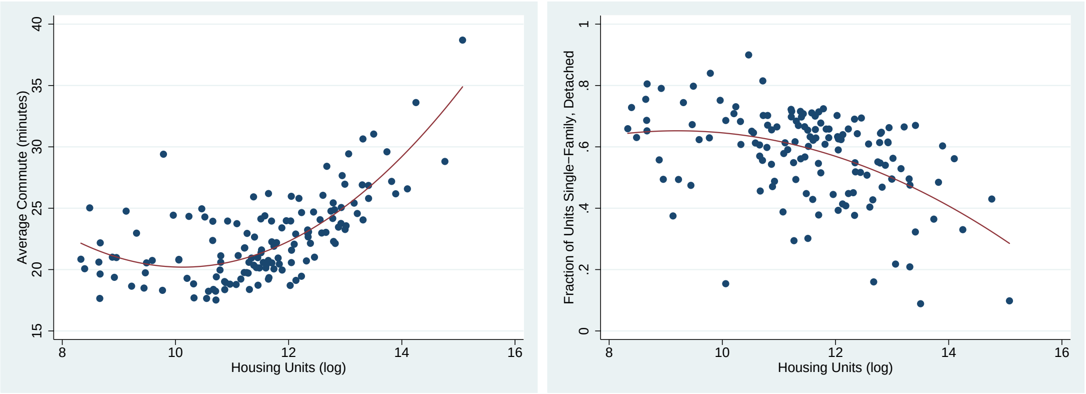
[1] Rappaport (2014) has independently developed another simulation model of an open city with endogenous population, housing supply and demand, and highway use and congestion. The model calibration and simulation results in this paper are remarkably similar to the model developed here.

[2] Some of these papers measure total energy use while others measure determinants of carbon emissions (see Brown, Southworth, and Sarzynski, 2008 and Glaeser and Kahn, 2010 for two recent examples). Generally, they are concerned with some rate of energy use, either energy per capita, energy per household, or energy per unit gross domestic product.

Figure 1: Stylized Facts


(a) Average Commute Time (b) Fraction Single Family, Detached

ning processes have a substantial effect on land use and transportation, particularly in larger cities.[3]

Current empirical studies fail to account for the energy embodied in consumption of the numeraire good.[4] The household iso-utility condition across cities implies that, as city size and cost-of-living increases, income rises. At the same time, the standard urban model predicts that as size rises, the fraction of income not spent on either housing or commuting must rise. Therefore, as the model in this paper demonstrates, consumption of the numeraire good is subject to both substitution and income effects, with important energy implications. In sum, the data generating process that produces cities is very messy and the natural experiment of doubling city size holding other things constant is never close to being performed.

Rather than attempting to correct for the effects of all these factors that confound empirical estimation of the partial relation between city size and energy consumption, a simulation model can create a city which holds technology, topography, planning, climate, population characteristics and preferences, and industrial structure constant as city size changes. It is then possible to introduce other factors, such as land use and transportation planning, to determine the partial effects of these factors on the relation between city size and per capita energy consumption. The simulation results can separate out the effects of different factors associated with changing city size on energy use. Robustness checks within the sim-

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[3] For example, Duranton and Turner (2011) find that the process of adding highways is sufficiently problematic that new road capacity has virtually no effect on congestion in U.S. cities.

[4] This includes Glaeser and Kahn (2010), who acknowledge their inability to produce a full energy accounting, and Borck (2014), who produces a closed city simulation model and shows that a height limit can lower energy consumption without accounting for the numeraire good.

ulation model allow the partial effects of variation in both model parameterization and city characteristics on energy consumption to be evaluated.

For example, the simulation can raise city size without a compensating variation in income to determine the partial effect of this necessary increase in income on the relation between city size and energy use. In the simulations performed here, holding income constant as city size doubles, there is a net fall in per capita energy consumption of about 3.7%.[5] However, this ignores the effects of the compensating rise in income needed to maintain utility as size increases, following the long quality-of-life literature beginning with Roback (1982), and more recently, Desmet and Rossi-Hansberg (2013). The compensating variation in income associated with a doubling of size is 2.4% and this has two effects on energy consumption.[6] First, it mitigates the fall in housing consumption due to the price increase. Second, the rise in income results in greater expenditure on the numeraire consumption good. When both these effects are considered, per capital energy consumption actually increases by 0.1% with city size.

This energy elasticity result is the first of two key findings in the paper. The second main finding lies in the ability of the model to simulate policies or market events that affect density, holding city population constant. As with the city size question, energy implications are determined, but the simulation also allows for welfare effects to be calculated. Because the model relies on an inter-regional equilibrium for both firms (zero profits) and households (a reservation utility level), welfare is measured following Sullivan (1985) as the change in aggregate land value minus the total cost of the compensating wage differential.

Simulation results suggest that a residential building height limit exacerbates sprawl, causing both energy increases and welfare reductions for a city of a given size, and that these effects grow larger as the city size increases. A greenbelt is also simulated, and the findings are reversed, with reductions in energy consumption and the counterintuitive result that welfare rises when the greenbelt is not severely binding. Welfare gains associated with imposition of a greenbelt appear to arise because it functions as a second-best response to unpriced congestion on the highways. This is consistent with Wheaton's (1998) theoretical demonstration of the effects of failing to price highway congestion.

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[5] This is the same assumption made in a regression with energy consumption per capita estimated as a linear function of both income and population. For instance, Brownstone and Golob (2009) estimate commuting and density as jointly determined, but with income as an exogenous, right-hand-side variable.

[6] The simulated compensating variation of 2.4% is similar to recent estimates of the urban cost elasticities with respect to city size of 1.6%-4.6% (depending on various assumptions) in French cities found by Combes, Duranton, and Gobillon (2012). The similarity is remarkable given their empirical approach to estimating the value versus the simulation approach found in this paper.

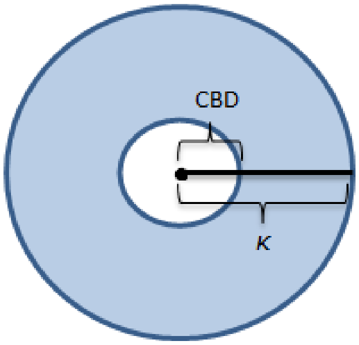
The remainder of this paper is organized as follows. First, the Urban Energy Footprint Model is extended in Section 2. The next section provides the parameter assumptions and calibration results of the simulation. Section 4 presents the simulation results and reports on the main findings in the paper. Section 5 concludes.

2 The Urban Energy Footprint Model (UEFM)

The standard urban model (SUM) has been described extensively in the literature, with notable contributions by Alonso (1964), Mills (1967), Muth (1969), and Wheaton (1974), and with an excellent summary by Brueckner (1987). The Urban Energy Footprint Model (UEFM) layers commuting and dwelling energy consumption parameters onto a standard urban model, building on the closed-city model of Larson, Liu, and Yezer (2012). This approach enables the measurement of the effects of parameters related to utility, profit, congestion, and regulatory characteristics of a city on the energy consumed by households.

There are several assumptions that are standard in urban simulation models. The first is the geographic set-up of the city. Cities are generally assumed to be monocentric and homogenous at a given radius k from the center. This allows the city to be expressed in radial terms. Cities have a center region providing employment called the Central Business District (CBD) where $0 < k \leq k_{CBD}$, a middle region with housing where $k_{CBD} < k \leq \kappa$, and an agricultural hinterland with neither housing nor employment where $k > \kappa$. As households commute to the CBD for work, roads become congested as the number of workers commuting through a given annulus rises.

Figure 2: A simple monocentric city



Second, households can change residential location costlessly, equalizing utility at all locations between the CBD and the city boundary, with a strictly lower utility in the agricultural area where there are no households. Third, land and housing prices adjust over space so that firms producing housing are in zero-profit equilibrium at all locations where they produce housing services, and profit is negative at locations outside the CBD where housing is not produced. In the common closed urban simulation model, these no-arbitrage equilibrium conditions hold within the city. In the open model constructed here, they hold across cities of different sizes

This section describes the open city model used in the numerical simulation. Beyond the general characteristics of usual models noted above, the model in this paper has two important features. The first is exogenous employment distributed outside of the CBD. Non-CBD employment gives more realistic commuting patterns in cities compared to a city with only CBD employment. The second key feature of this simulation is the commuting congestion function. The speed of commuters is related to the fraction of land dedicated to roads and the volume of traffic flowing through a particular annulus. As with the decentralized employment, this representation is meant to produce more realistic commuting and energy consumption patterns than exogenous and/or constant commuting speed functions.

2.1 Employment

Earnings, as well as the level and spatial distribution of city employment, are exogenous. Indeed, the wage of workers drives changes in city size. Employment location is divided between the CBD where it is uniformly distributed, and the portion of the city where it shares land use with housing. Hours of work are assumed to be fixed for all workers. In the range where employment exists alongside housing, employment is distributed according to a negative exponential, which McMillen (2004) has demonstrated is a reasonable parameterization of the spatial distribution of employment outside the CBD. This gives total employment as

$$E = E_{CBD} + E_{SRD} = E_{CBD} + \int_{k_{CBD}}^{k_E} \omega(k_{CBD}) e^{-gk} dk \tag{1}$$

Employment in the suburban residential district, E_{SRD}, is distributed from the edge of the CBD, k_{CBD}, to a maximum employment radius, k_E, following a negative exponential with density at the edge of the CBD of $\omega(k_{CBD})$ and a constant decay rate, g. The employment limit, k_E, rises in proportion to city size.

When city employment changes, an assumption must be made dictating the distribution of employment changes between CBD and SRD employment. In this model, the ratio of CBD and SRD employment does not change with city size. This is consistent with the empirical result that the employment density gradient does not vary with city size reported in Thurston and Yezer (1994).

2.2 Housing Production

Following customary practice in SUM models, housing is produced by developers in a perfectly competitive industry according to a CES production function with constant returns to scale.

$$H = A\left[\alpha_1 S^{\rho} + \alpha_2 L^{\rho}\right]^{1/\rho} \tag{2}$$

where H is housing production, S and L are structure and land inputs, respectively, α_1 and α_2 are distribution parameters, and the elasticity of substitution is $1/(1-\rho)$. Again, following custom, the elasticity of substitution between structure and land inputs is set at 0.75 and a fixed fraction, θ, of land in each annulus is available for residential development.[7]

2.3 Households

The household utility function is assumed to be CES for each of the N households in the city

$$U = \left[\beta_1 y^{\eta} + \beta_2 h^{\eta}\right]^{1/\eta} \tag{3}$$

where h is housing consumption, and y is the numeraire consumption good. β_1 and β_2 are distribution parameters, and the constant elasticity of substitution between housing and the numeraire is given by $1/(1-\eta)$. Each household inelastically supplies ϵ workers with combined earnings of w.[8] Given values for E and ϵ, this gives the city size in terms of the number of households, $N = E/\epsilon$. A household's budget constraint is

$$w = y(k) + r(k)h(k) + \epsilon T(k) \tag{4}$$

where T is the sum of both time and out-of-pocket commuting cost, r is the rental price of housing services, h is the quantity of housing services consumed, each varying with the distance from the center of the city.[9]

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[7] While the exact elasticity of substitution between structure and land inputs is debatable, it is known to be less than unity because the ratio of structure to land value falls as density rises toward the city center.

[8] In the current analysis, income from capital is ignored. Effectively, all of the housing and land rental payments that are not going towards energy use in the housing unit disappear from the model. This capital income effect would presumably increase energy consumption as housing and land prices rise, suggesting greater energy consumption in larger cities than in the model in this paper.

[9] Structure prices and the price of the numeraire good are assumed constant as city size changes within the

2.4 Commuting

All households either commute to the CBD via automobile with commuting costs $T(k)$ or commute costlessly without energy consumption to a job within their annulus in t_L units of time. The no arbitrage equilibrium of urban households requires the standard urban labor market assumption that wages in employment outside the CBD fall with the saving in commuting cost. Thus local households earnings are $w_L(k) = w - \epsilon T(k)$.

The velocity of automobiles commuting through an annulus is a positive function of the land fraction allotted to roads and is inversely related to the number of commuters. The congestion function follows the "Bureau of Public Roads" specification, which is widely adopted in the transportation literature. In this parameterization, velocity is a bounded nonlinear function

$$V(k) = \frac{1}{a + bM(k)^c} \tag{5}$$

where $M(k) = (N - N(k) - E(k))/R(k)$ is the ratio of traffic volume to roads, and a, b, and c are parameters that reflect the curvature of the function. The parameters a and b are defined such that the velocity at the edge of the CBD is $V(k_{CBD}) = v_{low}$ and the velocity at the edge of the city is $V(\kappa) = v_{high}$, where v_{low} and v_{high} are calibrated parameters. Parameter c is also calibrated. $N(k)$ is the population living inside of radius k and $E(k)$ is employment outside of radius k. $R(k)$ is exogenously and uniformly distributed as a constant fraction of land area.

Commuting costs include the fixed costs of owning a vehicle m_0, depreciation and maintenance costs related to the length of the commute $m_1 k$, fuel costs of travel, and the opportunity cost of time spent commuting. Miles per gallon, $G(V(k))$, is a function of vehicle velocity, and total fuel cost per mile is given by $p_g/G(V(k))$. The value of time spent commuting is a constant fraction, τ, of household earnings. Therefore, the total cost of commuting from radius k found in Equation 4 can be expressed as

$$T(k) = m_0 + m_1 k + p_g \int_0^k \frac{1}{G(V(M(k)))} d\kappa + \tau w \int_0^k \frac{1}{V(M(k))} d\kappa \tag{6}$$

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simulation. In reality, both will increase with city size as the price of labor and non-tradables rise, respectively (Balassa, 1964; Samuelson, 1964). The net result of these changes would be to shift the intensity of housing production to land; structure and numeraire good price increases would simply result in a higher wage given the reservation inter-regional utility assumption. On the other hand, larger cities may have a greater consumption amenity through variety (Glaeser, Kolko, and Saiz, 2001). The utility function and budget constraint in this paper implicitly assume that all of these effects are more or less offsetting.

Because traffic congestion lowers $V(k)$, it raises commuting cost by increasing both fuel and time costs.

2.5 Solving the model

The model is solved numerically following the methods of Muth (1975), Altmann and DeSalvo (1981), and McDonald (2009). Initialized wage and house price values at the edge of the CBD determine a house price gradient as a function of commuting costs. The other gradients for structure density, population density, and land rent follow recursively once commuting costs are determined.

The result is a two-equation system of nonlinear differential equations

$$\begin{bmatrix} \frac{dT(k)}{dk} \\ \frac{dN(k)}{dk} \end{bmatrix} = \begin{bmatrix} m_1 + p_g \frac{1}{G(V(M(k)))} + \tau w \frac{1}{V(M(k))} \\ 2\pi\theta k D(T(k)) \end{bmatrix} \tag{7}$$

with initial values

$$\begin{bmatrix} T(k_{CBD}) \\ N(k_{CBD}) \end{bmatrix} = \begin{bmatrix} m_0 + k_{CBD} \left[m_1 + p_g \frac{1}{G(v_{low})} + \tau w \frac{1}{v_{low}} \right] \\ 0 \end{bmatrix}$$

The first equation in the system gives commuting costs at radius k from the center of the city. The second equation gives $N(k)$, the number of households locating inside radius k, where $D(k)$ is the density at k.[10] The solution of this system, along with the exogenous employment gradient, gives commuting costs and population for each annulus in the city. Having solved for commuting costs, the rest of the gradients are known.

Two conditions must be met in order for the city to be in equilibrium. First, the price of land at the edge of the city must be equal to the agricultural reservation price of land per acre. This ensures land market equilibrium. If the land price is different than the reservation price, the CBD house price is re-initialized. Second, utility of households, computed based on commuting cost, wages, housing price, and the cost of the composite commodity, must be identical throughout the city and be equal to the exogenous utility level available elsewhere. If the utility of households is different than the regional reservation utility, the wage is re-initialized. If either value is re-initialized, then the entire simulation is re-computed.

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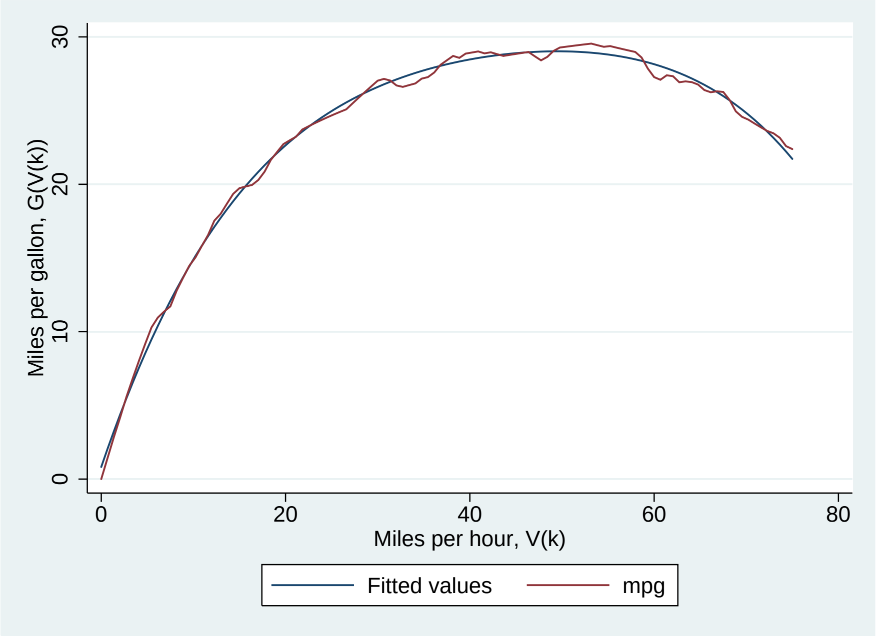
[10] All annuli inside of k_{CBD} have a constant commuting speed equal to v_{low}.

2.6 Calculating energy demand

With commute times, housing consumption and structure-land area ratios known for each annulus in the city, it is possible to calculate energy consumed commuting, in dwellings, and through the consumption of the numeraire good. Energy consumption in each category is measured as the sum of final energy consumption and intermediate energy consumption in the production and distribution of the energy.

Energy consumed while commuting follows from engineering relations. Gasoline consumption is a function of vehicle velocity, $G(V(k))$, though each annulus of the city. The specific form of this function is displayed in Figure 3 below.[11] It is assumed that each household in the city owns the same vehicle, and that vehicle is similar to an average vehicle in the U.S. fleet described by West et al. (1999), who conducted an automobile fuel efficiency study at the Fuels, Engines, and Emissions Research Center at the Oak Ridge National Laboratory. Using this fleet, they established the velocity-fuel economy relation shown in Figure 3.[12]

Figure 3: Velocity-Fuel Efficiency Relation



This function maps vehicle velocity to gasoline consumption, and assuming fully petroleum-

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[11] Acceleration/deceleration also affect fuel economy, but for simplification, it is assumed that fuel consumption is only related to velocity.

[12] Anas (2011) also finds that maximum fuel economy is between 40 and 45 miles per hour.

based gasoline (125,000 BTUs per gallon), gives energy consumed while commuting through each annulus.[13] The Energy Information Administration publishes a petroleum refining and distribution parameter in the Federal Register (2000) that is meant to be multiplied by intermediate energy consumption to arrive at a final use measure: *End Use Energy = Total Energy × Efficiency Parameter*. Dividing end-use energy consumption by the efficiency parameter gives the total energy consumed in the production, distribution, and final consumption. For gasoline, this parameter is equal to 0.83, giving 150,602 BTUs of energy consumption per gallon.

The calculation of dwelling energy consumption is somewhat more complicated. Three major factors determine dwelling energy consumption: income of the household, the square feet of interior space, and structure type.[14] An empirical model of energy demand using the 2005 Residential Energy Consumption Survey (RECS) provides estimates of 0.23 for the partial elasticity of household energy consumption with respect to interior space and 0.07 for the income elasticity. Single-family attached dwellings consume 7% less energy than single-family detached units. Multi-family units lower energy consumption 31% compared to single-family detached (see Larson, Liu, and Yezer, 2012). In the model, structure type is a function of the structure-land ratio $q = H/L$.[15]

Energy consumption within households comes entirely from electricity. It is known from the RECS that households that have only electricity available consume less energy than those that have access to natural gas, kerosene, and/or heating oil. However, when considering the full chain of production, distribution, and final energy use, the total energy content is similar for fossil fuel generated electricity as it is for fossil fuels consumed in the home.[16] There are two electricity efficiency parameters. The efficiency parameter for fossil fuel electricity production is 0.328, and the efficiency parameter for electricity transmission is 0.924 (Federal Register, 2000). Final electricity demand is divided by the product of these two measures

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[13] The equation is a 4th degree polynomial:

$$\textit{miles per gallon} = .822 + 1.833v - .0486v^2 + .000651v^3 - .00000372v^4$$

[14] There are many other factors that affect energy consumption, such as temperature, fuel type, and the age of the housing unit, but housing consumption, income, and structure type are the only variables that change within the energy footprint model.

[15] Structure types represented by values of q are calibrated with respect to the data to give $q_2 = 0.5$, $q_1 = 0.6$, and $q_0 = 0.7$ as cutoffs between single-family detached, single-family attached, 2-4 unit multifamily, and 5+ unit multifamily, respectively.

[16] The obvious exception is non-fossil fuel generated electricity such as nuclear, wind, hydroelectric, and solar, where the efficiency metrics often cease to be meaningful.

(0.303) to arrive at total energy consumption in dwellings.

For the numeraire good, the U.S. GDP-energy consumption ratio produced by the Energy Information Administration (2011) is used. This is a rough measure of the energy content of a consumption good, but it serves to represent all intermediate inputs in the production of consumption goods, including raw materials, intermediate product, final production, transportation, distribution, etc. Total BTU consumption in the U.S. divided by GDP in 2010 was 7,470.[17] Each dollar of numeraire consumption is therefore estimated to be associated with 7,470 BTUs of energy consumption.[18]

2.7 Income Changes and Welfare Effects

Because a city size increase raises housing prices and/or commuting costs, the iso-utility condition requires an increase in wages paid by firms. This is a well-known stylized fact and has been modeled in numerical urban simulations by Timothy and Wheaton (2001). In addition to this size effect on income, development policies that influence the cost of commuting and/or price of housing also require changes in earnings paid by firms, holding size constant. These income changes do not alter the utility level of households or the profit level of firms in an open city because each household receives the inter-regional utility level and each firm receives zero economic profit. Instead, land owners, who own an immobile asset, face welfare effects of various city-level policy and market effects.

Although real income is held constant as city size increases, nominal income changes cause substitution among consumption goods. A standard result of the open city version of the standard urban model is that the price of housing increases while the price of the composite commodity remains constant, with a resulting drop in housing consumption on the Hicksian housing demand curve. This substitution has important implications for energy consumption patterns as city size changes.

Effects of city size changes are presented both under constant utility (necessitating income changes), and constant income (with resulting utility changes) assumptions. This allows changes to be decomposed into those produced by the compensating variation in income and those what would arise if population were forced into the larger city regardless of utility.

Standard methods for making welfare comparisons among cities have been developed in the literature. Following Sullivan (1985), increases in aggregate land rent and decreases in

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[17] This is the inverse of "energy intensity," a metric measuring GDP per unit of energy consumed. See Sun (1998), for example.

[18] Expenditures for non-gasoline commuting costs (m_0+m_1k) are assumed to have the same energy content as the numeraire goods for purposes of computing energy consumption.

earnings needed to maintain household utility are counted as surplus for the social planner controlling the city. The change in land rent is based on an area large enough to include the developed area under the largest alternative considered. For example if the urbanized area is smaller under one alternative, then the agricultural land rent in the area not urbanized must be counted.

3 Model Parameter Calibration

The standard method of calibrating a numerical urban simulation model is to first select a city or group of cities as calibration targets. Parameters come from the established literature on housing production and consumption relationships, engineering relationships concerning fuel use and energy content of vehicles at various speeds, and pure calibration when no guidance is given from any of the previous sources.

Because this is the first open city model to attempt to simulate a doubling of city size, there are some special issues of calibration not confronted previously. The model is first calibrated to a city size of 1 million and then allowed to simulate a city of 2 million with the same homogeneous household type. When this is done, it is anticipated, *a priori*, that the simulated city of 2 million will depart in a systematic fashion from an actual city of 2 million.[19]

The most important difference occurs because the relative price of housing rises with city size. Larger cities attract households for which housing consumption is a relatively smaller fraction of total expenditure for two main reasons. First, higher housing prices select smaller households as noted in Black et al. (2002). Second, the fact that the income elasticity of demand for a primary residence is less than unity means that skill intensity of the population (ratio of more to less educated workers) rises with city size. A crucial feature of the simulation is that household type is held constant, because different household types consume energy in different amounts. Therefore, it is anticipated that when city size is doubled in the simulation, the housing consumption increase observed in actual cities will be slightly larger than that which is generated by the simulation where household size and education are held constant.[20] The house price effect on skill intensity also means that the increase in median income in actual cities will exceed that in the simulation model.

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[19] Rappaport's (2014) results show that a model calibrated to an initial population size of 2 million can, by varying wages paid in the CBD, simulate cities ranging from about 500,000 to 4 million.

[20] The relation between city size, house prices, and skill level is discussed in detail in Kim, Liu, and Yezer (2009).

Another difference is that the rise in city radius with size in the simulation will be smaller than the actual increase in simulation. As has been reported in Altmann and DeSalvo (1981), this is also a product both of the restricting the model to homogenous households. In evaluating the final model calibration exercise, it is important to recall that it is expected *a priori* that maintaining a homogenous household type will cause systematic differences in the relation between the actual effects of doubling city size and the simulated effect of a well calibrated model. However, because this is a first attempt to simulate open cities of different sizes, calibration issues may differ from those encountered previously in the literature.

3.1 Calibration Targets

Target outputs from the model are based on a composite of five cities each with population of approximately 1 million and another group of five cities with population of 2 million. Cities are defined as the sum of the principal cities within a CBSA. Principal cities are used instead of CBSAs in order for the simulation to reflect the main urban areas and close suburbs in a city while excluding the outer suburbs and rural satellite counties. Composite cities are used instead of empirical city size relations because of the small number of cities with large enough populations.[21] Cities are selected for geographic diversity in order for the samples to represent a plausible average of a city of a given size. Additionally, cities with substantial topological impediments to uniform development are avoided.

Characteristics of the five-city composites for both the 1 million and 2 million population cities can be seen in Table 1. While there is significant diversity among cities of a given size, the average characteristics are consistent with expectation based on the standard urban model of what happens when the population increases from about 1 million to 2 million. The footprint of the city increases from 369 to 591 square miles, median income rises from 49,000 to 52,000, structures become denser, with the share single-family detached dwellings falling from 57% to 49%, individual units become smaller, falling from 1,548 to 1,513 square feet, and average commute times rise from 24.4 minutes to 28.5 minutes.[22]

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[21] In the 2011 American Community Survey (ACS), there are 30 MSAs with a number of principal city households greater than 300,000.

[22] This commuting result is consistent with Anas (2011), who find that a doubling of MSA employment results in an increase of commute times of 10%.

3.2 Calibration Parameters

Parameters follow the established literature on cities and numerical urban simulations, as well as from well-established relationships in the physical sciences. Parameter notation, values, and notes are shown in Table 2. Housing production function parameters are all from the literature with the exception of the production technology parameter A, which is calibrated to fit the target cities. CES share and elasticity parameters for both the housing production and utility functions are based on Muth (1975) and Altmann and DeSalvo (1981).

The fraction of city area used for housing is 25%, which is similar to the values used in Muth (1975). Land used for roads is 25%.[23] Land used for employment is based on a fixed ratio of one worker per thousand square feet of land. Three quarters of all employment is located in the CBD. The small-city baseline CBD radius is 1 mile.

Single-family detached units' 90th percentile structure-land area ratio is 0.45 in the 2011 American Housing Survey (AHS). Based on this value, the cutoff for such units in the simulation is set to be 0.5 versus single-family attached. No land measures are given for non-single-family detached units, so cutoff parameters must be calibrated, and are set at 0.6 and 0.7 for 2-4 and 5+ unit apartments, respectively.

3.3 Calibration Results

Calibration results obtained by simulating the model to accommodate households totaling 440,000 and 880,000, corresponding to populations of 1 and 2 million respectively, are reported in Tables 1 and 3. The differences between actual and simulated values agree well with expectations. Constraining households to be identical is known to produce a smaller simulated city radius. Because household composition is uniform and constant, actual values for a city of 2 million are expected to depart slightly and systematically from the simulated values. In all cases, these differences are observed. Specifically, the fall in housing unit size is larger and the rise in median income smaller in the simulation than for the actual cities because the skill intensity ratio rises endogenously with city size. For similar reasons, the rise in density of housing in the simulated scenario is larger than observed in reality. This illustrates why use of actual data on changes associated with city size can yield false conclusions regarding the true size effect due to the confounding influence of household composition effects.

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[23] It is common to see values ranging from 1/4 to 2/3 of urban land dedicated to housing in the literature. This variation is primarily due to the treatment of roads, which are often included in the fraction of land to housing, and missing land due to interrupted development. Here, land and roads are treated separately.

The elasticity of time to work with respect to city size in the sample of actual cities is 0.15 and this is slightly smaller than the simulated value of 0.16. However, this is a case in which the sample of actual cities has, for some reason, an unusually high elasticity of travel time. Anas (2011) has recently estimated the elasticity of travel time with respect to size for a large number of U.S. cities at 0.11. It appears that the simulated value of travel time elasticity lies between these two possible standards for the actual value.

Overall, the agreement between the simulated cities and actual mean values for the reference cities is close and the differences are in the direction anticipated for a model in which population is homogeneous as city size increases. Finally, some variation between the UEFM simulation and actual cities is to be expected due to the effects of zoning and land use planning which causes actual cities to depart from the unregulated market solution.

The general spatial characteristics of the simulated city are displayed in Figure 4, which shows baseline simulations for cities of 440,000 and 880,000 households. These functions are consistent with both stylized facts and previous simulation models, with the exception of vehicle velocity, which is a highly non-linear function of distance from the CBD rather than constant as in most other simulations. Increasing city size raises the house price, household density, commuting time and structure density functions and lowers the lot size, and housing consumption functions.

The spatial pattern of energy consumption is displayed in Figure 5. Jumps in the function relating energy use in dwellings are due to the discrete changes in type of unit (single-family detached, to attached, to low-density multifamily, to high-density multi-family) density associated with changes in the structure-land ratio. Both the residential energy use equations estimated to support the UEFM and previous literature, including controlled experiments on households, have demonstrated that the discrete switch from detached to semi-detached to multifamily units is associated with a significant discrete shift in energy use. The gentle positive slope between jumps is due to the effect of increasing unit size on energy consumption. Jumps in consumption of energy due to changes in the numeraire good or composite commodity are due to the implications for the household budget of shifts in energy consumption as structure type changes.

4 Simulation Results and Implications

Application of the UEFM to the relation between city size or density and energy use in housing and commuting requires solving for city characteristics under alternative scenarios and comparing the results. Based on previous literature on energy use in cities, a number of comparisons are possible. First, it is possible to disaggregate energy use into commuting, housing, and the composite commodity. Second, the UEFM simulation can be operated as a closed city model and force population into a larger city with no compensating change in income. Third, for changes within a city size category, it is possible to compute welfare effects by subtracting the compensating variation in income from the change in aggregate land rent. In subsequent tables of results, all these alternatives are considered.

Because the literature on city size and energy tends to be merged with discussions of density effects, both of these issues will be discussed here. Obviously, residential density increases with city size as the relative price of housing rises compared to the composite commodity. This means that density and size vary directly in data on cities and the UEFM reproduces this relation faithfully.

A further advantage of the UEFM is that it can be used to examine the effects of urban development policies that change density artificially, holding size constant. Two density changing policies that are observed in cities are examined here. The first is a height limit on residential development, implemented as a maximum floor-area ratio similar to Borck (2014) or Bertaud and Brueckner (2005), who examine the effects of height limits on housing and welfare in Bangalore, India using a closed city model. The UEFM results here complement these findings nicely by extending them to an open model and adding energy implications. The second is the addition of a greenbelt in which residential development is not allowed. By restricting the supply of urban land, the greenbelt raises residential densities. In practice, many greenbelt policies allow some residences on very large plots; for example, five acre minimum lot zoning is common. There are also cases in which residential development "jumps" the greenbelt. All these various alternatives could be simulated with the UEFM but significant modifications in the simulation would be required.[24]

4.1 City Size Effect with Unregulated Density

The first scenario evaluated using the UEFM is the "pure" city size effect. In an unregulated housing market, city size is doubled, from 440,000 households (approximately one million people) to 880,000 households (two million people). The effects of this doubling of population in which income is raised to keep utility constant are presented in Table 4 (labeled "Baseline"

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[24] In particular, housing in greenbelts with large lot zoning is generally occupied by higher income households and realistic simulation in a model with one household type would be challenging. Finally, this greenbelt housing constitutes a tiny fraction of all housing in cities.

or Scenario 1). Average density, i.e. population per square mile, rises by 36% with the population increase.[25] The two characteristics most related to energy use, commuting time to work and fraction housed in 5+ unit structures both rise with city size. Of course these variables have opposite effects on energy consumption and the net effect on energy use is based on the difference in changes associated with size.

Comparing the energy consequences of doubling city size, and increasing density 36%, the importance of the compensating variation in income is obvious. Considering only commuting and dwelling energy utilization, there is a 2.6% fall in energy use. However, the increase in consumption of the numeraire associated with the compensating rise in income net of housing and commuting expenditure implies additional energy consumption of 2.2%. The net effect of these two effects is an increase in energy use of 0.1%, so the energy implication of substituting one city of two million for two cities of one million, when households are fully compensated for the change, is essentially zero. As Table 5 shows, this result is notably robust to changes in simulation parameters. While the level of energy use varies substantially with parameter values, the partial effect of each on the simulated city size elasticity is nearly zero.

This finding of no city size effect on per-capita energy consumption when household utility is constant and households are indifferent about the change in city size may appear counterintuitive. One reason that this result is surprising is that energy use other than housing and commuting is usually ignored. Table 6 contains the results of Scenario 2 simulations in which there is no income compensation and households are forced into the larger city by simulating the UEFM as a closed city model. Under this scenario, the city of one million is unchanged from the baseline in Table 4 but the larger city is predictably denser, occupies less land area, has shorter commuting time and smaller housing units. Per household energy used in commuting now only rises 16.3% rather than 17.8% while dwelling energy use falls 6.2% rather than 6.4%. Finally the sign of the change in numeraire good consumption is reversed as the higher house prices and unchanged income cause a reduction in non-housing consumption. Accordingly the change in city size is associated with a significant 2.1% fall in per-household energy consumption.

Some of those who imagine that a rise in city size is associate with a fall in energy consumption may have in mind an experiment in which there is no compensating variation in income and where and household utility falls as a result. Of course, such a change could be effected simply by raising taxes on households and reducing consumption generally without

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[25] Specifically, average density increases from 1,281 to 1,737 households per square mile where households occupy land.

resort to city size policy.[26]

4.2 Increasing Density with a Greenbelt

The greenbelt proposal considered here involves prohibiting urban development beyond a given radius. The greenbelt is placed at a radius of nine miles in each case so that it is more binding on the larger city. This allows analysis of the effects of more or less binding greenbelts as well as issues that could arise as a city grows into an increasingly binding greenbelt restriction.

Before discussing the simulation results it is important to consider market failures in the baseline city. Roads are provided without tolls or fees. Indeed, there is no attempt to relate road capacity to benefits and costs.[27] There is substantial congestion in these cities as evidenced by the travel velocity function. Congestion increases with city size because of rising number and length of commuting trips while land used for highways at any radius is fixed. As first demonstrated by Muth (1975), a system of congestion tolls based on marginal congestion cost would substantially increase urban densities. Accordingly, the greenbelt, by raising residential densities, can be a second best reaction to the market failure caused by the lack of highway pricing in the baseline model.

Comparing the baseline with the greenbelt simulation results for each city size category in Table 7, it is clear that the effects on the larger city are substantial because the greenbelt regulation is more binding.[28] The effect on overall energy consumption in the smaller city is negligible. This is in part due to the compensating variation in income which raises consumption of the numeraire good. Even in the case of the larger city where the greenbelt has a substantial effect on density, housing consumption, and commuting time, the decline in overall per-household energy use is only about 1.1% because the compensating variation in income raises energy consumption embodied in the composite commodity by approximately 0.7%. Taken together, these changes suggest that the effects of a greenbelt on energy consumption, as city size changes, are rather small.

However, the welfare analysis of the greenbelt policy at the bottom of Table 7 produces what may appear to some to be a remarkable result. For both city sizes, the compensating

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[26] This assumes that the revenue is not used, directly or indirectly, for activities that result in energy consumption.

[27] This may be a fair description of the way urban roads are provided in the real world. In the UEFM simulations, it would be possible to vary road capacity with opportunity cost of the land and benefits from congestion reduction as well as to simulate the effects of congestion charges.

[28] A greenbelt at 9 miles reduces radius about 14% for the smaller city and 30% for the larger city.

variation in earnings associated with the greenbelt is positive. However, the rise in land rent, including both developed residential land and the agricultural land included in the greenbelt that would have been developed in the absence of regulation, is substantial. This results in a net welfare change associated with imposition of greenbelt regulation that is positive in the small city but negative in the large city. The amounts are not large, \$13.68 per year per capita for the smaller city and -\$25.72 per year per capita in the large city, but the fact that a greenbelt can increase welfare at all in an otherwise laissez-faire city is worthy of some discussion. Clearly this is a case of the theory of the second best in which, given the failure to price transportation congestion, the baseline city is too large and not sufficiently dense. As the city grows, however, the greenbelt, having a fixed radius, becomes ever more restrictive, tipping the city into welfare loss. Comparing energy use in Tables 4 and 7, the greenbelt has a significant effect on energy consumption per household. Energy use falls 0.5% in the small and 1.7% in the larger city where the greenbelt is more binding.

4.3 Decreasing Density with a Height Limit

Effects of the other most common density regulation, a building height, structure density, or floor area ratio regulation, are displayed in Table 8. This type of regulation, sometimes referred to as residential down-zoning, can be imposed anywhere in the city. For purposes of this simulation exercise, the height limit is made uniform across the city and it is only binding in the central area. The limit is identical across city sizes and hence it is more binding in the larger city. Thus, Table 8 gives some insight into the effects of city size and of more or less binding height limits.

Overall, the height limit has negligible effects on the smaller city but consequences for the larger city are significant. A binding height limit or floor area ratio increases city radius and housing consumption, raises housing prices, and increases commuting time versus the baseline simulations in Table 4. Most important, the height limit requires a substantial compensating variation in earnings needed to induce workers to live in the city. Comparing energy consumption in the smaller city with that in the larger city where the height limit is more binding, there is now a small increase in energy consumption associated with city size. Holding size constant, imposition of a binding height limit in a large city raises energy consumption in both commuting and dwellings while the numeraire good is essentially constant. Therefore the partial effect of a binding height limit on a city is to increase energy consumption significantly.[29]

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[29] These findings echo recent work by Borck (2014), who considers a closed city with no numeraire energy

Welfare analysis of the height limit for the small and large city shows that there is a substantial compensating variation in earnings required and the rise in rents on urban land is partially offset by the fall in agricultural land as the city spreads. The annualized net welfare effect for the small city where the limit is barely binding is negative but small, -\$2.03 per person, while that for the large city where the limit is significantly binding is substantial, -\$11.35 per person. Given that the height limit has negative welfare effects and raises energy consumption in these simulation results, its economic rationale would necessarily lie in some aesthetic gain from urban form that is omitted from the UEFM.

5 Conclusions

The goal of this paper is to demonstrate that an open city simulation model with a single parameterization can simulate cities of different size with homogenous population, industrial composition, topography, and technology. In addition to usual features, such a model must have endogenous congestion of transportation systems because this is an important characteristic of changing city size. For a variety of reasons, cities with homogenous population differ from actual cities because population heterogeneity is a function of city size. Considering this, the model is quite effective in replicating the characteristics of cities over the 1 to 2 million population range. The model has a further advantage of facilitating computation of the welfare implications of alternative patterns of urban development.

While this type of urban simulation model is potentially valuable in studying a variety of issues, the relation between city size and energy consumption is illustrated here. This is a challenging application because energy use varies with vehicle velocity in commuting, so that commuting distance and time are both important, and the precise density of structures matters. Direct empirical estimation of the effect of city size on energy use is hampered by the modest number of large cities available, the quality of data on aggregate energy use, and the substantial heterogeneity of larger cities along a number of dimensions.

The final result of the inquiry into energy implications of city size is surprising and remarkably simple: the elasticity of energy consumption with respect to city size (population) is approximately unity. While this result contrasts with much of the empirical literature on density and energy use, there are seeds of the result found elsewhere. For example, Gaigne, Riou, and Thisse (2012) theoretically demonstrate that it is possible for households in larger cities to consume more energy than households in smaller cities, and Glaeser and Kahn

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accounting.

(2012) notably express the desire to undertake a full energy accounting of consumption, rather than focusing only on commuting and dwelling energy consumption, but leave this endeavor for further research.

The ability to model both the welfare and energy use implications of specific urban development policies is illustrated by considering greenbelts and residential height limits. First, density reductions through a binding greenbelt can achieve very modest reductions in energy use per capita, and perhaps counterintuitively, can in some cases, increase city welfare, where welfare is defined as the change in aggregate land value minus the total change in the cost of the compensating wage differential. Second, height or density limits raise energy consumption per capita and unambiguously lower welfare.

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Table 1: Calibration Cities

Calibration: Large Cities

CBSA Code	CBSA Name	Area (sq. mi.)	Population	Housing units	Median Income	Units/ sq. mi.	S.F. Detached	S.F. Attached	2-4 Units	5+ Units	Avg. Commute
19820	Detroit, MI	400	1,453,257	668,183	43,409	1,671	67%	7%	9%	18%	25.8
41860	San Francisco, CA	350	2,181,589	924,550	69,248	2,639	36%	11%	16%	36%	29.6
26420	Houston, TX	761	2,354,020	1,003,954	44,269	1,319	48%	5%	6%	41%	27.2
38060	Phoenix, AZ	937	2,490,498	1,079,290	51,820	1,152	60%	6%	7%	27%	26.2
16980	Chicago, IL	504	3,572,223	1,542,968	51,264	3,060	33%	5%	27%	35%	33.6
AVERAGE		591	2,410,317	1,043,789	52,002	1,968	49%	7%	13%	31%	28.5

Calibration: Small Cities

CBSA Code	CBSA Name	Area (sq. mi.)	Population	Housing units	Median Income	Units/ sq. mi.	S.F. Detached	S.F. Attached	2-4 Units	5+ Units	Avg. Commute
28140	Kansas City, MO-KS	515	778,945	360,109	49,001	700	64%	7%	7%	22%	22.3
38900	Portland, OR	223	926,981	410,431	49,682	1,845	56%	6%	10%	29%	25.1
19740	Denver, CO	341	981,125	439,483	48,237	1,290	50%	9%	6%	36%	27.0
41700	San Antonio, TX	505	1,385,147	547,627	43,586	1,085	66%	3%	7%	24%	24.6
33460	Minneapolis, MN	264	995,852	441,488	56,124	1,674	49%	8%	10%	33%	23.3
AVERAGE		369	1,013,610	439,828	49,326	1,318	57%	6%	8%	29%	24.4

Table 2: Simulation Parameters

Parameter	Baseline	Description	Source
ρ	0.75	Elast. of substitution in the housing prod. function	Altmann and DeSalvo (1981)
α_1	1.1	Structure share parameter in housing production function	Muth (1975); Altmann and DeSalvo (1981)
α_2	0.9	Land share parameter in housing production function	Muth (1975); Altmann and DeSalvo (1981)
A	0.32	Housing production technology parameter	Calibrated
η	0.75	Elasticity of substitution in the utility function	Altmann and DeSalvo (1981)
β_1	1	Numeraire share parameter in utility function	Numeraire
β_2	0.1056	Housing share parameter in utility function	Altmann and DeSalvo (1981)
p_L^a	500	Reservation agricultural price per acre of land	Bertaud and Brueckner (2005)
θ	0.25	Fraction of land used for housing	Muth (1975)
θ_R	0.25	Fraction of land used for roads	Muth (1975)
$\omega(k_{CBD})$	13000	Density of employment at the center of the CBD	Calibrated
g	0.4	Exponential in the employment density gradient	McMillen (2004)
E_{CBD}/E	0.75	Fraction of households commuting to the CBD	ACS(2011)
k_{CBD}	1	Small city radius of the CBD	Calibrated
t_L	12	Time of commute for local workers	ACS (2011)
v_{low}	5	Minimum commuting speed	Muth (1975)
v_{high}	45	Maximum commuting speed	Muth (1975)
c	1.75	Curvature parameter in speed function	Muth (1975)
τ	0.5	Time cost of commuting (fraction of wage)	Bertaud and Brueckner (2005)
p_g	3.5	Gasoline price per gallon	EIA (2011)
m_0	2123	Fixed cost of commuting	American Automobile Association (2007)
m_1	0.222	Dollars of depreciation per mile	American Automobile Association (2007)
$\bar{U}$	15500	Reservation utility	Calibrated
q_0	0.7	5+ unit building floor-area ratio cutoff	Calibrated
q_1	0.6	2-4 unit building floor-area ratio cutoff	Calibrated
q_2	0.5	sf. attached floor-area ratio cutoff	AHS (2011)

Note: Values are approximate to those from the cited source.

Table 3: Simulation Calibration

Variable	Small City Actual	Small City Simulated	Large City Actual	Large City Simulated
Lot Size (acre) – Occupied Units[1]	0.278	0.190	0.250	0.183
Unit (square feet) – Occupied Units[1]	1,548	1,560	1,513	1,514
Area (sq. miles)[2]	369	346	591	511
Radius (assuming circle)[2]	10.8	10.5	13.7	12.8
Median Income[2]	$49,326	$49,924	$52,002	$51,147
Total Occupied Units[2]	439,828	440,000	1,043,789	880,000
Time to work[2]	24.4	23.6	28.5	27.4
Fraction housed in 1 unit structures[2]	57%	54.0%	49%	36.4%
Fraction housed in 2-4 unit structures[2]	14%	21.2%	20%	17.0%
Fraction housed in 5+ unit structures[2]	29%	24.8%	31%	46.5%
Energy consumed in dwelling, per capita (mmBTUs)[3]	49.84	42.59	*	39.85

[1] Source for actual values: AHS (2011)

[2] Source for actual values: ACS (2010)

[3] Source for actual values: RECS (2009) households with 100% electricity consumption

* Energy consumption per dwelling is not given in the RECS by city size, so it is presented here only for the small city.

Table 4: Scenario 1, Baseline

Variable	Small City	Large City	Difference	Elasticity
Urban Form				
Total Occupied Units	440,000	880,000	440,000	
Lot Size (acre) – Detached Units	0.190	0.183	-0.007	-0.036
Unit (square feet) – All Units	1,560	1,514	-46	-0.030
City Area (sq. miles)	346	511	165	0.475
City Radius (assuming circle)	10.5	12.8	2.3	0.214
Residential Struct./Land ratio (CBD)	1.20	1.69	0.49	0.405
Residential Density (hh per sq. mile)	1,281	1,737	456	0.356
Time to work	23.6	27.4	3.9	0.164
Fraction housed in 1 unit structures	54.0%	36.4%	-17.6%	-0.326
Fraction housed in 2-4 unit structures	21.2%	17.0%	-4.1%	-0.195
Fraction housed in 5+ unit structures	24.8%	46.5%	21.7%	0.875
Income/Expenditure Accounting				
Base Income	$ 49,924	$ 51,147	$ 1,223	0.024
Numeraire Expenditure	$ 36,782	$ 37,212	$ 430	0.012
Housing Services Expenditure	$ 9,885	$ 10,113	$ 228	0.023
Housing Expenditure	$ 8,387	$ 8,712	$ 324	0.039
Dwelling Energy Expenditure	$ 1,498	$ 1,401	$ -96	-0.064
Commuting Gasoline Expenditure	$ 758	$ 893	$ 135	0.178
Income Reductions*	$ 2,499	$ 2,929	$ 430	0.172
Total Energy Consumption (billion BTUs)				
Commuting	11,478	27,045	15,567	1.356
Dwelling	61,831	115,706	53,874	0.871
Commuting and Dwelling	73,310	142,751	69,441	0.947
Numeraire	94,181	192,499	98,318	1.044
Total	167,491	335,250	167,760	1.002
Energy Consumption per Household (million BTUs)				
Commuting	26.1	30.7	4.6	0.178
Dwelling	140.5	131.5	-9.0	-0.064
Commuting and Dwelling	166.6	162.2	-4.4	-0.026
Numeraire	214.0	218.8	4.7	0.022
Total	380.7	381.0	0.3	0.001
Welfare Accounting				
Total Wages ($m) (-)	$ 21,966	$ 45,009	$ 23,043	1.049
Residential Land Rent ($m) (+)	$ 438	$ 1,003	$ 565	1.292
Agriculture Land Rent ($m) (+)**	$ 115	$ 63	$ -53	-0.457

* Consists of time-cost of commuting and reduced income from non-CBD employment

** Measured up to a 15 mile radius

Table 5: Simulation Parameter Sensitivity

Baseline Per-Capita Energy Estimate (million BTUs): Small: 380.75; Large: 381.06; Elasticity: 1.0008

Parameter	Baseline	Description	*Per-capita energy use when parameter in row is increased by 10%*		
			Small City	Large City	Elasticity
ρ	0.75	Elast. of substitution in the housing prod. function	393.37	393.16	0.9995
α_1	1.1	Structure share parameter in housing production function	393.35	393.84	1.0012
α_2	0.9	Land share parameter in housing production function	384.93	385.34	1.0011
A	0.32	Housing production technology parameter	375.37	375.83	1.0012
η	0.75	Elasticity of substitution in the utility function	567.75	569.52	1.0031
β_1	1	Numeraire share parameter in utility function	-	-	-
β_2	0.1056	Housing share parameter in utility function	399.46	399.89	1.0011
p_L^a	500	Reservation agricultural price per acre of land	380.67	380.99	1.0008
θ	0.25	Fraction of land used for housing	381.07	381.17	1.0003
θ_R	0.25	Fraction of land used for roads	382.58	383.23	1.0017
$\omega(k_{CBD})$	13000	Density of employment at the center of the CBD	380.74	381.08	1.0009
g	0.4	Exponential in the employment density gradient	380.72	381.05	1.0009
E_{CBD}/E	0.75	Fraction of households commuting to the CBD	380.69	381.02	1.0009
k_{CBD}	1	Small city radius of the CBD	381.82	382.27	1.0012
t_L	12	Time of commute for local workers	380.75	381.06	1.0008
v_{low}	5	Minimum commuting speed	379.69	379.79	1.0003
v_{high}	45	Maximum commuting speed	381.55	381.76	1.0006
c	1.75	Curvature parameter in speed function	380.55	380.78	1.0006
τ	0.5	Time cost of commuting (fraction of wage)	379.79	380.18	1.0010
p_g	3.5	Gasoline price per gallon	380.33	380.65	1.0009
m_0	2123	Fixed cost of commuting	380.78	381.09	1.0008
m_1	0.222	Dollars of depreciation per mile	379.96	380.34	1.0010
$\bar{U}$	15500	Reservation utility	410.65	411.03	1.0009
q_0	0.7	5+ unit building floor-area ratio cutoff	382.08	382.29	1.0005
q_1	0.6	2-4 unit building floor-area ratio cutoff	380.75	381.06	1.0008
q_2	0.5	sf. attached floor-area ratio cutoff	381.20	381.41	1.0005

Note: Baseline parameter values are those used in the baseline simulation shown in Table 4 and Figures 4 and 5. The elasticity column shows the elasticity of energy use with respect to city size under the parameter in the row multiplied by 1.1. The elasticity of elasticity column shows the elasticity of the city size elasticity with respect to the parameter in the row.

Table 6: Scenario 2, No Income Compensation

Variable	Small City	Large City	Difference	Elasticity	Large City Δ vs. baseline
Urban Form					
Total Occupied Units	440,000	880,000	440,000		
Lot Size (acre) – Detached Units	0.190	0.178	-0.012	-0.063	-0.0052
Unit (square feet) – All Units	1,560	1,471	-89	-0.057	-43
City Area (sq. miles)	346	491	145	0.418	-20
City Radius (assuming circle)	10.5	12.5	2.0	0.191	-0.25
Residential Struct./Land ratio (CBD)	1.20	1.71	0.51	0.422	0.02
Residential Density (hh per sq. mile)	1,281	1,807	526	0.410	70.21
Time to work	23.6	27.3	3.7	0.156	-0.2
Fraction housed in 1 unit structures	54.0%	35.8%	-18.2%	-0.336	-0.6%
Fraction housed in 2-4 unit structures	21.2%	16.8%	-4.3%	-0.204	-0.2%
Fraction housed in 5+ unit structures	24.8%	47.3%	22.5%	0.906	0.8%
Income/Expenditure Accounting					
Base Income	$ 49,924	$ 49,924	$ 0	0.000	$ -1,223
Numeraire Expenditure	$ 36,782	$ 36,282	$ -499	-0.014	$ -929
Housing Services Expenditure	$ 9,885	$ 9,847	$ -38	-0.004	$ -266
Housing Expenditure	$ 8,387	$ 8,461	$ 73	0.009	$ -251
Dwelling Energy Expenditure	$ 1,498	$ 1,387	$ -111	-0.074	$ -15
Commuting Gasoline Expenditure	$ 758	$ 881	$ 124	0.163	$ -11
Income Reductions*	$ 2,499	$ 2,913	$ 414	0.165	$ -17
Total Energy Consumption (billion BTUs)					
Commuting	11,478	26,700	15,222	1.326	-345
Dwelling	61,831	114,493	52,662	0.852	-1,212
Commuting and Dwelling	73,310	141,193	67,884	0.926	-1,557
Numeraire	94,181	186,840	92,659	0.984	-5,659
Total	167,491	328,034	160,543	0.959	-7,216
Energy Consumption per Household (million BTUs)					
Commuting	26.1	30.3	4.3	0.163	-0.4
Dwelling	140.5	130.1	-10.4	-0.074	-1.4
Commuting and Dwelling	166.6	160.4	-6.2	-0.037	-1.8
Numeraire	214.0	212.3	-1.7	-0.008	-6.4
Total	380.7	372.8	-7.9	-0.021	-8.2
Welfare Accounting					
Total Wages ($m) (-)	$ 21,966	$ 43,933	$ 21,966	1.000	$ -1,076
Residential Land Rent ($m) (+)	$ 438	$ 981	$ 543	1.242	$ -22
Agriculture Land Rent ($m) (+)**	$ 115	$ 69	$ -46	-0.402	$ 6
Total Surplus ($m, vs. Baseline)					$ 1,061

* Consists of time-cost of commuting and reduced income from non-CBD employment

** Measured up to a 15 mile radius

Table 7: Scenario 3, Greenbelt (city radius maximum of 9 miles)

Variable	Small City	Large City	Difference	Elasticity	Small City Δ vs. baseline	Large City Δ vs. baseline
Urban Form						
Total Occupied Units	440,000	880,000	440,000			
Lot Size (acre) – Detached Units	0.139	0.087	-0.052	-0.371	-0.051	-0.096
Unit (square feet) – All Units	1,542	1,461	-81	-0.052	-19	-53
City Area (sq. miles)	254	254	0	0.000	-92	-257
City Radius (assuming circle)	9.0	9.0	0.0	0.000	-1.5	-3.8
Residential Struct./Land ratio (CBD)	1.25	1.87	0.62	0.497	0.05	0.18
Residential Density (hh per sq. mile)	1,751	3,520	1,770	1.011	469.22	1,783.30
Time to work	23.2	26.3	3.1	0.133	-0.4	-1.2
Fraction housed in 1 unit structures	48.1%	16.0%	-32.1%	-0.668	-5.9%	-20.4%
Fraction housed in 2-4 unit structures	22.5%	19.2%	-3.3%	-0.148	1.3%	2.1%
Fraction housed in 5+ unit structures	29.4%	64.8%	35.4%	1.205	4.6%	18.3%
Income/Expenditure Accounting						
Base Income	$ 50,012	$ 51,431	$ 1,419	0.028	$ 88	$ 284
Numeraire Expenditure	$ 36,846	$ 37,421	$ 574	0.016	$ 65	$ 209
Housing Services Expenditure	$ 9,967	$ 10,367	$ 399	0.040	$ 82	$ 254
Housing Expenditure	$ 8,495	$ 9,058	$ 563	0.066	$ 108	$ 346
Dwelling Energy Expenditure	$ 1,472	$ 1,309	$ -164	-0.111	$ -25	$ -93
Commuting Gasoline Expenditure	$ 731	$ 816	$ 85	0.116	$ -27	$ -77
Income Reductions*	$ 2,467	$ 2,828	$ 361	0.146	$ -32	$ -101
Total Energy Consumption (billion BTUs)						
Commuting	11,069	24,710	13,641	1.232	-409	-2,335
Dwelling	60,783	108,057	47,274	0.778	-1,048	-7,649
Commuting and Dwelling	71,852	132,768	60,916	0.848	-1,458	-9,983
Numeraire	94,828	196,790	101,961	1.075	647	4,290
Total	166,680	329,557	162,877	0.977	-810	-5,693
Energy Consumption per Household (million BTUs)						
Commuting	25.2	28.1	2.9	0.116	-0.9	-2.7
Dwelling	138.1	122.8	-15.3	-0.111	-2.4	-8.7
Commuting and Dwelling	163.3	150.9	-12.4	-0.076	-3.3	-11.3
Numeraire	215.5	223.6	8.1	0.038	1.5	4.9
Total	378.8	374.5	-4.3	-0.011	-1.8	-6.5
Welfare Accounting						
Total Wages ($m) (-)	$ 22,005	$ 45,259	$ 23,254	1.057	$ 39	$ 250
Residential Land Rent ($m) (+)	$ 460	$ 1,145	$ 685	1.487	$ 23	$ 142
Agriculture Land Rent ($m) (+)**	$ 145	$ 145	$ 0	0.000	$ 29	$ 82
Total Surplus ($m, vs. Baseline)					$ 13.68	$ -25.72

* Consists of time-cost of commuting and reduced income from non-CBD employment

** Measured up to a 15 mile radius

Table 8: Scenario 4, Height Limit (floor-area ratio maximum of 1)

Variable	Small City	Large City	Difference	Elasticity	Small City Δ vs. baseline	Large City Δ vs. baseline
Urban Form						
Total Occupied Units	440,000	880,000	440,000			
Lot Size (acre) – Detached Units	0.190	0.183	-0.007	-0.038	0.0000	-0.0004
Unit (square feet) – All Units	1,561	1,517	-44	-0.028	0	3
City Area (sq. miles)	347	526	178	0.514	1	15
City Radius (assuming circle)	10.5	12.9	2.4	0.230	0.01	0.18
Residential Struct./Land ratio (CBD)	1.00	1.00	0.00	0.000	-0.20	-0.69
Residential Density (hh per sq. mile)	1,279	1,689	410	0.321	-2.70	-48.42
Time to work	23.6	28.0	4.4	0.186	0.1	0.6
Fraction housed in 1 unit structures	54.1%	37.2%	-16.9%	-0.313	0.1%	0.8%
Fraction housed in 2-4 unit structures	21.2%	17.5%	-3.7%	-0.174	0.0%	0.5%
Fraction housed in 5+ unit structures	24.7%	45.3%	20.6%	0.835	-0.1%	-1.2%
Income/Expenditure Accounting						
Base Income	$ 49,933	$ 51,228	$ 1,295	0.026	$ 9	$ 81
Numeraire Expenditure	$ 36,783	$ 37,236	$ 453	0.012	$ 2	$ 25
Housing Services Expenditure	$ 9,884	$ 10,097	$ 213	0.022	$ -1	$ -16
Housing Expenditure	$ 8,386	$ 8,690	$ 304	0.036	$ -2	$ -22
Dwelling Energy Expenditure	$ 1,499	$ 1,407	$ -91	-0.061	$ 1	$ 6
Commuting Gasoline Expenditure	$ 760	$ 922	$ 162	0.213	$ 2	$ 29
Income Reductions*	$ 2,505	$ 2,973	$ 468	0.187	$ 6	$ 44
Total Energy Consumption (billion BTUs)						
Commuting	11,513	27,924	16,411	1.425	35	879
Dwelling	61,857	116,170	54,313	0.878	26	464
Commuting and Dwelling	73,371	144,094	70,723	0.964	61	1,343
Numeraire	94,170	192,235	98,065	1.041	-11	-264
Total	167,541	336,329	168,788	1.007	51	1,079
Energy Consumption per Household (million BTUs)						
Commuting	26.2	31.7	5.6	0.213	0.08	1.0
Dwelling	140.6	132.0	-8.6	-0.061	0.06	0.5
Commuting and Dwelling	166.7	163.7	-3.0	-0.018	0.14	1.5
Numeraire	214.0	218.4	4.4	0.021	-0.03	-0.3
Total	380.8	382.2	1.4	0.004	0.11	1.2
Welfare Accounting						
Total Wages ($m) (-)	$ 21,970	$ 45,081	$ 23,110	1.052	$ 4	$ 71
Residential Land Rent ($m) (+)	$ 440	$ 1,068	$ 628	1.428	$ 2	$ 65
Agriculture Land Rent ($m) (+)**	$ 115	$ 58	$ -57	-0.496	$ -0.2	$ -5
Total Surplus ($m, vs. Baseline)					$ -2.03	$ -11.35

* Consists of time-cost of commuting and reduced income from non-CBD employment

** Measured up to a 15 mile radius

Figure 4: Baseline simulations: Urban Form

Blue line: 440,000 household city
Red line: 880,000 household city

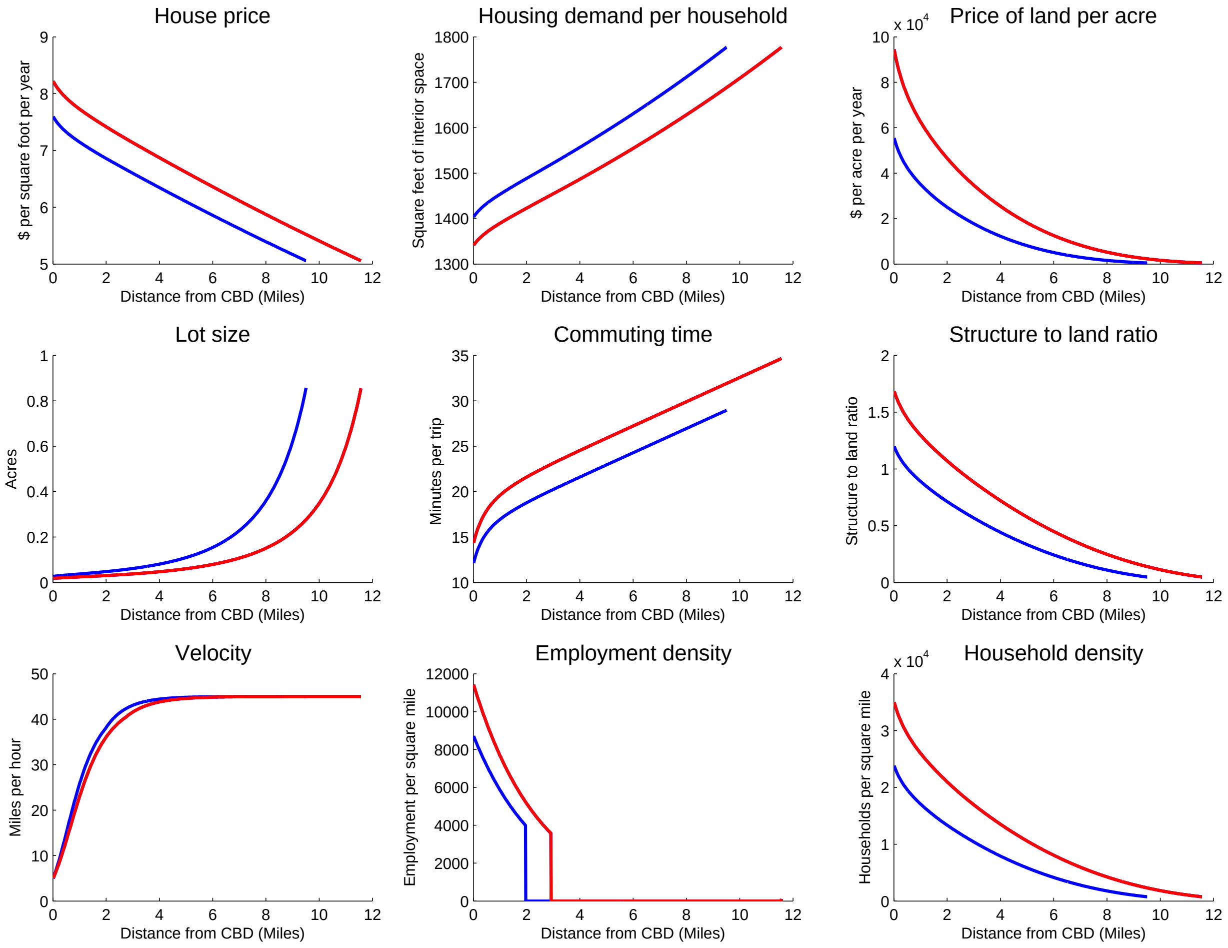

Figure 5: Baseline simulations: Energy Consumption

Blue line: 440,000 household city
Red line: 880,000 household city